ACROSS-WIND RESPONSE OF A TALL BUILDING WITH ACTIVE TUNED MASS DAMPER USING LINEAR-QUADRATIC-GAUSSIAN CONTROLLER

Jang-Youl You
Department of Architecture Engineering Songwon University, Gwangju, 61756, South Korea

Young-Moo Sohn
Central Connecticut State University, Assistant Professor School of Engineering & Technology Department of Engineering, USA

Young-Moon Kim, Ki-Pyo You
Department of Architectural Engineering, Chonbuk National University, Long-span Steel Frame System Research Center, South Korea

ABSTRACT

Modern tall buildings use lighter construction materials that have high strength and lower stiffness, and are thus susceptible to excessive wind-induced excitations, which can result in occupant discomfort and structural safety concerns. Recently, a number of studies have been conducted to investigate buildings that use actuators as an active control force based on the modern control theory of linear quadratic optimum algorithms. The optimal control law of a linear-quadratic-Gaussian (LQG) controller with an active tuned mass damper (ATMD) is used to reduce the across-wind response of a tall building. The ATMD model consisted of a second mass with optimum parameters for the tuning frequency and damping ratio of the tuned mass damper (TMD) under a stationary random load. A fluctuating across-wind load acting on a tall building was treated as a stationary Gaussian white noise and simulated numerically in the time domain using the across-wind load spectra proposed by Kareem (1982). This simulated across-wind load was used to calculate the across-wind responses of tall buildings with and without ATMD using an LQG controller. By comparing the respective root mean square (rms) responses, it was found that the numerically simulated across-wind responses without ATMD provided a good approximation to the closed form response and the reduced responses with ATMD and LQG controllers were estimated by varying the values of the control design parameters.

Key words: Wind-induced excitations, Numerical simulation; ATMD; LQG; Across-wind response.
1. INTRODUCTION

Most modern tall buildings use lighter construction materials that have high strength and lower stiffness (i.e., are more flexible); however, this can lead to wind-induced vibrations, resulting in occupant discomfort and concerns over structural safety [1]. It is therefore necessary to reduce wind-induced displacement and acceleration at the top floor level of tall buildings [2-4].

Although a number of studies have been conducted on estimating and mitigating wind-induced structural vibrations in tall buildings, leading to significant improvements, no mechanisms for regulating the complicated interactions between fluctuating atmospheric flows and the sides of a building have been established [5,6,7]. The fluctuating along-wind load that almost approaches the turbulent flow velocity can be estimated theoretically, and hence the along-wind response of a tall building can also be estimated, for example through the use of a gust factor method [8,9,10]. Mitigation of such excessive wind-induced vibrations has been studied extensively over several decades [2,3,4] and, in recent years, modern optimal control theory and devices have been used to obtain the required control force and reduce the vibrational response [3,4,11].

In 1972, Yao introduced modern control theory into vibration control of civil engineering structures [12]. As modern tall buildings subjected to fluctuating wind loads oscillate at the fundamental natural frequency of the building, which is known, modern control theory and auxiliary devices can be applied to control wind-induced excitations in tall buildings. One of the most common control devices for reducing structural responses is a tuned mass damper (TMD) system, first posited by Den Hartog (1947). This consists of an auxiliary mass with properly tuned springs and damping devices, which increases damping and reduces the response in the main structure [13]. A number of further studies have since been conducted on the behavior and effectiveness of TMD, and a number of TMDs have been installed in tall buildings to control wind-induced vibrations [3,14]. When the wind load is modeled as stationary Gaussian white noise, the TMD parameter for reducing wind-induced excitations in a building can be derived using the method discussed by McNamara [2]. The Center Point Tower in Sydney is one of the first examples of TMD use in a building, and a 400-ton TMD is installed in the Citicorp Center, a 274-m-tall office building in New York; and another TMD has been designed for the John Hancock Tower in Boston. These TMDs have been installed to reduce wind-induced vibrations [3,14]. In the 1990s, multiple tuned mass dampers (MTMD), consisting of many TMDs with natural frequencies distributed around the natural frequency of the main structure, were studied extensively, and MTMD systems were shown to be more effective than simple TMD systems [15,16,17,18]. It is generally accepted that the performance of a TMD can be improved by incorporating a feedback controller through the use of an active control force in the TMD design, known as an active tuned mass damper (ATMD) [3,4,11,19].

