ANALYSIS OF A THIN AND THICK WALLED PRESSURE VESSEL FOR DIFFERENT MATERIALS

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ABSTRACT

In the present work the problem of calculation of the stress developed in the thin and thick cylindrical pressure vessels is numerically solved by using software in C++. The analysis has been done for two different materials pressure vessels. The variations in the thickness of the pressure vessels have been considered for the analysis for different internal pressures. The common characteristic of the pressure vessels solved is that the radial and tangential stresses vary in the same nature of curve for different thickness of vessels. Lame’s equations, maximum normal stress theory, maximum shear stress theory have been applied for the analysis of the thick walled pressure vessels of brittle and ductile materials. Analysis also performed for open and closed end cylinders by using Burnie’s equation and shear strain theory. Barlow’s equation has been applied for the calculation and analysis of the high pressure oil and gas pipe analysis. The yield point stress and ultimate stresses have been considered for the ductile and brittle material respectively. The modeling details of the methodology, employed in the analysis, are extensively discussed and the numerical approach is proven to be very efficient for the software developed of pressure vessel for thin and thick cylinders.

Keywords: Thick Walled Cylinder, Pressure Vessels, Thin Walled Cylinder, Ductile Material, Brittle Material.

1. INTRODUCTION

The design of pressure vessels for operation at very high pressures is a complex problem involving many considerations including definition of the operating and permissible stress levels, criteria of failure, material behavior, etc. For the purpose of developing the design philosophy and the relative operational limitations of various approaches, the elastic strength or yielding pressure of the vessel will be used as the criterion of failure. It should be noted, however, that some designs can be used at pressures in excess of that at which yielding of one or more components is predicted.
Generally, however, the use of vessels beyond the yielding pressure will depend upon the amount of plastic strain permissible and the ductility of the materials involved. At first glance there is little in common between an aircraft fuselage, a gas cylinder and a beer can, they are in fact all pressure vessels which must be designed to meet very specific requirements of integrity and cost, although the exact match of these vary widely. Also, the impacting issues of life expectancy, environmental effects, effect of cyclic loading, inspection during manufacture and use, together with product liability considerations all have to be taken into account. Fuel tanks, rocket motor cases, pipes are some examples of pressure vessels made of composite materials. Ever increasing use of this new class of materials in conventional applications is coupled with problems that are intrinsic to the material itself. Difficulties are many folded. Determination of material properties, mechanical analysis and design, failure of the structure are some examples which all require a non-conventional approach. Numerous applications concurrently are accompanied by various researches in the related field. Majority of the studies in the analysis of pressure vessels finds their origins in Lethnitskii’s approach [1]. H. Al-Gahtani et al., [2] have numerically investigated the feasibility of a proposed local pressure testing to verify structural integrity of nozzle-to-shell junctions in repaired/ altered spherical pressure vessels. The application of the theory given in this book is later applied to laminated composite structures in tubular form Tsai [3]. The studies followed consider also different loading and environmental conditions. Recently, there are some studies involved directly with tubes under internal pressure [4, 5]. In the study by Xia et al.,[5], the combined effect of thermo mechanical loading in addition to internal pressure is considered. Cassandra A. Latimer, M. S. et al., [6] have examined the impact of vascular proteins on bipolar seal performance found that collagen and elastin (CE) content within porcine arteries was a significant predictor of a vessel’s burst pressure (VBPr). Abhay K. Jha et al., [7], carried out the detailed metallurgical investigation to understand the cause of failure. High strength low alloy (HSLA) steel with a nominal composition of 0.15C–1.25Cr–1Mo–0.25V is being extensively used in space programme.

