DOA ESTIMATION AND ADAPTIVE NULLING IN 5G SMART ANTENNA ARRAYS FOR COHERENT ARRIVALS USING SPATIAL SMOOTHING

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ABSTRACT

Spatial smoothing is a pre-processing technique and is a prerequisite to be able to perform accurate direction-of-arrival (DOA) estimation of coherent sources if subspace-based DOA methods are employed. In this work, a large-scale 5G uniform linear array (ULA) of sixty elements is considered. Signals from five sources, which are uniformly separated in space by an angle of 2°, impinge upon the array. The first two sources are coherent to each other whereas the rest are uncorrelated. Root-MUSIC and ESPRIT were considered for analysis. MATLAB simulations reinforce the fact that spatial smoothing enables the array to accurately estimate coherent sources. The root mean square error (RMSE) between the true and estimated DOAs was used as a measure of performance. RMSE decreased with an increase in signal-to-noise ratio (SNR), array size, and separation between the DOAs. ESPRIT was faster than Root-MUSIC in execution, and therefore, its output was fed, as an input, to the Least Mean square (LMS) null steering algorithm to achieve smart antenna functionality. The aperture lost due to spatial smoothing is negligible in 5G arrays since it accounts only for a small fraction of the array’s total aperture.

Key words: Coherent sources, Direction-of-Arrival (DOA) estimation, Linear Antenna Arrays, Null Steering, Smart antennas and Spatial smoothing.


1. INTRODUCTION

Direction-of-Arrival (DOA) estimation involves finding out the directions (spatial angles) from which the source signals strike the sensor array. DOA methods have been classified into three broad approaches, namely, the classical methods, the subspace methods, and the maximum-likelihood (ML) methods. Classical methods (such as Capon’s method) are based on beam forming and have a simple implementation. Such methods suffer from poor
resolution. ML-based methods provide accurate DOA estimation and possess robust characteristics but are computationally intensive owing to the non-convex and multi-dimensional search space. Subspace methods offer a reasonable compromise over the other two types, in terms of the computational cost and the achievable resolution [1].

Subspace-based methods mainly involve the Eigen Value Decomposition (EVD) of the array covariance matrix into signal and noise subspaces. The Pisarenko Harmonic Decomposition (PHD), the MUltiple SIgnal Classification (MUSIC), the Minimum-Norm, the Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT), and their variants are based on the noise subspace. PHD, MUSIC, Minimum-Norm, and a few others employ peak-searching on the pseudospectrum to estimate the DOAs. Root-MUSIC and Root-Min-Norm avoid spectral peak-searching and are based on a polynomial rooting procedure. ESPRIT makes use of subarrays within the main array to exploit the translational shift between the subarrays which manifests as a rotational invariance in the Eigenvector space [2].

Other methods for DOA estimation include the Direct Data Domain (DDD) methods such as the Matrix Pencil Method (MPM) and the Method of Direct Estimation (MODE) that directly operate on the received data and therefore avoid the computation of the array covariance matrix and its EVD. Such methods can also handle coherent sources. However, we limit our focus only to subspace methods in the presence of coherent signals.

Beamforming is the process of focusing the array radiation into specific spatial directions by making use of a tailored radiation pattern, obtained by applying appropriate weights at the individual elements. The weight vector is computed using some optimization criterion such as maximizing the signal-to-noise ratio (SNR), minimizing the mean square error, or maximizing the likelihood of detection. There are many types of beamforming such as analog versus digital, switched beam versus adaptive beam, cooperative versus co-located beamforming and so on. A detailed review of the types and importance of beamforming for 5G millimeter systems is given in [3]. Digital beamforming is also known as adaptive beamforming and makes use of adaptive algorithms such as the least mean squares (LMS), the sample matrix inversion (SMI), the recursive least squares (RLS), and their numerous variants [4].

Two signals are coherent if one is a scaled and shifted version of the other. Multipath interference and intentional jamming are the main causes for sources being coherent to each other [5]. When coherent sources arrive at the array, the source covariance matrix becomes non-diagonal, singular, and rank-deficient, that is, its rank would be less than the number of incoming signals [6]. As a result, the signal eigenvector diverges into the noise subspace, thereby jeopardizing the functionality of subspace methods, the mainstay of which is the Eigen decomposition of the array covariance matrix. Adaptive and optimum beamformers that synthesize a desired array radiation pattern are also vulnerable to the trap of coherent signals [7], [8]. Spatial smoothing comes to rescue in both the cases and is a prerequisite to deal with coherent arrivals. It is a pre-processing scheme that restores the rank of the source covariance matrix by decorrelating coherent sources through appropriate array manipulations [6], [9], [10].

A smart antenna system (SAS) is one which can perform both DOA estimation and digital beamforming. A complete SAS simulated in [11] makes use of a modified Bessel Beamformer for weight adaptation, and, Root-MUSIC for DOA estimation. Hardware prototypes of DOA-based smart antennas were studied in [12]–[14]. A smart antenna based wireless push system is demonstrated in [15].
The present study is an extended and improved version of our earlier work [16], in which Root-MUSIC and ESPRIT were used for estimating the DOAs of uncorrelated sources in millimeter wave ULAs. In the present work, we demonstrate how spatial smoothing functions as a handy tool in restoring the ability of large-scale millimeter arrays for accurate angle estimation of coherent sources. Spatially smoothed Root-MUSIC and ESPRIT are used for analysis. Additionally, we showcase the results of null steering using the LMS algorithm. The LMS also needs a spatially smoothed version of the array covariance matrix for proper operation.