An ATMD design for mitigating wind-induced vibrations in a tall building with a linear quadratic regulator (LQR) controller using a deterministic harmonic wind load was presented by Chang and Soong [14], in one of the first active control studies for reducing wind-induced
vibrations in tall buildings. Since then, a number of studies of optimal control algorithms, including LQR, linear-quadratic-Gaussian (LQG), $H_2$, and $H_\infty$, to determine the optimal control force for reducing wind-induced vibrations in tall buildings have been published [11,19,20,21,22-34]. However, the superiority of ATMD over TMD for reducing wind-induced vibrations in tall buildings is still disputed [11].

From the perspective of modern optimal control theory, fluctuating across-wind loads acting on a tall building can be considered as stationary Gaussian white noise processes with a constant power spectral density function. Therefore, a linear control system that has the system and measurement noise characteristics of Gaussian white noise can be considered as a stochastic linear control system, and LQG controllers have hence been used to investigate the effectiveness of ATMD for reducing the across-wind response of tall buildings [21,23,24]. In this study, fluctuating across-wind loads acting on a tall building were simulated numerically in the time domain using the across-wind load spectra proposed by Kareem [5]; the simulation procedures used were taken from Deodatis [35]. Using this simulated fluctuating across-wind load, the across-wind responses of a tall building, without ATMD and with ATMD and an LQG controller, are estimated, and the effectiveness of ATMD with an LQG controller for reducing the across-wind response of a tall building is investigated.

2. EQUATIONS OF MOTION

A tall building installed with ATMD at the top floor level and with an active control force, such as an actuator, is shown in Figure 1. The building is modeled as an equivalent single degree of freedom system with a generalized mass $m_1$, damping $c_1$ and stiffness $k_1$, which correspond to the first mode modal mass, damping, and stiffness of the building.

$$m_1 \ddot{y}_1(t) + c_1 \dot{y}_1(t) + k_1 y_1(t) = C_d r(t) + k_d r(t) + f(t) - u(t)$$

(1)

$$m_d \ddot{r}(t) + C_d \dot{r}(t) + k_d r(t) = u(t) - m_d \ddot{y}_1(t)$$

(2)

where $r(t) = y_d(t) - y_1(t)$ is the displacement of $m_d$ relative to $m_1$.

This equation can be written in terms of the state-space formulation as:
Across-Wind Response of a Tall Building with Active Tuned Mass Damper Using Linear-Quadratic-Gaussian Controller

\[
\dot{X}(t) = AX(t) + B\dot{u}(t) + Hf(t)
\]

where \( X(t) = [\dot{y}_1, x_1, \dot{x}_1]^T \) denotes the state vector of the system, with

\[
A = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & \frac{k_i}{m_i} & \frac{k_d}{m_i} & -\frac{c_i}{m_i} \\
\frac{k_i}{m_i} & -\left(\frac{k_d}{m_d} + \frac{k_d}{m_i}\right) & \frac{c_i}{m_i} - \left(\frac{c_d}{m_d} + \frac{c_d}{m_i}\right) & 1
\end{bmatrix}
\]

as a system dynamic matrix,

\[
B = \begin{bmatrix}
0 & 0 & -\frac{1}{m_i} & \frac{1}{m_i} + \frac{1}{m_d}
\end{bmatrix}^T
\]

as the location vector of \( u(t) \), and

\[
H = \begin{bmatrix}
0 & 0 & \frac{1}{m_i} & -\frac{1}{m_i}
\end{bmatrix}^T
\]

as the location vector of \( f(t) \) [10].