The procedure is based on the classical laminated plate theory. A cylindrical shell having a number of sub-layers, each of which is cylindrically orthotropic, is treated as in the state of plane strain. Internal pressures, axial force, body force due to rotation in addition to temperature and moisture variation throughout the body are considered as loading. In the study of Katırçı [8], these parameters are compared with the experimental results. M. Walker and P.Y. Tabakov [9], have provided an original in-depth analysis of the problem and then a new technique for determining the optimal design of engineering structures, with manufacturing tolerances accounted for, is proposed and demonstrated. The numerical examples used to demonstrate the technique involve the design optimization of anisotropic fibre-reinforced laminated pressure vessels. Jacek KruŚelecki and Rafał Proszowski [10], have investigated the problem of shape and thickness optimization of thin-walled pressure vessel heads. The optimal geometry of a closure, which minimizes the design objective containing both depth and capacity or both depth and volume of the material of a closure (two variants) is looked for in the class of the uniform strength structures. Three types of optimization problems are considered: the optimal shape is sought for a prescribe wall thickness; the optimal wall thickness is sought for a prescribed shape of a closure and the case when we look for both shape functions.

2. MATERIALS AND METHODS

2.1 Pressure Vessel Design Model for Cylinders

2.1.1 Thick Wall Theory

Thick-wall theory is developed from the Theory of Elasticity which yields the state of stress as a continuous function of radius over the pressure vessel wall. The state of stress is defined relative
to a convenient cylindrical coordinate system: $\sigma_t$, $\sigma_r$, $\sigma_l$. These are tangential, radial and longitudinal stresses respectively.

Stresses in a cylindrical pressure vessel depend upon the ratio of the inner radius to the outer radius ($r_o / r_i$) rather than the size of the cylinder.

Principal Stresses ($\sigma_1$, $\sigma_2$, $\sigma_3$)

1. Determined without computation of Mohr’s Circle;
2. Equivalent to cylindrical stresses ($\sigma_t$, $\sigma_r$, $\sigma_l$)

Applicable for any wall thickness-to-radius ratio

**Cylinder under Pressure**

Consider a cylinder, with capped ends, subjected to an internal pressure, $p_i$, and an external pressure, $p_o$.

![Figure 1: Cross sectional view of Cylinder with cap ends](image)

The cylinder geometry is defined by the inside radius, $r_i$, the outside radius, $r_o$, and the cylinder length, $l$. In general, the stresses in the cylindrical pressure vessel ($\sigma_t$, $\sigma_r$, $\sigma_l$) can be computed at any radial coordinate value, $r$, within the wall thickness bounded by $r_i$ and $r_o$, and will be characterized by the ratio of radii, $\zeta = r_o / r_i$. These cylindrical stresses represent the principal stresses and can be computed directly using equations 1 and 3. Thus we do not need to use Mohr’s circle to assess the principal stresses.

**Tangential Stress:**

$$\sigma_t = \frac{p_o r_i^2 - p_o r_o^2 - r_i r_o^2 (p_o - p_i)}{r_o^2 - r_i^2} \quad \text{for } r_i \leq r \leq r_o \quad (1)$$

**Radial Stress:**

$$\sigma_r = \frac{p_o r_i^2 - p_o r_o^2 - r_i r_o^2 (p_o - p_i)}{r_o^2 - r_i^2} \quad \text{for } r_i \leq r \leq r_o \quad (2)$$
Longitudinal Stress:

- Applicable to cases where the cylinder carries the longitudinal load, such as capped ends.
- Only valid far away from end caps where bending, nonlinearities and stress concentrations are not significant.

\[ \sigma_l = \frac{p_0 r_l^2 - p_a r_i^2}{r_o^2 - r_i^2} \text{ for } r_i \leq r \leq r_o \]

Two Mechanical Design Cases

1. Internal Pressure Only \((p_o = 0)\)
2. External Pressure Only \((p_i = 0)\)

Design Case 1: Internal Pressure Only

Only one case to consider — the critical section which exists at \(r_i/r_o\)

Substituting \(p_o = 0\) in to eqs. (1 and 2) and incorporating \(\zeta = r_o/\gamma_i\), the largest value of each stress component is found at the inner surface:

\[ \sigma_t(r = r_i) = \sigma_{t,max} = p_i \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} = p_i \frac{\zeta^2 + 1}{\zeta^2 - 1} C_{ti} \]

Where

\[ C_{ti} = \frac{\zeta^2 + 1}{\zeta^2 - 1} = \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} \text{ is a function of cylinder geometry only} \]

\[ \sigma_r(r = r_i) = \sigma_{r,max} = -p_i \]

