FREEZE LINING IN FURNACE A HEAT TRANSFER MODEL USING FINITE DIFFERENCE METHOD AND MAT LAB

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ABSTRACT

Freeze lining technique is now recommended in furnace operation. It is a solidified layer of molten metal on the refractory wall of furnace. Generally furnace walls are constructed with refractory material protected with steel shell and graphite layer. In the process of smelting of Ferro-alloy through an electrical submerged arc furnace (SAF), a small layer adjacent to the refractory wall of liquid molten metal get solidifies due to heat loss through the furnace wall. This layer is called freeze lining also well known as skull. This freeze lining can protects the refractory from chemical attack, erosion, thermal stresses and it assures long life time of the furnace. This layer should have optimum thickness otherwise it may effect on production volume. In this present project the thickness of freeze lining or skull was evaluated by heat transfer analysis using finite difference method (FDM) and MAT LAB, for 16.5 MVA, SAF of 12 tons capacity which produce SiMn alloy. Even though the complex solidification phenomena exits in the furnace a simple approach that an approximate model is only demonstrated.

Key word: Freeze Lining, Furnace, Finite Difference Method and Mat Lab


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1. INTRODUCTION

Submerged arc furnace is mainly used for smelting of ferro alloys such as ferrosilicon, ferromanganese, ferrochrome and silico-manganese alloys. The heat generated by the arcs and by joule’s heat formed on the passage of the current through the charge. Continuous charge in the furnace the smelting reaction takes place in the slag layer and tapping of slag through a perforated hole (tap hole) is carried out at regular time intervals. The heat generated in SAF is protected by insulated furnace walls consists of refractory brick wall with outer shell as steel sheath and inside with graphite or carbon paste. The refractory is gradually destroyed by hot molten ferro alloy and slag due to physical and chemical erosion during the smelting process. Due to heat loss in the furnace through the refractory brick walls the liquid molten metal gets solidifies and forms a solid layer on refractory wall inside the cylinder develops in phase change processes liquid to solids because of heat loss through conduction. This solid layer on inner surface of refractory is called as freeze lining. The freeze lining concept in the SAF is accepted since 1995 in ferro alloy industry.

In this present study the freeze lining thickness is evaluated in one dimensional heat transfer analysis by using transient heat conduction equation and is solved using one dimensional finite difference approximation method. A very good mathematical tool, mat lab is used to estimate the temperature profile across the furnace walls.

![Figure 1](image-url) Refractory wear profile of a 48MVA SAF, used for SiMn production in South Africa excavated in 2013

2. PHASE CHANGE PROBLEM

The phenomena of solidification and melting are associated with many practical applications. They occur in a diverse range of industrial processes, such as metal processing, solidification of castings, environmental engineering and thermal energy storage system in a space station. In these processes, matter is subject to a phase change.

2.1 Analytical Methods: Stefan’s solution with constant thermophysical properties shows that the rate of melting or solidification in a semi-infinite region is governed by a dimensionless number, known as the Stefan number (St),

\[ St = \frac{[C_l(T_l - T_m)]}{H_f} \]  \hspace{1cm} (1)

Where \(C_l\) is the heat capacity of the liquid,
\(H_f\) is the latent heat of fusion and
\(T_l\) and \(T_m\) are the temperatures of the surrounding and melting point respectively.
Neumann extended the Stefan’s solution to the two-phase problem. The initial state of the phase change material is assumed to be solid, for a melting process, but its initial temperature is not equal to the phase change temperature, and its temperature during the melting is not maintained at a constant value. If melting of a semi-infinite slab \(0 < x < \infty\) is considered, initially solid at a uniform temperature \(T_s \leq T_m\) and a constant temperature is imposed on the face \(x = 0\), with assumptions of constant thermophysical properties, the problem can be mathematically expressed as follows:

Heat conduction in liquid region

\[
\frac{\partial^2 T_l}{\partial t^2} = \alpha_l \frac{\partial^2 T_l}{\partial x^2} \quad \text{for } 0, x, X(t), t > 0 \quad (2)
\]

Heat conduction in solid region

\[
\frac{\partial^2 T_s}{\partial t^2} = \alpha_s \frac{\partial^2 T_s}{\partial x^2} \quad \text{for } X(t) < x; t > 0 \quad (3)
\]

Interface temperature

\[
T(X(t); t = T_m) \quad t > 0 \quad (4)
\]

Stefan condition

\[
k_s \frac{\partial T_s}{\partial t} - k_l \frac{\partial^2 T_l}{\partial x^2} = H \rho \frac{dx}{dt} \quad \text{for } x = X(t), t > 0 \quad (5)
\]

Initial conditions

\[
T(x, 0) = T_s < T_m \quad \text{for } x > 0; X(0) = 0 \quad (6)
\]

Boundary conditions

\[
T(0, t) = T_l > T_m \quad \text{for } t > 0 \quad (7)
\]

\[
T(x, t) = T_s \quad \text{for } x \to \infty, t > 0 \quad (8)
\]

Where \(X(t)\) is the position of the melting interface (moving boundary). Figure 2 illustrates this problem more clearly. However, the Neumann’s solution is available only for moving

![Figure 2](image-url)  
**Figure 2** Two phase Stefan space time problem boundary problems in the rectangular coordinate system.
3. NUMERICAL METHODS FOR SOLVING THE PURE HEAT CONDUCTION EQUATION WITH A PHASE CHANGE INVOLVED

3.1. Fixed grid methods

The heat flow equation is approximated by finite difference replacements for the derivatives in order to calculate values of temperature $T_{i,n}$ at $x_i = i\Delta x$ and time $t_n = n\Delta t$ on a fixed grid in the $(x, t)$ plane. At any time $t_n = n\Delta t$, the moving boundary will be located between two adjacent grid points; for instance, between $i_b\Delta x$ and $(i_b+1)\Delta x$.

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The new temperature is calculated from temperatures of the previous step on the basis of the following formulation:

$$T_{i,n+1} = T_{i,n} + \left( \frac{\Delta t}{\Delta x^2} \right) \left( T_{i-1,n} - 2T_{i,n} + T_{i+1,n} \right)$$

The temperature at $x = i_b\Delta x$ is computed as

$$T_{ib,n+1} = T_{ib,n} + \left( \frac{\Delta t}{\Delta x^2} \right) \left( \frac{1}{\rho_n + 1} T_{i-1,n} - 2T_{i,n} + \frac{1}{\rho_n} T_{i+1,n} \right)$$

The variation of the location of the moving boundary is

$$P_{n+1} = P_n - \left( \frac{\Delta t}{\rho H\Delta x^2} \right) \left( \frac{\rho_n}{\rho_n + 1} T_{ib,n} - \frac{1}{\rho_n} T_{ib,n} \right)$$

3.2. Variable grid methods

The fixed grid methods sometimes break down as the boundary moves a distance larger than a space increment in a time step. This constraint that depends on the velocity of the moving boundary may largely increase the array size (i.e. memory) and the CPU-time if computations are to be performed for extended times. Instead of applying a fixed time step and searching for the location of the moving boundary, Variable grid is intended to determine a variable time step, as part of the solution, such that the moving boundary coincides with a grid line in space. The fully implicit finite difference equations were used.

The substantial temperature derivative of each grid point is

$$\frac{dT}{dr} \bigg|_i = \frac{\partial T}{\partial x} \bigg|_i \frac{dx}{dr} \bigg|_i + \frac{\partial T}{\partial t} \bigg|_i \frac{dt}{dr}$$

Where $\frac{dx}{dr}$ the moving rate of each grid point is related to the moving boundary by

$$\frac{dx}{dr} \bigg|_i = \frac{x}{X(t)} \frac{dX}{dt}$$

By substituting equations, the governing equation for one-dimensional problems becomes...
The position of the moving boundary $X(t)$ is updated at each step by using a finite difference form of the Stefan condition on the moving boundary.