3. OPTIMAL PARAMETERS OF ATMD

While the basic mechanism of how a TMD reduces the main body of structural responses is well established, the optimal parameters of a TMD may differ for different structures and external loading conditions [38]. Warburton investigated the effectiveness of a TMD for reducing the random responses of a main structure under a random load with a Gaussian white noise spectrum of constant value. The optimum parameters for the tuning frequency (\( f_{opt} \)) and the damping ratio (\( \xi_{opt} \)) of the passive TMD have been derived under stationary random excitations with a white noise spectrum, and the optimum TMD parameters of \( f_{opt} \) and \( \xi_{opt} \) have been determined for a mass ratio \( \mu \).

\[
f_{opt} = \sqrt{\frac{1 + \frac{3}{2} \mu}{1 + \mu}}
\]

\[
\xi_{opt} = \frac{\mu(1 + \frac{3}{4} \mu)}{4(1 + \mu)(1 + \frac{1}{2} \mu)}
\]

where \( \mu \) is the mass ratio of \( m_d/m_i \).

The optimal \( f_{opt} \) ratio is greater for random excitations than for regular harmonic excitations, however, the optimal \( \xi_{opt} \) is smaller for random excitations than for harmonic excitations. In addition, the optimum parameters for ATMD are identical to those for passive TMD.
4. NUMERICAL SIMULATION OF A FLUCTUATING A ACROSS-WIND LOAD

A fluctuating across-wind load can be treated as a random process of stationary Gaussian white noise that can be simulated numerically in the time domain using across-wind load power spectral density data. This is particularly useful for some response estimations that are more or less narrow-banded random processes, such as the across-wind response of a tall building. The numerical simulation procedure presented in this work is taken from Deodatis [35],

\[
f(t) = \sum_{k=1}^{N} 2 \sqrt{S_{f}(f_k) \Delta \omega} \cos(\omega_k t + \phi_k)
\]

where \( S_{f}(f_k) \) is the value of the spectral density of the across-wind load corresponding to the first modal resonant frequency.

\[
\Delta \omega = \omega_u - \omega_1 / N
\]

\[
\omega_k = \omega_1 + \left( k - \frac{1}{2} \right) / N
\]

\( \omega_u \) = upper frequency of \( S(\omega) \), \( \omega_1 \) = lower frequency of \( S(\omega) \)

\( \Phi_t = \) uniformly distributed random numbers between 0~2\( \pi \), \( N = \) number of random numbers

The across-wind load power spectral density used in equation (9) is that derived by Kareem[5], described below.

Kareem used a 5 sq in. (127 mm\(^2\)) square, 20-in. (508 mm\(^2\)) tall prism model and eight pressure transducers, integrating eight simultaneously monitored channels of pressure data on a building model surface, and thus obtained the normalized across-wind load spectra of the across-wind forcing function on a square across-section tall building exposed to urban and suburban atmospheric flow conditions, as shown below:

**Figure 2** Normalized reduced spectra of across setting-wind load in a suburban (BL1) and urban (BL3) environment
5. LINEAR-QUADRATIC-GAUSSIAN CONTROLLER

A tall building is subjected to a fluctuating along-wind load that has a stationary white noise spectrum, and can be considered a linear dynamic system with system and measurement noise. If the system and measurement noises are zero-mean Gaussian white noises with constant covariance intensity, they are stationary white noise spectra matrices. If the external random load acting on a tall building (such as a fluctuating along-wind load) can be considered as a system noise with the constant power spectral density of Gaussian white noise, we can formulate plant model dynamics with an LQG controller in modern optimal control theory as follows [36]:

\[ \dot{X}(t) = AX(t) + Bu(t) + \omega(t) \quad (10) \]

\[ Y(t) = CX(t) + Du(t) + v(t) \quad (11) \]

where \( D = 0 \) for simplicity. \( X(t) \) and \( Y(t) \) are the state and output vectors, and \( \omega(t) \) and \( v(t) \) are the system and measurement noises, respectively, which are assumed to be uncorrelated zero mean Gaussian white noises with covariance intensity matrices \( W \) and \( V \), respectively. That is:

\[ E[\omega(t)\omega(t + \tau)^T] = W\delta(\tau) \quad (12) \]

\[ E[v(t)v(t + \tau)^T] = V\delta(\tau) \quad (13) \]

and

\[ E[\omega(t)v(t + \tau)^T] = 0, \quad E[v(t)\omega(t + \tau)^T] = 0 \quad (14) \]

where \( E \) is the expectation operator and \( \delta(q - \tau) \) is a delta function.

