SUPRA PAIRWISE CONNECTED AND PAIRWISE SEMI-CONNECTED SPACES

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ABSTRACT
The aim of this paper is to introduce the notion of supra pairwise separated sets (briefly, $S_\tau$-p-separated) in supra bitopological spaces. Based on this notion we introduced the notion of $S_\tau$-p-connected and $S_\tau$-ps-connected spaces. Further, we study some of their characterizations and properties.

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1. INTRODUCTION
The study of bitopology was initiated by Kelly[7] in 1963. The concept of connectedness in bitopological space has been introduced by Pervin[10]. The study of supra topology was introduced by Mashhour[8] in 1983. In topological space the arbitrary union condition is enough to have a supra topological space. Gowri and Rajayal[3] are introduced the notion of supra bitopological spaces. Gowri and Jegadeesan[2] studied the concept of pairwise connectedness in soft biČech’ closure spaces. The purpose of this article is to introduce and exhibit some results of supra pairwise connectedness and supra pairwise semi-connectedness in supra bitopological spaces.

2. PRELIMINARIES
Definition 2.1 [8] $(X,S_\tau)$ is said to be a supra topological space if it is satisfying these conditions:
Definition 2.2 [8] Each element \( A \in S_\tau \) is called a supra open set in \((X, S_\tau)\), and its complement is called a supra closed set in \((X, S_\tau)\).

Definition 2.3 [8] If \((X, S_\tau)\) is a supra topological spaces, \( A \subseteq X, A \neq \emptyset, S_\tau A \) is the class of all intersection of \( A \) with each element in \( S_\tau \), then \((A, S_\tau A)\) is called a supra subspace of \((X, S_\tau)\).

Definition 2.4 [8] The supra closure of the set \( A \) is denoted by \( S_\tau -cl(A) \) and is defined as \( S_\tau -cl(A) = \bigcap\{B : B \text{ is a supra closed and } A \subseteq B\} \).

Definition 2.5 [8] The supra interior of the set \( A \) is denoted by \( S_\tau -int(A) \) and is defined as \( S_\tau -int(A) = \bigcup\{B : B \text{ is a supra open and } B \subseteq A\} \).

Definition 2.6 [3] If \( S_{\tau_1} \) and \( S_{\tau_2} \) are two supra topologies on a non-empty set \( X \), then the triplet \((X, S_{\tau_1}, S_{\tau_2})\) is said to be a supra bitopological space.

Definition 2.7 [3] Each element of \( S_{\tau_i} \) is called a supra \( \tau_i \)-open sets (briefly, \( \tau_i \)-open) in \((X, S_{\tau_1}, S_{\tau_2})\). Then the complement of \( S_{\tau_i} \)-open sets are called a supra \( \tau_i \)-closed sets (briefly, \( \tau_i \)-closed sets), for \( i = 1, 2 \).

Definition 2.8 [3] If \( (X, S_{\tau_1}, S_{\tau_2}) \) is a supra bitopological space, \( Y \subseteq X, Y \neq \emptyset \) then \((Y, S_{\tau_1}, S_{\tau_2})\) is a supra bitopological subspace of \((X, S_{\tau_1}, S_{\tau_2})\) if \( S_{\tau_i} = \{U \cap Y ; U \text{ is a } S_{\tau_i} \text{ -open in } X\} \) and \( S_{\tau_i} = \{V \cap Y ; V \text{ is a } S_{\tau_i} \text{ -open in } X\} \).

Definition 2.9 [3] The \( \tau_i \)-closure of the set \( A \) is denoted by \( S_{\tau_i} -cl(A) \) and is defined as \( S_{\tau_i} -cl(A) = \bigcap\{B : B \text{ is a } \tau_i \text{-closed and } A \subseteq B\} \).

Definition 2.10 [3] The \( \tau_i \)-interior of the set \( A \) is denoted by \( S_{\tau_i} -int(A) \) and is defined as \( S_{\tau_i} -int(A) = \bigcup\{B : B \text{ is a } \tau_i \text{-open and } B \subseteq A\} \).

Proposition 2.11 [4] Let \((X, S_{\tau_1}, S_{\tau_2})\) be supra bitopological spaces, if \( U \) and \( V \) are \( S_{\tau_1} \)-open sets then \( U \cup V \) also \( S_{\tau_1} \)-open, for \( i = 1, 2 \).

Proposition 2.12 [4] Let \((X, S_{\tau_1}, S_{\tau_2})\) be supra bitopological spaces, if \( U \) and \( V \) are \( S_{\tau_1} \)-closed sets then \( U \cap V \) also \( S_{\tau_1} \)-closed, for \( i = 1, 2 \).

Definition 2.13 [5] A subset \( A \) of a supra bitopological space \((X, S_{\tau_1}, S_{\tau_2})\) is called an \( S_{\tau_1} \)-semi-open (briefly, \( S_{\tau_1} \)-s-open) if \( A \subseteq S_{\tau_1} -cl(S_{\tau_i} -int(A)) \). Where \( i \neq j, i,j = 1,2 \). The complement of \( S_{\tau_1} \)-s-open set is said to be \( S_{\tau_1} \)-s-closed set.

The family of \( S_{\tau_1} \)-s-open (resp. \( S_{\tau_1} \)-s-closed) sets of \( X \) is denoted by \( S_{\tau_1} \)-sO\((X)\) (resp. \( S_{\tau_1} \)-sC\((X)\)), Where \( i \neq j, i,j = 1,2 \).