### 3.3. Apparent heat capacity methods

In this method, the latent heat is accounted for by increasing the heat capacity of the material in the phase change temperature range. For instance, if the latent heat is released uniformly in the phase change temperature range, the apparent heat capacity can be defined as

$$C_{app} = \begin{cases} 
  c_s & T < T_s \\
  c_{in} & T_s < T < T_l \\
  c_{ls} & T > T_l 
\end{cases} \quad \text{solid / liquid phase}$$

In terms of the definition of the apparent heat capacity, the energy equation in one dimension becomes

$$\rho C_{app} \frac{\partial T}{\partial t} - \frac{\partial}{\partial x} (k \frac{\partial T}{\partial t})$$

Equation can easily be discretized and solved numerically. The procedure for calculating the apparent heat capacity is as follows. (i) in the explicit finite difference formulation, $C_{app}$ is determined using the temperatures at the grid points from the previous time step; (ii) in the implicit formulation, two ways are available, the first is to evaluate $C_{app}$ based on the previous time step temperatures (as in the explicit case) and the second is according to the present time step temperatures by an iterative scheme.

### 3.4. Source based method

This method allows any additional heat from either a heat or a heat to be introduced into the general form of the energy equation as an extra term, that is, the source term. For the illustration of this method, a general source based method recently developed by Voller and Swaminathan will be presented as follows. In this general source based method derived from a standard enthalpy formulation, the sensible heat and latent heat are separated in the transient term of the energy

$$\rho \frac{\partial (cT + Hf)T}{\partial t} = \frac{\partial}{\partial x} (k \frac{\partial T}{\partial t})$$

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} (k \frac{\partial T}{\partial t}) = S$$

where the latent heat is now included in the source term $S$ as

$$S = -\rho \frac{\partial Hf}{\partial t}$$

where $V_P$ is the volume associated with the grid point P, ‘a’ is the coupling coefficient, the superscript ‘o’ as well as the subscripts ‘P’ and ‘nb’ refer to the value at the previous time step, the grid point under consideration and the neighbouring grid points, respectively.

### 3.5. Enthalpy method

The essential feature of the basic enthalpy methods is that the evolution of the latent heat is accounted for by the enthalpy as well as the relationship between the enthalpy and temperature.
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The method can be illustrated by considering a one dimensional heat conduction-controlled phase problem. An appropriate equation for such a case can be expressed as

\[ \rho \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) \]

The relationship between the enthalpy and temperature can be defined in terms of the latent heat release characteristics of the phase change material. This relationship is usually assumed to be a step function for isothermal phase change problems and a linear function for non-isothermal

\[ h = \begin{cases} C_s T & T \leq T_m \quad \text{solid phase} \\ C_s T + H_f & T > T_m \quad \text{liquid phase} \end{cases} \]

The relationship between the enthalpy and temperature can be defined in terms of the latent heat release characteristics of the phase change material. This relationship is usually assumed to be a step function for isothermal phase change problems and a linear function for non-isothermal

\[ h = \begin{cases} \frac{C_s T}{T_m} & T < T_s \quad \text{solid phase} \\ \frac{C_s T + \frac{H_f(T-T_s)}{(T_l-T_s)}}{T_l-T_s} & T_s \leq T \leq T_l \quad \text{solid / liquid phase} \\ C_l T + H_f = C_l n (T_l-T_s) & T \geq T_l \quad \text{liquid phase} \end{cases} \]