The optimal controller \( u(t) \) in equation(10) is obtained when all states \( X(t) \) of the system and the output \( Y(t) \) are a combination of all states. However, in practice, all states \( X(t) \) are not available and the system and output measurements are driven by stochastic disturbances called “noises” which have the constant power spectral density of Gaussian white noise. In such situations, we need a state estimator or an observer to estimate all states of the system. Then, the design of the state observer can be performed using a Kalman filter, which is an optimal state observer for a stochastic dynamic system[36].

Let \( X(t) \) be the state estimate and let \( Xe(t) = X(t) - X(t) \) denote the estimation error; then, the state can be estimated using a Kalman filter as:

\[ \dot{X}(t) = AX(t) + Bu(t) + L(Y(t) - CX(t)) \quad (15) \]

where the observer gain matrix \( L \) is given by

\[ L = \Gamma C V^{-1} \quad (16) \]

where \( \Gamma = E[Xe(t) Xe(t + \tau)] \)

and \( \Gamma \) satisfies the filter algebraic Riccati equation (FARE)

\[ A\Gamma + \Gamma A^T + W - \Gamma C V^{-1}C\Gamma = 0 \quad (17) \]
In the LQG controller problem, the optimal controller $u(t)$ in equation (10) that minimizes the cost functional $J$ of equation (18) is subject to the constrained equation (15) when determined separately as the deterministic LQR controller problem.

As with the deterministic LQR problem, an optimal control law that minimizes the same quadratic cost functional with a trade-off between the state cost and control cost can be found \[35\]. However, for a stochastic dynamic system, such as the optimal control problem of wind-induced vibration in a tall building, the cost functional of the deterministic LQR problem cannot be employed because of the stochastic nature of the state space formulation of the stochastic differential equation of (10). That is to say, the ensemble average over all possible realizations of the excitation is considered, so the cost functional $J$ is given by

$$J = E\left\{ \lim_{T \to \infty} \frac{1}{T} \int_0^T \left[ X^T(t)Qx(t) + u^T(t)Ru(t) \right] dt \right\}$$  \hspace{1cm} (18)$$

where $Q$ and $R$ are the positive semi-definite and positive definite weighting matrices, respectively. The term $X^T(t)Qx(t)$ in equation (18) is a measure of control accuracy and the term $u^T(t)Ru(t)$ is a measure of control effort. Minimizing $J$ by keeping the system response and the control effort close to zero requires appropriate choices of the weighting matrices $Q$ and $R$ [20,36]. If it is desirable that the system response be small, then large values for the elements of $Q$ should be chosen by selecting the matrix to be diagonal and making the diagonal elements large values, such that any respective state variable will be small [20]. If it is required that the control energy be small, then large values of the elements of $R$ should be chosen.

The unique state-feedback optimal controller $u(t)$ that minimizes the cost functional $J$ of equation (18) is determined as follows [36,37]:

$$u(t) = -K\dot{X}(t)$$  \hspace{1cm} (19)$$

where \( K = RBP \)

and $P$ is the unique symmetric, positive semi-definite solution to the algebraic Riccati equation (ARE) given by

$$AP + PA^T - PBR^{-1}B^TP + Q = 0$$  \hspace{1cm} (20)$$

We can determine the LQR controller feedback gain matrix $K$ and the observer gain Kalman filter matrix $L$ independently; this is the so-called separation principle [36].

In the LQR control problem, the optimal controller is found by tuning some weighting matrices $Q$ and $R$. If the value of $Q$ is relatively large compared to $R$, the state vector $X(t)$ is small relative to the control $u(t)$; that is, more control force will be applied to the main structure. This means that larger values of $Q$ result in the poles of the closed-loop system matrix $(A-B*K)$ being far left in the s-plane, so the state decays faster to the zero state [37].

### 6. NUMERICAL EXAMPLE

This numerical example is from the "Numerical Example" described by Kareem[5]. The height of a tall building $H$ is 180 m, the width $B$ is 30 m, the depth $D$ is 30 m, the natural frequency $f_1$ is 0.2 Hz, the critical $\xi_{opt}$ is 0.01, the air density is 0.973 kg/m$^3$, the hourly mean wind speed at the building height $V_h$ is 24.4 m/s, the reduced velocity $V_h/f_{1w}$ corresponding to the mean hourly wind speed is 4.0, the generalized mass of a fundamental mode shape $m_1$ is
Across-Wind Response of a Tall Building with Active Tuned Mass Damper Using Linear-Quadratic-Gaussian Controller

10,942,500 kg, and \( S(f_n) \) is 3.149*10^8; all other data are as given in Kim et al. [31-34]. The optimum parameters for ATMD are identical to those for passive TMD: a \( \mu \) of 0.01, \( f_{opt} \) of 1.0 and \( \xi_{opt} \) of 0.05.