Throughout this paper, For a subset \( A \) of \((X, S_{\tau_1}, S_{\tau_2})\), \( S_{\tau_1} -scl(A) \) denote the semi closure of \( A \) with respect to the topology \( S_{\tau_1} \).

3. SUPRA PAIRWISE CONNECTED SPACES

In this section we introduce the concept of supra pairwise connectedness in supra bitopological spaces.

Definition 3.1 Two non-empty subsets \( U \) and \( V \) of a supra bitopological space \((X, S_{\tau_1}, S_{\tau_2})\) are said to be supra pairwise separated (briefly, \( S_\tau \)-p-separated) if and only if \( U \cap S_{\tau_1} -cl(V) = \emptyset \) and \( S_{\tau_2} -cl(U) \cap V = \emptyset \).
Remark 3.2 In other words two non-empty sets U and V of a supra bitopological space \((X, S_{\tau_1}, S_{\tau_2})\) are said to be \(S_\tau\)-p-separated if and only if \([U \cap S_{\tau_1}-\text{cl}(V)] \cup [S_{\tau_2}-\text{cl}(U) \cap V] = \emptyset\).

Definition 3.3 A supra bitopological space \((X, S_{\tau_1}, S_{\tau_2})\) is said to be supra pairwise disconnected (briefly, \(S_\tau\)-p-disconnected) if it can be written as two disjoint nonempty subsets U and V such that \(S_{\tau_2}-\text{cl}(U) \cap S_{\tau_1}-\text{cl}(V) = \emptyset\) and \(S_{\tau_2}-\text{cl}(U) \cup S_{\tau_1}-\text{cl}(V) = X\).

Example 3.4 Let \(X = \{a, b, c, d\}\),
\[S_{\tau_1} = \{\emptyset, X, \{a, b\}, \{b, c, d\}\},\]
\[S_{\tau_2} = \{\emptyset, X, \{c, d\}, \{a, b, d\}\}.\]
Here \(U = \{a, b\}\) is \(S_{\tau_1}\)-open and \(V = \{c, d\}\) is \(S_{\tau_2}\)-open. Then \(U \cup V = \{a, b\} \cup \{c, d\} = X\) and \(U \cap V = \{a, b\} \cap \{c, d\} = \emptyset\). Therefore \((X, S_{\tau_1}, S_{\tau_2})\) is \(S_\tau\)-p-disconnected.

Definition 3.5 A supra bitopological space \((X, S_{\tau_1}, S_{\tau_2})\) is said to be supra pairwise connected (briefly, \(S_\tau\)-p-connected) if it is not \(S_\tau\)-p-disconnected.

Example 3.6 Let \(X = \{a, b, c, d\}\),
\[S_{\tau_1} = \{\emptyset, X, \{a\}, \{a, b\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}\},\]
\[S_{\tau_2} = \{\emptyset, X, \{c\}, \{b, d\}, \{b, c, d\}, \{a, c, d\}\}.\]
Here \(U = \{a\}\) is \(S_{\tau_1}\)-open and \(V = \{a, c, d\}\) is \(S_{\tau_2}\)-open. Then \(U \cup V = \{a\} \cup \{a, c, d\} \neq X\) and \(U \cap V = \{a\} \cap \{a, c, d\} = \{a\} \neq \emptyset\). Therefore \((X, S_{\tau_1}, S_{\tau_2})\) is \(S_\tau\)-p-connected.

Theorem 3.7 In a supra bitopological space \((X, S_{\tau_1}, S_{\tau_2})\) every subsets of \(S_\tau\)-p-separated sets are also \(S_\tau\)-p-separated.

Proof: Let \((X, S_{\tau_1}, S_{\tau_2})\) be a supra bitopological space. Let U and V are \(S_\tau\)-p-separated sets. Let \(A \subset U\) and \(B \subset V\).
Therefore, \(U \cap S_{\tau_1}-\text{cl}(V) = \emptyset\) and \(S_{\tau_2}-\text{cl}(U) \cap V = \emptyset\). \(\rightarrow (1)\)
Since, \(A \subset U\) \(\Rightarrow S_{\tau_2}-\text{cl}(A) \subset S_{\tau_2}-\text{cl}(U)\)
\[= S_{\tau_2}-\text{cl}(A) \cap B \subset S_{\tau_2}-\text{cl}(U) \cap B\]
\[= S_{\tau_2}-\text{cl}(A) \cap B \subset S_{\tau_2}-\text{cl}(U) \cap V\]
\[= S_{\tau_2}-\text{cl}(A) \cap B \subset \emptyset\ \text{by \( (1)\)}\]
\[= S_{\tau_2}-\text{cl}(A) \cap B = \emptyset\]
Since, \(B \subset V\) \(\Rightarrow S_{\tau_1}-\text{cl}(B) \subset S_{\tau_1}-\text{cl}(V)\)
\[= S_{\tau_1}-\text{cl}(B) \cap A \subset S_{\tau_1}-\text{cl}(V) \cap A\]
\[= S_{\tau_1}-\text{cl}(B) \cap A \subset S_{\tau_1}-\text{cl}(V) \cap U\]
\[= S_{\tau_1}-\text{cl}(B) \cap A \subset \emptyset\ \text{by \( (1)\)}\]
\[= S_{\tau_1}-\text{cl}(B) \cap A = \emptyset\]
Hence, A and B are \(S_\tau\)-p-separated sets.