4. FINITE DIFFERENCE FORMULATION FOR 1-D HEAT DIFFUSION EQUATION

Consider the one-dimensional transient heat conduction equation

\[ \rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) \]

where \( \rho \) is density, \( c_p \) heat capacity, \( k \) thermal conductivity, \( T \) temperature, \( x \) distance and \( t \) time. If the thermal conductivity, density and heat capacity are constant over the model domain, the equation can be simplified to

\[ \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \]

where \( \alpha \) is the thermal diffusivity

The derivative of temperature versus time can be approximated with a forward finite difference approximation as

\[ \frac{\partial T}{\partial t} \approx \frac{T_{i+1}^{n+1} - T_{i}^{n+1}}{\Delta t} = \frac{T_{i}^{n+1} - T_{i}^{n}}{\Delta t} = \frac{T_{i}^{\text{new}} - T_{i}^{\text{current}}}{\Delta t} \]

where \( n \) represents the temperature at the current time step whereas \( n+1 \) represents the new (future) temperature. The subscript \( i \) refers to the location. Both \( n \) and \( i \) are integers; \( n \) varies from 1 to \( n_t \) (total number of time steps) and \( i \) varies from 1 to \( n_x \) (total number of grid points in \( x \)-direction).

The spatial derivative is replaced by a central finite difference approximation, i.e.

\[ \frac{\partial^2 T}{\partial x^2} \approx \frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right) = \frac{T_{i+1}^{n} - 2T_{i}^{n} + T_{i-1}^{n}}{\Delta x^2} \]

Substituting equations we get overall equation to find the temperatures at required time and distance steps

\[ \frac{T_{i}^{n+1} - T_{i}^{n}}{\Delta t} = \kappa \left( \frac{T_{i+1}^{n} - 2T_{i}^{n} + T_{i-1}^{n}}{\Delta x^2} \right) \]

The third and last step is a rearrangement of the discretized equation, so that all known quantities (i.e. temperature at time \( n \)) are on the right hand side and the unknown quantities on the left-hand side (properties at \( n+1 \)). This results in:


\[ T_{i+1}^n = T_i^n + \kappa \Delta t \left( \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2} \right) \]

Because the temperature at the current time step \((n)\) is known, we can use above equation to compute the new temperatures

### 5 RESULTS AND DISCUSSIONS

The one-dimensional phase-change problem for a High temperature furnace is depicted in Fig.4. The inner lining of the brick wall is coated with a protective freeze lining whose thickness represents the position of the solidification points for the Phase Change Material. The outer surface of the brick wall is cooled with water stream. The outside temperature is \(T_{\infty}\) and the convective heat transfer coefficient \(h_{\infty}\). At a time-varying heat flux \(q'(t)\) is imposed over the time interval \(\Delta T\) is 200sec up to 4800 sec was evaluated

The proposed mathematical model rests on the following assumptions

1. The temperature gradients in the \(x\) direction are much larger than those in the other directions. As a result, a one dimensional analysis can be applied.
2. The heat transfer inside the liquid phase of the PCM is conduction dominated.
3. The thermal properties of the phase change material (PCM) are temperature independent.

\[ \rho c_p \frac{\partial T}{\partial x} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) - \delta H \frac{\partial f}{\partial t} \]

Where \(\delta H\) and \(f\) are the enthalpy and the liquid fraction, respectively. The enthalpy \(\delta H\) is defined as

\[ \delta H = \rho C_{p, \text{liquid}} - C_{p, \text{solid}} \cdot T + \rho \lambda \]

The liquid fraction \(f\) varies linearly between the solidus \(T_{\text{sol}}\) and the liquidus \(T_{\text{liq}}\) in the following manner

\[
\begin{cases}
0 & T \leq T_{\text{sol}} \\
\frac{T - T_{\text{sol}}}{T_{\text{liq}} - T_{\text{sol}}} & T_{\text{sol}} < T < T_{\text{liq}} \\
1 & T \geq T_{\text{liq}}
\end{cases} 
\]