Natural Boundary Condition

Longitudinal stress depends upon end conditions

\(\sigma_l = p_i C_{ti}\) for capped ends

\(\sigma_l = 0\) for uncapped ends

Where \(C_{ti} = \frac{1}{\zeta^2 - 1}\)

2.1.2 The Thin-walled Pressure Vessel Theory

An important practical problem is that of a cylindrical or spherical object which is subjected to an internal pressure \(p\). Such a component is called a pressure vessel, fig. 2. Applications arise in many areas, for example, the study of cellular organisms, arteries, aerosol cans, scuba-diving tanks and right up to large-scale industrial containers of liquids and gases.

In the present paper assume that

(i) The material is isotropic
(ii) The strains resulting from the pressures are small
(iii) The wall thickness \(t\) of the pressure vessel is much smaller than some characteristic radius
Because of (i, ii), the isotropic linear elastic model is used. Because of (iii), it will be assumed that there is negligible variation in the stress field across the thickness of the vessel.

As a rule of thumb, if the thickness is less than a tenth of the vessel radius, then the actual stress will vary by less than about 5% through the thickness, and in these cases the constant stress assumption is valid.

Thin-wall theory is developed from a Strength of Materials solution which yields the state of stress as an average over the pressure vessel wall.

Use restricted by wall thickness-to-radius ratio:

- According to theory, Thin-wall Theory is justified for \( \frac{t}{r} \leq \frac{1}{20} \)
- In practice, typically use a less conservative rule, \( \frac{t}{r} \leq \frac{1}{10} \)

State of Stress Definition

1. Hoop Stress, \( \sigma_t \), assumed to be uniform across wall thickness.
2. Radial Stress is insignificant compared to tangential stress, thus, \( \sigma_r = 0 \)
3. Longitudinal Stress, \( \sigma_l \)
   - Exists for cylinders with capped ends;
   - Assumed to be uniformly distributed across wall thickness;
   - This approximation for the longitudinal stress is only valid far away from the end-caps.

4. These cylindrical stresses \( (\sigma_t, \sigma_r, \sigma_l) \) are principal stresses \( (\sigma_t, \sigma_r, \sigma_l) \) which can be determined without computation of Mohr’s circle plot.
   - Analysis of Cylinder Section
The internal pressure exerts a vertical force, $F_V$, on the cylinder wall which is balanced by the tangential hoop stress, $F_{Hoop}$.

\[ F_V = pA_{proj} = p(d_i)(1) = pd_i \]  
\[ F_{Hoop} = \sigma_t A_{stressed} = \sigma_t(t)(1) = \sigma_t t \]  
\[ \sum F_y = 0 = F_V - 2F_{Hoop} = pd_i - 2\sigma_t t \]

Solving for the tangential stress,

Comparison of state of stress for cylinder under internal pressure versus external pressure:

**Internal Pressure Only**

\[ \sigma_t = \frac{pd_i}{2t} \quad \text{Hoop stress} \]
\[ \sigma_r = 0 \quad \text{By definition} \]
\[ \sigma_l = \frac{pd_i}{4t} = \frac{\sigma_t}{2} \quad \text{Capped case} \]

**External Pressure Only**

\[ \sigma_t = \frac{pd_o}{2t} \quad \text{Hoop stress} \]
\[ \sigma_r = 0 \quad \text{By definition} \]
\[ \sigma_l = \frac{pd_o}{4t} = \frac{\sigma_t}{2} \quad \text{Capped case} \]

3. RESULTS AND DISCUSSIONS

The results of the present work have been listed in the following paragraphs in the form of graphs. The effects of internal pressure of the fluid in the pressure vessels and their thickness have been considered to find the optimum condition of the design. Software has been developed in C++ for the analysis of the cylinders. Afterwards, the graphs have been plotted in origin 50, a menu driven software. The design for both the thin and thick wall cylinders have been taken into account in the present work. The thin walled cylinders have limited application while the thick walled cylinders are useful for high pressure fluid flow, especially in the oil and gas pipe design.