Definition 3.8 A subset \(Y\) of a supra bitopological space \((X, S_{\tau_1}, S_{\tau_2})\) is called \(S_\tau\)-p-connected if the space \((Y, S_{\tau_1}, S_{\tau_2})\) is \(S_\tau\)-p-connected.

Theorem 3.9 Let \((Y, S_{\tau_1}, S_{\tau_2})\) be a supra bitopological subspace of a supra bitopological space \((X, S_{\tau_1}, S_{\tau_2})\) and let \(U, V \subset Y\), then U and V are \(S_\tau\)-p-separated in X if and only if U and V are \(S_\tau\)-p-separated in Y.
Proof: Let \((X, S_{t_1}, S_{t_2})\) be a supra bitopological space and \((Y, S_{t_1}, S_{t_2})\) be a supra bitopological subspace of \((X, S_{t_1}, S_{t_2})\). Let \(U, V \subset Y\). Assume that, \(U\) and \(V\) are \(S_t\)-p-separated in \(X\) implies that \(U \cap S_{t_1}\text{-cl}(V) = \emptyset\) and \(S_{t_2}\text{-cl}(U) \cap V = \emptyset\).

That is, \((U \cap S_{t_1}\text{-cl}(V)) \cup (S_{t_2}\text{-cl}(U) \cap V) = \emptyset\).

Now, \([U \cap S_{t_1}\text{-cl}(V)) \cup (S_{t_2}\text{-cl}(U) \cap Y] \cup ((S_{t_2}\text{-cl}(U)) \cap Y \cap V\] = \([U \cap S_{t_1}\text{-cl}(V)) \cup (S_{t_2}\text{-cl}(U)) \cap Y \cap V\] = \emptyset.

Therefore, \(U\) and \(V\) are \(S_t\)-p-separated in \(Y\).

Conversely, let us assume that \(U\) and \(V\) are \(S_t\)-p-separated in \(Y\).

Implies that \(U \cap S_{t_1}\text{-cl}(V) = \emptyset\) and \(S_{t_2}\text{-cl}(U) \cap V = \emptyset\).

That is, \((U \cap S_{t_1}\text{-cl}(V)) \cup (S_{t_2}\text{-cl}(U) \cap V) = \emptyset\).

Hence \(U\) and \(V\) are \(S_t\)-p-separated in \(X\).

Remark 3.10 The following example shows that \(S_t\)-p-connectedness in supra bitopological space does not preserves hereditary property.

Example 3.11 In Example 3.6, the supra bitopological space \((X, S_{t_1}, S_{t_2})\) is \(S_t\)-p-connected. Consider \((Y, S_{t_1}, S_{t_2})\) be the supra bitopological subspace of \(X\), Such that \(Y = \{a, b, c\}\). Taking \(U = \{a, b\}\) and \(V = \{c\}\), \(S_{t_2}\text{-cl}(U) \cap S_{t_1}\text{-cl}(V) = \emptyset\) and \(S_{t_2}\text{-cl}(U) \cup S_{t_1}\text{-cl}(V) = Y\). Therefore, the supra bitopological subspace \((Y, S_{t_1}, S_{t_2})\) is \(S_t\)-p-disconnected.

Theorem 3.12 Let \((X, S_{t_1}, S_{t_2})\) be a supra bitopological space then the following conditions are equivalent.

- \((X, S_{t_1}, S_{t_2})\) is \(S_t\)-p-connected.
- \(X\) cannot be expressed as the union of two non-empty disjoint sets \(U\) and \(V\) such that \(U\) is \(S_{t_1}\)-open and \(V\) is \(S_{t_2}\)-open.
- \(X\) contains no non-empty proper subset which is both \(S_{t_1}\)-open and \(S_{t_2}\)-closed.

Proof: Case (i): (1) \(\Rightarrow\) (2)

Let \((X, S_{t_1}, S_{t_2})\) be \(S_t\)-p-connected. Assume that (2) is not true.

Let \(X = U \cup V\), Where \(U\) and \(V\) are non-empty disjoint sets such that \(U\) is \(S_{t_1}\)-open and \(V\) is \(S_{t_2}\)-open.

\(U \cap V = \emptyset\)

\(\Rightarrow U \subset X - V\)

\(\Rightarrow S_{t_2}\text{-cl}(U) \subset S_{t_2}\text{-cl}(X - V)\)

Hence \((S_{t_2}\text{-cl}(U)) \cap V = \emptyset\).

Similarly, \(U \cap (S_{t_2}\text{-cl}(V)) = \emptyset\).

Thus \([U \cap S_{t_1}\text{-cl}(V)) \cup (S_{t_2}\text{-cl}(U) \cap V] = \emptyset\).

So we have a \(S_t\)-p-separation of \(X\). Which is a contradiction to (1). Hence (2) holds. Therefore (1) \(\Rightarrow\) (2).

Case (ii): (2) \(\Rightarrow\) (3)

If possible, \(N\) be a proper non-empty subset of \(X\) which is \(S_{t_1}\)-open and \(S_{t_2}\)-closed. Then \(X - N\) is a proper non-empty subset of \(X\) which is \(S_{t_2}\)-open.