The numerically simulated across-wind load and response rates without ATMD are shown in Figure 3 and Figure 4. The root mean square (rms) displacement response without ATMD, as shown in Figure 3, is 0.0047 m, which is a good approximation to that of Kareem’s closed form response of 0.0040 m.

![Figure 3 Simulated across-wind load in the time domain.](image)

![Figure 4 Across-wind response without ATMD (rms = 0.0047 m).](image)

If the relative displacement response of ATMD compared to that of the main structure is five to ten times larger than that of main structure, then the ATMD works well; hence, the values for the diagonal elements of the weighting matrix \( Q \) and \( R \) should be selected as

\[
Q = 1.0 * 10^8 \cdot \begin{bmatrix}
100 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 100 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}, \quad R = 1.0 * 10^{-12}
\]

Using these values of \( Q \) and \( R \) and the assumed value of \( V \), which is 1.0*10-8, the estimated reduced response with ATMD using a LQG controller is shown in Figure 5. The rms value of the estimated reduced response is 0.0041 m, which represents a reduction of about 10 % compared to the response without ATMD. The estimated relative displacement response of ATMD to that of the main structure is shown in Figure 6, with an rms value of 0.0.0434 m; this is about 10 times larger than that of the main structure. The active control force of the LQG controller is shown in Figure 7.
If a value of $V = 1.0 \times 10^{-4}$ is used as an increased measurement noise, then the estimated reduced response with ATMD using an LQG controller is as shown in Figure 8. The rms value of the estimated reduced response is 0.0012 m, which represents a reduction of about 70% compared to the response without ATMD. The estimated relative displacement response of ATMD compared to that of the main structure is shown in Figure 9, with an rms value of 0.0103 m, about nine times larger than that of main structure. Finally, the active control force of the LQG controller is shown in Figure 10.
Across-Wind Response of a Tall Building with Active Tuned Mass Damper Using Linear-Quadratic-Gaussian Controller

If the value of $V$ is 1.0, then the estimated reduced response with ATMD using an LQG controller is as shown in Figure 11. The rms value of the estimated reduced response is 0.000019 m, which represents a reduction of about 99% compared to the response without ATMD. The estimated relative response of ATMD compared to that of the main structure is shown in Figure 12, with an rms value of 0.00016 m, about eight times larger than that of the main structure. The active control force of the LQG controller is shown in Figure 13.

The reduced responses of the main structure, and the relative displacement response of ATMD compared to that of the main structure with values of the elements of $Q$ 100 times larger and the same value of $R$, are shown in Figs 14 and 15. The control force is shown in Figure 16.

$$Q = 1.0 \times 10^{10} \times \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad R = 1.0 \times 10^{-12}.$$
As shown in the above results, the reduced responses of the main structure and the relative displacement response of ATMD compared to that of the main structure decrease further as the values of the elements of the weighting matrix $Q$ and the intensity of the covariance of the measurement noise increase.

7. CONCLUSIONS

The optimal control technique for LQG for obtaining reductions across all responses of a tall building with ATMD is investigated. Fluctuating across-wind loads acting on a tall building are simulated numerically and the across-wind responses of a tall building without ATMD, and with ATMD using an LQG controller, are calculated. The estimated across-wind response without ATMD is a good approximation to that of the closed form response. The reduced across-wind responses using ATMD with an LQG controller are estimated under different values of the LQG control design parameters. Therefore, an ATMD system using an LQG controller is an effective and useful design for mitigating wind-induced vibrations in a tall building.

ACKNOWLEDGMENTS

This work was supported by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MSIP) (NRF-2016R1A2B4015364)

REFERENCES


Across-Wind Response of a Tall Building with Active Tuned Mass Damper Using Linear-Quadratic-Gaussian Controller