At each time-step, the liquid fraction \(f\) is updated iteratively in the following manner

\[ f^{t+1} \approx f^t + \left( \frac{dF}{dT} \right)^t \left( T^{t+1} - F^{-1}(f^t) \right) \]

As far as the lining is concerned, submerged arc furnaces generate heat in a somewhat concentrated area in front of the electrodes while the area between electrodes has a relatively low heat load. The system must be engineered to provide the proper quantity and distribution, but simply put, there should be a continuous water film on every part of the shell, with no dry or steaming areas.
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![Diagram of furnace structure](image)

**Figure 4** Sub merged arc furnace of 16.5 MVA, 12 tons capacity

**Table 1** Material properties

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Material</th>
<th>Thermal conductivity K (W/m K)</th>
<th>Sp. Heat Cp (KJ/Kg K)</th>
<th>Density ρ (Kg/m³)</th>
<th>Diffusivity α (m²/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Steel</td>
<td>43.3</td>
<td>0.47</td>
<td>7801</td>
<td>11.72x10⁻⁶</td>
</tr>
<tr>
<td>2</td>
<td>Refractory</td>
<td>1.68</td>
<td>0.875</td>
<td>2400</td>
<td>2.7x10⁻⁶</td>
</tr>
<tr>
<td>3</td>
<td>Graphite</td>
<td>17.5</td>
<td>0.712</td>
<td>2300</td>
<td>3.6x10⁻⁶</td>
</tr>
<tr>
<td>4</td>
<td>SiMn</td>
<td>22</td>
<td>0.48</td>
<td>7440</td>
<td>4.4x10⁻⁶</td>
</tr>
<tr>
<td>5</td>
<td>Copper</td>
<td>386</td>
<td>0.383</td>
<td>8954</td>
<td>112.34x10⁻⁶</td>
</tr>
<tr>
<td>6</td>
<td>Copper nikel</td>
<td>29</td>
<td>0.4</td>
<td>8940</td>
<td>6.7x10⁻⁶</td>
</tr>
</tbody>
</table>

In this present problem the outer shell is changed the steel, copper and cupronickel and the solution is done both conventional and with Matlab. The following plot show how the temperature distributed and also cooling require at the outer shell generally with water and its flow rate.

![Temperature Profile in Furnace wall when Outer shell is steel](image)

**Figure 5** Temperature Profile in Furnace wall when Outer shell is steel

http://www.iaeme.com/IJMET/index.asp 588  editor@iaeme.com
Figure 6 Temperature Profile in Furnace wall when Outer shell is Copper

Figure 7 Temperature Profile in Furnace wall when Outer shell is Cupronickel

(a)      (b)
We would like to estimate the freeze lining thickness from data provided, by firstly estimating the heat flow through the entire domain in the radial. If the cooling water flow rate in the above the heat transfer rate has been estimated as 5462 W/m², this figure could be validated using the outer surface of the furnace sidewall, and the cooling water data discussed earlier.

6. CONCLUSIONS
A one dimensional transient heat transfer analysis was presented for predicting the thickness of freeze lining. The time varying thickness of protection layer on refractory wall inside the submerged arc furnace was estimated using finite difference method and MATLAB. This methodology is possible to evaluate the skull formation efficiently. The analysis is carried out by assuming the furnace wall as multilayer including the solidified layer of slag. This layer protects the furnace wall from thermal shock, erosion and wear. The optimum thickness obtained as 32 to 80 mm with outer shell as steel. The analysis also presented by replacing outer shell steel with copper and also copper nickel. Freeze lining technology has become popular to maintain Furness walls to prevent degradation and to increase over the entire lifetime.
7. FUTURE SCOPE
This system can be extended to 2D, 3D analysis and also solidification phenomena, as phase change problem that includes such as movement in the slag bath solidification. This will model the system more realistic calculation for its actual phenomena. The operator must be cautious, however, as some systems that claim to be freeze linings do not include all the necessary elements of a True Freeze Lining.

REFERENCES