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Also, \(X = N \cup (X - N)\). Thus, \(X\) is expressed as the union of two nonempty disjoint sets such that one is \(S_{τ_1}\)-open and the other \(S_{τ_2}\)-open. Which is a contradiction to (2). Hence \(X\) does not have proper non-empty set which is both \(S_{τ_1}\)-open and \(S_{τ_2}\)-closed.

Case (iii): (3) \(⇒\) (1)

Suppose \(X\) is \(S_τ\)-p-disconnected. That is, \(X = U \cup V, U \cap S_{τ_1}\text{-cl}(V) = \emptyset, S_{τ_2}\text{-cl}(U) \cap V = \emptyset\).

Therefore, \(U \cap V = \emptyset\). That is \(U = X - V, S_{τ_1}\text{-cl}(U) \cap V = \emptyset\). Therefore \(S_{τ_1}\text{-cl}(U) = X - V\).

Hence \(U = S_{τ_1}\text{-cl}(U)\). That is \(U\) is \(S_{τ_2}\)-closed. Similarly, \(V = S_{τ_2}\text{-cl}(V)\). That is \(S_{τ_2}\)-closed.

This implies that \(U\) is \(S_{τ_2}\)-open. Then there exists a non-empty proper subset \(U\) which is \(S_{τ_1}\)-closed and \(S_{τ_2}\)-open. This is a contradiction to (3). Therefore, (3) \(⇒\) (1).

**Theorem 3.13** Let \(C\) be a \(S_τ\)-p-connected subset of a supra bitopological space \((X, S_{τ_1}, S_{τ_2})\).

If \(X\) has a \(S_τ\)-p-separation \(X = U / V\), then \(C \subseteq U\) or \(C \subseteq V\).

**Proof:** Let \(X = U \cup V\).

Then \(X = U \cup V\) and \([U \cap S_{τ_1}\text{-cl}(V)] \cup [S_{τ_2}\text{-cl}(U) \cap V] = \emptyset\) \(⇒\) (1). From (1) implies

\(U \cap V = \emptyset\). Thus, \(U = X - V\) or \(V = X - U\). Then we have, \([C \cap U] \cap S_{τ_1}\text{-cl}(C \cap V)] \cup [S_{τ_2}\text{-cl}(C \cap U) \cap (C \cap V)] \subseteq [(U \cap S_{τ_1}\text{-cl}(V)) \cup [S_{τ_2}\text{-cl}(U) \cap V] = \emptyset\).

That is, \(C = (C \cap U)/(C \cap V)\) is a \(S_τ\)-p-separation of \(C\). But \(C\) is \(S_τ\)-p-connected.

Then we have \(C \cap U = \emptyset\) or \(C \cap V = \emptyset\).

\(C \subseteq (X - U)\) (or) \(C \subseteq (X - V)\). Therefore, \(C \subseteq U\) (or) \(C \subseteq V\).

**Theorem 3.14** If \(C\) is \(S_τ\)-p-connected set and \(C \subseteq F \subseteq S_{τ_1}\text{-cl}(C) \cap S_{τ_2}\text{-cl}(C)\) then \(F\) is \(S_τ\)-p-connected set.

**Proof:** Assume that \(F\) is not \(S_τ\)-p-connected. Then we have a \(S_τ\)-p-separation \(F = U / V\) and \(F = U \cup V\) with \([U \cap S_{τ_1}\text{-cl}(V)] \cup [S_{τ_2}\text{-cl}(U) \cap V] = \emptyset\) \(⇒\) (1).

And \(U\) and \(V\) are non-empty. \(⇒\) (2)

By Theorem 3.13, \(C \subseteq U\) (or) \(C \subseteq V\). Suppose \(C \subseteq U\). Then \(V = V \cap V \subseteq V \cap F \subseteq V \cap S_{τ_1}\text{-cl}(C) \subseteq S_{τ_2}\text{-cl}(U) = \emptyset\). Which is a contradiction to (1). Then \(\emptyset \subset V \subset \emptyset\).

Thus, \(V = \emptyset\) is a contradiction to (2). Therefore, \(F\) is \(S_τ\)-p-connected.

**Remark 3.15** It is clear that the above Theorem 3.15, the relation of belonging to a \(S_τ\)-p-connected subset divides up any set into its disjoint maximal \(S_τ\)-p-connected subsets which we shall call the supra components of the set. The next theorem generalizes the fact that the supra components of a supra topological space are closed.

**Theorem 3.16** Any supra component \(D\) of a supra bitopological space \((X, S_{τ_1}, S_{τ_2})\) satisfies the equation \(D = S_{τ_1}\text{-cl}(D) \cap S_{τ_2}\text{-cl}(D)\).

**Proof:** Let \(D\) be a supra component and suppose that \(q \notin D\). Then \(D \cup \{q\}\) is not connected and we have some separation \(D \cup \{q\} = U / V\). By Theorem 3.13, either \(D \subseteq U\) and \(\{q\} \subseteq V\) or \(D \subseteq V\) and \(\{q\} \subseteq U\). Thus, \(D \cup \{q\} = D \cup \{q\}\) or \(D \cup \{q\}\)

\(= \{q\} / D\). Hence \(\{q\}\) is \(S_{τ_2}\)-open or \(\{q\}\) is \(S_{τ_1}\)-open. And so \(\notin S_{τ_2}\text{-cl}(D)\) or \(\notin S_{τ_1}\text{-cl}(D)\). This is equivalent to saying that \(q \notin S_{τ_1}\text{-cl}(D) \cap S_{τ_2}\text{-cl}(D)\). Then we have \(S_{τ_1}\text{-cl}(D) \cap S_{τ_2}\text{-cl}(D)\) \subseteq D. Clearly, \(D \subseteq S_{τ_1}\text{-cl}(D) \cap S_{τ_2}\text{-cl}(D)\) and the equation is satisfied.