Figure 3 represents the variations of internal pressure with the thickness of the pressure vessel for different materials. The brittle material cannot withstand with the pressure above 30 N/mm². The thickness of the cylinder has to be increases for the given input values. On the other hand, ductile material has more capability to withstand with the increase in internal pressure. It also requires increasing the thickness by a small amount on increasing the pressure.
Figure 3: Variations of internal pressure with the thickness for different materials

Figure 4 represents the variation of internal pressure with thickness of the wall of cylinder for open end cylinder design of ductile and brittle material. From the figure, it is clearly shown that the brittle material requires more thickness of the cylinder as compared to the ductile material for the same input values.

Figure 4: Variation of Internal pressure with thickness of the wall of cylinder

Figure 5 represents the variation of internal pressure with thickness of the wall of cylinder for closed end cylinders. The value of thickness is lower than the open end cylinder for the same input conditions.

Figure 5: Variation of Internal pressure with thickness of the wall of cylinder
Figure 6 shows the variation of internal pressure with thickness of the wall of high pressure vessels specially used in oil and gas industries for transportation of fluids. In such type of analysis, it has been observed that the increase in the thickness on increasing the internal pressure dominated by brittle material.

![Figure 6: Variation of Internal pressure with thickness of the wall of high pressure cylinder](image1)

Figure 7 represents the variation of internal pressure with thickness of the wall of a thin walled cylinder. The brittle and ductile materials have been chosen for the analysis of the thin walled cylinder. The increase in the thickness of the brittle material again dominated for the increases in the internal pressure. This analysis has been performed for the less thick vessel as compared to previous result.

![Figure 7: Variation of Internal pressure with thickness of the wall of a thin walled cylinder](image2)

Figure 8 shows the variation of internal radius with tangential stress of a thick cylinder for various internal pressures. On increasing the internal pressure, the tangential stresses also increase at a given radius of the vessel. On the other hand, it has also been observed that the tangential stress decreases on the location beyond the centre line of the vessel. It has minimum value at the outer surface of the vessel and maximum at the inner surface of the vessel.

![Figure 8: Variation of Internal radius with tangential stress of a thick cylinder](image3)
Figure 8: Variation of Internal radii with tangential stress of a thick cylinder

Figure 9: Variation of Internal radii with radial stress of a thick cylinder

Figure 9 represents the radial stresses variation with the increases in the internal radius of the pressure vessel at different internal pressure of the fluid. The radial stresses are negative at the inner surface of the pressure vessel and it has maximum value at the outer surface and that is equal to zero. At high internal pressure, it has minimum value at the inner surface and minimum at outer surface.

Figure 10: Variation of change in volume with internal pressure of a thin cylinder
Figure 10 represents the variation of change in volume with internal pressure of a thin cylinder for four different types of material. The material has different properties. Due to these material properties, it has been observed that the increases in the volume of pressure vessels made of aluminium are dominated by the other materials on increases in the internal pressure. Among all the four materials pressure vessels, steel has minimum influenced of change in volume due to increase in the internal pressure of the fluid. The other two materials, namely brass and copper, exist between steel and aluminium. The input conditions are the same for the analysis.

4. CONCLUSION

In this paper, optimization study of pressure vessels is conducted considering a model of the entire pressure vessel with the help of software in C++. A method has been applied and implemented for the optimization of the thickness of pressure vessels on the basis of its thickness variation. The results are calculated for thick and thin cylinders and applied maximum normal stress theory for brittle material and maximum shear stress theory for ductile material. The analysis has also been made for open end and closed end cylinders. Birnie’s equations and maximum strain theory applied for closed and open end cylinders design respectively. For very high pressure oil and gas pipes, Barlow’s equations have been applied. A calculation for thin walled pressure vessel has been made for a fixed length of pipe i.e. 10000 mm.

For both the ductile and brittle material, the thickness of the vessels should be increases on increasing the internal pressure of the fluid. For all types of design, the brittle material should be thicker than that of ductile material. By comparing the standard result of radial and tangential stresses for thick and thin cylinders, it has been found that the nature of curve is similar to the present result. The property of material is different; hence one can say that the material optimization is correct and authentic.

REFERENCES


AUTHOR’S DETAIL

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