**Definition 3.17** If \((X, S_{τ_1}, S_{τ_2})\) is supra pairwise totally disconnected (briefly, \(S_τ\)-pt-disconnected) if for each pair of points of \(X\) can be separated by a \(S_τ\)-p-separation of \(X\), that is given two distinct points \(x\) and \(y\) of \(X\) there is a \(S_τ\)-p-separations, \(X = U / V\) such that \(x \in U\), \(y \in V\).
Definition 3.18 If \((X, S_{\tau_1}, S_{\tau_2})\) is supra pairwise weakly totally disconnected (briefly, \(S_{\tau}\)-pwt-disconnected) if for each pair of points of \(X\) can be separated by a \(S_{\tau}\)-pseparation of \(X\), that is given two distinct points \(x\) and \(y\) of \(X\) there is a \(S_{\tau}\)-p-separations, \(X = U / V \) such that \(x \in U, y \in V \) or \(y \in U \) and \(x \in V \).

Definition 3.19 A supra bitopological space \((X, S_{\tau_1}, S_{\tau_2})\) is said to be supra pairwise feebly disconnected (briefly, \(S_{\tau}\)-pf-disconnected) if it can be written as two nonempty disjoint subsets \(U\) and \(V\) such that \(U \cap S_{\tau_1}\)-cl\((V) = S_{\tau_2}\)-cl\((U) \cap V = \emptyset\) and \(U \cup S_{\tau_1}\)-cl\((V) = S_{\tau_2}\)-cl\((U) \cup V = X\).

Result 3.20 Every \(S_{\tau}\)-p-disconnected in supra bitopological space \((X, S_{\tau_1}, S_{\tau_2})\) is \(S_{\tau}\)-pf-disconnected but the following example proves that the converse is not true.

Example 3.21 In Example 3.6, consider \(U = \{a\}\) and \(V = \{b, c, d\}\) which satisfies the condition \(U \cap S_{\tau_1}\)-cl\((V) = S_{\tau_2}\)-cl\((U) \cap V = \emptyset\) and \(U \cup S_{\tau_1}\)-cl\((V) = S_{\tau_2}\)-cl\((U) \cup V = X\).

Therefore, the supra bitopological space \((X, S_{\tau_1}, S_{\tau_2})\) is \(S_{\tau}\)-pf-disconnected. But, the supra bitopological space \((X, S_{\tau_1}, S_{\tau_2})\) is \(S_{\tau}\)-p-connected.

4. SUPRA PAIRWISE SEMI CONNECTED SPACES

In this section we introduce the concept of supra pairwise semi connectedness in supra bitopological spaces.

Definition 4.1 Let \((X, S_{\tau_1}, S_{\tau_2})\) be a supra bitopological space, \(A \subseteq X\), \(A\) is said to be supra pairwise semi open (briefly, \(S_{\tau}\)-ps-open) set if it is \(S_{\tau}\)-open set and \(S_{\tau}\)-open set.

The complement of \(S_{\tau}\)-ps-open set is called \(S_{\tau}\)-ps-closed set in \(X\).

Example 4.2 Let \(X = \{a, b, c, d\}\),
\[S_{\tau_1} = \emptyset, X, \{a, c\}, \{a, b, d\}, \{a, c, d\}\],
\[S_{\tau_2} = \emptyset, X, \{c, d\}, \{a, b, c\}, \{b, c, d\}\],
\[S_{\tau_{12}}\)-sO\((X) = \emptyset, X, \{a, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\],
\[S_{\tau_{21}}\)-sO\((X) = \emptyset, X, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\].

Here \(A = \{a, b, c\}\) is \(S_{\tau_{12}}\)-s-open set but it is not \(S_{\tau_{21}}\)-open set. Therefore \(A\) is \(S_{\tau}\)-ps-open.

Remark 4.3 Every \(S_{\tau_1}\)-open set is \(S_{\tau}\)-ps-open but the converse is false in the following example.

Example 4.4 Let \(X = \{a, b, c, d\}\),
\[S_{\tau_1} = \emptyset, X, \{a, b\}, \{a, b, c\}, \{b, c, d\}\],
\[S_{\tau_2} = \emptyset, X, \{b\}, \{a, b, d\}, \{a, c, d\}\],
\[S_{\tau_{12}}\)-sO\((X) = \emptyset, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\].

Here \(a, b, d\) is \(S_{\tau_{12}}\)-s-open set but it is not \(S_{\tau_1}\)-open set.

Definition 4.5 The \(S_{\tau_{ij}}\)-semi-closure of a set \(A\) is denoted by \(S_{\tau_{ij}}\)-scl\((A)\) and defined as \(S_{\tau_{ij}}\)-scl\((A) = \cap \{B : B\) is a \(S_{\tau_{ij}}\)-s-closed and \(A \subseteq B\), for \(i, j = 1, 2\}\).
Theorem 4.7 Finite union of $S_{ij}$-s-open sets is $S_{ij}$-s-open set.

Proof: Let $C$ and $D$ be to $S_{ij}$-s-open sets. Then $C \subseteq S_{ij}$-cl($S_{ij}$-int($C$)) and $D \subseteq S_{ij}$-cl($S_{ij}$-int($D$)). By Proposition 2.11, implies that $C \cup D \subseteq S_{ij}$-cl($S_{ij}$-int($C \cup D$)). Therefore, $C \cup D$ is a $S_{ij}$-s-open set.

Remark 4.8 Finite intersection of $S_{ij}$-s-open sets may fail to be $S_{ij}$-s-open set as seen from the following example.

Example 4.9 In Example 4.4, $S_{ij}$-sO($X$) = $\emptyset$, $X$, $\{a\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, $\{a, b, d\}$, $\{b, c, d\}$, $\{a, b, c\}$, $\{a, b, d\}$, $\{a, c, d\}$, $\{b, c, d\}$, $\{a, b, c, d\}$, $\{a, b, d, e\}$, $\{b, c, d, e\}$, $\{a, b, c, d, e\}$.

Here $\{a, b, d\}$ and $\{b, c, d\}$ are $S_{ij}$-s-open sets but their intersection $\{b, d\}$ is not $S_{ij}$-s-open set.

Theorem 4.10 Finite intersection of $S_{ij}$-s-closed sets is $S_{ij}$-s-open set.

Proof: Let $C$ and $D$ be to $S_{ij}$-s-closed sets. Then $S_{ij}$-int($S_{ij}$-cl($C$)) $\subseteq C$ and $S_{ij}$-int($S_{ij}$-cl($D$)) $\subseteq D$. By Proposition 2.12, implies that $C \cap D \subseteq S_{ij}$-int($S_{ij}$-cl($C \cap D$)). Therefore, $C \cap D$ is a $S_{ij}$-s-closed set.

Remark 4.11 Finite union of $S_{ij}$-s-closed set is not $S_{ij}$-s-closed set as from the following example.

Example 4.12 In Example 4.4, $S_{ij}$-sC($X$) = $\emptyset$, $X$, $\{a\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, $\{a, b, d\}$, $\{b, c, d\}$, $\{a, b, c\}$, $\{a, b, d\}$, $\{a, c, d\}$, $\{b, c, d\}$, $\{a, b, c, d\}$.

Here $\{a\}$ and $\{c\}$ are $S_{ij}$-s-closed sets but their intersection $\{a, c\}$ is not $S_{ij}$-s-closed set.

Definition 4.13 Two non-empty subsets $U$ and $V$ of $(X, S_{ij}, S_{ij})$ are said to be supra pairwise semi-separated (briefly, $S_{ij}$-ps-separated) if and only if $U \cap S_{ij}$-scl($V$) = $\emptyset$ and $S_{ij}$-scl($U$) $\cap V$ = $\emptyset$.

If $X = U \cup V$ such that $U$ and $V$ are $S_{ij}$-ps-separated sets, then $U$, $V$ form a $S_{ij}$-ps-separation of $X$ and it is denoted by $X = U \cup \{a, b, c\}$.

Definition 4.14 A supra bitopological space $(X, S_{ij}, S_{ij})$ is said to be supra pairwise semi disconnected (briefly, $S_{ij}$-ps-disconnected) if it can be written as the disjoint non-empty subsets $U$ and $V$ such that $S_{ij}$-scl($U$) $\cap S_{ij}$-scl($V$) = $\emptyset$ and $S_{ij}$-scl($U$) $\cup S_{ij}$-scl($V$) = $X$.

Example 4.15 Let $X = \{a, b, c\}$.

$S_{ij} = \{\emptyset, X, \{a\}, \{a, c\}, \{b, c\}\}$,

$S_{ij}$-sO($X$) = $\emptyset$, $X$, $\{a\}$, $\{a, c\}$, $\{b, c\}$,

$S_{ij}$-sO($X$) = $\emptyset$, $X$, $\{a\}$, $\{a, c\}$, $\{b, c\}$.

Here $U = \{a\}$ is $S_{ij}$-s-open and $V = \{b, c\}$ $S_{ij}$-s-open. Then $U \cup V = \{a, b, c\} = X$ and $U \cap V = \emptyset$. Therefore, $(X, S_{ij}, S_{ij})$ is $S_{ij}$-ps-disconnected.

Definition 4.16 A supra bitopological space $(X, S_{ij}, S_{ij})$ is said to be supra pairwise semi connected(briefly, $S_{ij}$-ps-connected) if it is not $S_{ij}$-ps-disconnected.

Example 4.17 Let $X = \{a, b, c, d\}$.

$S_{ij} = \{\emptyset, X, \{a, b\}, \{a, d\}, \{a, b, d\}, \{a, b, c\}, \{a, b, c, d\}\}$,

$S_{ij}$-sO($X$) = $\emptyset$, $X$, $\{a, b\}$, $\{a, c\}$, $\{a, b, d\}$, $\{a, b, c\}, \{a, c, d\}$, $\{b, c, d\}$,

$S_{ij}$-sO($X$) = $\emptyset$, $X$, $\{a, b\}$, $\{a, c\}$, $\{a, b, d\}$, $\{a, b, c\}$, $\{a, c, d\}$, $\{b, c, d\}$.
Here \( U = \{a, b\} \) is \( S_{\tau_1} \)-s-open and \( V = \{b, c\} \) \( S_{\tau_2} \)-s-open. Then \( U \cup V = \{a, b, c\} \neq X \) and \( U \cap V = \{b\} \neq \emptyset \). Therefore, \((X, S_{\tau_1}, S_{\tau_2})\) is \( S_{\tau} \)-ps-connected.

**Proposition 4.18** Every \( S_{\tau_i} \)-closed set is \( S_{\tau_i} \)-s-closed set in \((X, S_{\tau_1}, S_{\tau_2})\), for \( i = 1, 2 \).

**Proof:** Let \( A \) be \( S_{\tau_1} \)-closed set, for \( i = 1, 2 \). Then \( A \subseteq S_{\tau_1} \)-int(\( S_{\tau_1} \)-cl(\( A \))).

Hence, \( A \) is \( S_{\tau_1} \)-s-closed set in \( X \).

**Theorem 4.19** If \( A \) and \( B \) are \( S_{\tau} \)-p-separated in \((X, S_{\tau_1}, S_{\tau_2})\), then \( A, B \) are \( S_{\tau} \)-ps-separated in \( X \).

**Proof:** Let \( A, B \) are \( S_{\tau} \)-p-separated sets in supra bitopological space \((X, S_{\tau_1}, S_{\tau_2})\). This implies, \( A \cap S_{\tau_1} \)-cl(\( B \)) = \emptyset \) and \( S_{\tau_2} \)-cl(\( A \)) \( \cap B = \emptyset \), by Proposition 4.18, since every \( S_{\tau} \)-closed set is \( S_{\tau} \)-s-closed in \( X \). Hence, \( A \cap S_{\tau_1} \)-scl(\( B \)) = \emptyset \) and \( S_{\tau_2} \)-scl(\( A \)) \( \cap B = \emptyset \).

Therefore, \( A \) and \( B \) are \( S_{\tau} \)-ps-separated in \( X \).

**Definition 4.20** A subset \( Y \) of a supra bitopological space \((X, S_{\tau_1}, S_{\tau_2})\) is called \( S_{\tau} \)-ps-connected if the space \((Y, S_{\tau_1}, S_{\tau_2})\) is \( S_{\tau} \)-p-disconnected.

**Remark 4.21** The following example shows that \( S_{\tau} \)-ps-connectedness in supra bitopological space does not preserves hereditary property.

**Example 4.22** Let \( X = \{a, b, c, d\} \), \( S_{\tau_1} = \{\emptyset, X, \{c\}, \{a, b\}, \{a, b, c\}\}, S_{\tau_2} = \{\emptyset, X, \{b, c\}, \{a, c, d\}, \{b, c, d\}\} \), \( S_{\tau_1} = \{\emptyset, Y, \{c\}, \{b\}, \{b, c\}\}, S_{\tau_2} = \{\emptyset, Y, \{b, c\}, \{c, d\}\} \).

Here the supra bitopological space \((X, S_{\tau_1}, S_{\tau_2})\) is \( S_{\tau} \)-ps-connected. Let \( Y = \{b, c, d\} \subset X \). Taking \( U = \{b\} \) is \( S_{\tau_1} \)-s-open and \( V = \{c, d\} \) is \( S_{\tau_2} \)-s-open. Thus, \( U \cup V = Y \) and \( U \cap V = \emptyset \). Therefore, the supra bitopological subspace \((Y, S_{\tau_1}, S_{\tau_2})\) is \( S_{\tau} \)-ps-disconnected.

**Theorem 4.23** Let \((X, S_{\tau_1}, S_{\tau_2})\) be a supra bitopological space then the following conditions are equivalent.

- \((X, S_{\tau_1}, S_{\tau_2})\) is \( S_{\tau} \)-ps-connected.
- \( X \) cannot be expressed as the union of two non-empty disjoint sets \( U \) and \( V \) such that \( U \) is \( S_{\tau_1} \)-s-open and \( V \) is \( S_{\tau_2} \)-s-open.
- \( X \) contains no non-empty proper subset which is both \( S_{\tau_1} \)-s-open and \( S_{\tau_2} \)-s-closed.

**Proof:** The proof of the theorem is similar to proof of the Theorem 3.12.

**Theorem 4.24** Let \( A \) be a \( S_{\tau} \)-ps-connected subset of a supra bitopological space \((X, S_{\tau_1}, S_{\tau_2})\). If \( X \) has a \( S_{\tau} \)-ps-separation \( X = U / V \), then \( A \subset U \) or \( A \subset V \).

**Proof:** The proof is similar to the proof of the Theorem 3.13.

**Definition 4.25** Let \((X, S_{\tau_1}, S_{\tau_2})\) be a supra bitopological space and \( x \in X \). The supra semi component of \( x \) (briefly, \( S_{\tau} \)-sc(\( x \))) is the union of all \( S_{\tau} \)-ps-connected subset of \( X \) containing \( x \).

Further, if \( E \subset X \) and if \( x \in E \), then the union of all \( S_{\tau} \)-ps-connected sets containing \( x \) and contained in \( E \) is called \( S_{\tau} \)-sc(\( E \)).

**Theorem 4.26** In a supra bitopological space \((X, S_{\tau_1}, S_{\tau_2})\) each \( S_{\tau} \)-sc(\( x \)) satisfies the equation \( S_{\tau} \)-sc(\( x \)) = \( S_{\tau_1} \)-scl(\( S_{\tau} \)-sc(\( x \))) \( \cap S_{\tau_2} \)-scl(\( S_{\tau} \)-sc(\( x \))).

**Proof:** Let \( x \) be any point in \( X \) and let \( S_{\tau} \)-sc(\( x \)) be its supra semi component. Suppose that a point \( p \in X \) does not belongs to \( S_{\tau} \)-sc(\( x \)). Then \( S_{\tau} \)-sc(\( x \)) \( \cup \{p\} \) is not \( S_{\tau} \)-ps-connected and
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hence there exist a $S_{\tau}$-ps-separation $U / V$ in $X$ such that $S_{\tau}$-sc$(x) \cup \{p\} = U / V$. By Theorem 4.24, either $S_{\tau}$-sc$(x) \subseteq U$ and $\{p\} \subseteq V$ or $S_{\tau}$-sc$(x) \subseteq V$ and $\{p\} \subseteq U$. Thus, $S_{\tau}$-sc$(x) \cup \{p\} = S_{\tau}$-sc$(x) / \{p\}$ or $S_{\tau}$-sc$(x) \cup \{p\} = \{p\} / S_{\tau}$-sc$(x)$. Hence $p \notin S_{\tau}$-scl($S_{\tau}$-sc$(x)$) or $p \notin S_{\tau}$-scl($S_{\tau}$-sc$(x)$).

**Definition 4.27** If $(X, S_{\tau_{1}}, S_{\tau_{2}})$ is supra pairwise totally semi disconnected (briefly, $S_{\tau}$-pts-disconnected) if for each pair of points of $X$ can be separated by a $S_{\tau}$-psseparation of $X$, that is given two distinct points $x$ and $y$ of $X$ there is a $S_{\tau}$-psseparations, $X = U / V$ such that $x \in U$, $y \in V$.

**Definition 4.28** If $(X, S_{\tau_{1}}, S_{\tau_{2}})$ is supra pairwise weakly totally semi disconnected (briefly, $S_{\tau}$-pwts-disconnected) if for each pair of points of $X$ can be separated by a $S_{\tau}$-ps-separation of $X$, that is given two distinct points $x$ and $y$ of $X$ there is a $S_{\tau}$-ps-separations, $X = U / V$ such that $x \in U$, $y \in V$ or $y \in U$ and $x \in V$.

**Definition 4.29** A supra bitopological space $(X, S_{\tau_{1}}, S_{\tau_{2}})$ is said to be supra pairwise weakly semi-$T_{2}$-space (briefly, $S_{\tau}$-pws-$T_{2}$-space) if for every pair of distinct points of $X$, at least one belongs to a $S_{\tau_{1}}$-s-open set and the other belongs to a $S_{\tau_{2}}$-s-open set satisfying $U \cap V = \emptyset$.

**Theorem 4.30** Let $(X, S_{\tau_{1}}, S_{\tau_{2}})$ be a $S_{\tau}$-pw-$T_{2}$-space. If $S_{\tau_{1}}$ has a base whose members are also $S_{\tau_{2}}$-s-closed or $S_{\tau_{2}}$ has a base whose members are also $S_{\tau_{1}}$-s-closed, then the space $X$ is $S_{\tau}$-pwts-disconnected.

**Proof:** Suppose $S_{\tau_{1}}$ has a base whose sets are also $S_{\tau_{2}}$-s-closed. Consider two distinct points of $X$. Since the space is $S_{\tau}$-pw-$T_{2}$-space, one of the points $x$ has $S_{\tau_{1}}$-open neighbourhood $G$ which does not contain the other point $y$ and $y$ has a $S_{\tau_{2}}$-open neighbourhood $H$ which does not contain $x$. Then there exist a $S_{\tau_{1}}$-basic open set $V$ such that $V$ is also $S_{\tau_{2}}$-s-closed and $x \in V \subseteq G$. Then $X = V / (X - V)$ is a $S_{\tau}$-ps-separation of $X$ such that $x \in V$ and $y \in X - V$. Similarly, we obtain a $S_{\tau}$-ps-separation $X = U / (X - U)$ with $x \in X - U$, $y \in U$. Thus $(X, S_{\tau_{1}}, S_{\tau_{2}})$ is $S_{\tau}$-pwts-disconnected.

**Theorem 4.31** Let $(X, S_{\tau_{1}}, S_{\tau_{2}})$ be a $S_{\tau}$-p-$T_{2}$-space. If $S_{\tau_{1}}$ has a base whose members are also $S_{\tau_{2}}$-s-closed or $S_{\tau_{2}}$ has a base whose members are also $S_{\tau_{1}}$-s-closed, then the space $X$ is $S_{\tau}$-pts-disconnected.

**Proof:** The proof is similar to the proof of the Theorem 4.30.

**5. CONCLUSIONS**

In this paper, some results of supra pairwise connectedness and semi connectedness in supra bitopological spaces have been discussed. We plan to extend our research work to supra $\delta$ semi connectedness and compactness in supra bitopological space.

**REFERENCES**