Root-MUSIC and Root-Min-Norm are computationally lighter than their spectral counterparts and are, therefore, preferred. In addition, Root-MUSIC has better probability of resolution than spectral MUSIC and its asymptotic variance is always lesser than or equal to the asymptotic variance of Root-Min-Norm [17]. Root-Min-Norm is no better than the Root-MUSIC in terms of the computational complexity. Hence, we consider only Root-MUSIC and ESPRIT for analysis.

Overall, a DOA-based 5G smart antenna beamforming system that can tackle coherent sources is simulated in which the DOAs obtained from spatially smoothed ESPRIT algorithm (DOA block) are fed, as input angles, into the LMS algorithm (beamforming block). To the best of our knowledge and available evidence from the literature, no previous attempts were made to simulate DOA estimation and beamforming in the presence of coherent signals for 5G systems using subspace-based DOA estimation algorithms.

Newer interpretations are drawn in relation to the array aperture. Given the fact that large-scale millimeter-wave arrays have a wide aperture (thanks to the possibility of housing many an antenna within a small area), the array aperture lost due to spatial smoothing turns out to be a negligible fraction of the total available aperture and, therefore, can be overlooked. Moreover, the computational overhead incurred due to the process of spatial smoothing is quite low and affordable compared to the computational load incurred due to operations such as the EVD, Polynomial root finding, and/or Peak-Searching [18].

The rest of the paper has been organized as follows: - Section 2 describes the signal and the array model. Section 3 mentions a brief outline of the techniques and algorithms. Section 4 describes the simulation parameters and the methodology followed for the study. Section 5 explains the simulation results obtained from MATLAB. Section 6 provides the conclusion and scope for future enhancements.

2. SIGNAL AND ARRAY MODEL
Consider a uniform linear array (ULA) operating at a frequency of 30 GHz consisting of $M$ antenna elements with an inter-element spacing of $d = 0.5\lambda$. Signals from $D$ ($D < M$) narrowband sources located in the far-field of the ULA, traveling as plane waves, are intercepted by the array elements. The number of sources is assumed to be known thereby avoiding the need for any information theoretic criteria to estimate the model order or the number of sources. It is well-known that criteria such as the Akaike information criteria (AIC) test, minimum description length (MDL) test, and their variants are susceptible to failure when the signals are coherent [19]. The angle $\theta_i, i \in [1,D]$, denotes the unknown spatial direction from which the signal corresponding to the $i^{th}$ source hits the array. $K$ snapshots of the received signal are assumed to be available. Figure 1 shows the array geometry.

Pair-wise coherent sources occur in a two-ray multipath propagation model or in a situation where there is exactly one jammer per source signal. Multipath can occur in undersea acoustic channels (Sonar), aero-acoustic channels (due to reverberation), radars, and wireless communications. Two sources become coherent when they are fully correlated (have
perfect correlation). We assume that the first two sources $S_1$ and $S_2$ are pair-wise coherent and the remaining sources have no correlation either among themselves or with $S_1$ and $S_2$.

![Figure 1](image_url) The $M$-element uniform linear array

The received signal, the steering vectors, the array manifold, the correlation matrix, and the algorithmic steps for the Root-MUSIC, the ESPRIT, and the LMS have been clearly described in [2]. We, therefore, maintain the same variables, expressions, and models. Denoting the wave number by $\beta$ instead of $k$ is the only change made. However, a few formulations are worthy enough to be repeated and are given hereunder:

The signal received at the array is given by an $M \times 1$ column vector

$$\bar{x}(k) = \bar{A} \bar{s}(k) + \bar{n}(k)$$  \hspace{1cm} (1)

where $\bar{s}(k)$ is the $D \times 1$ column vector denoting the incoming signals from each source, at time $k$. $\bar{n}(k)$ denotes the zero-mean noise vector of size $M \times 1$. $\bar{A} = [\bar{a}(\theta_1) \; \bar{a}(\theta_2) \; \ldots \; \bar{a}(\theta_D)]$ is the $M \times D$ matrix known as the array manifold. Its columns denote the steering vectors $\bar{a}(\theta_i)$; $i = 1 \; to \; D$. The steering vector for a given angle of arrival $\theta_i$ is given by the column vector

$$\bar{a}(\theta_i) = [1 \; e^{j\beta d \sin \theta_i} \; e^{j2\beta d \sin \theta_i} \; \ldots \; e^{j(M-1)\beta d \sin \theta_i}]^T$$  \hspace{1cm} (2)

where $\beta = \frac{2\pi}{\lambda}$ is the wavenumber.

The covariance matrix is a mean subtracted correlation matrix and is equal to the correlation matrix for zero-mean processes. The array correlation matrix is given by

$$\bar{R}_{xx} = \bar{A} \bar{R}_{ss} \bar{A}^H + \bar{R}_{nn}$$  \hspace{1cm} (3)

where $\bar{R}_{ss}$ and $\bar{R}_{nn}$ denote the source and noise correlation matrices, respectively. For practical purposes, the signal and noise processes are assumed to be ergodic and hence expectation operators (the ensemble averages) are replaced by temporal statistics (the time averages).

In case of digital beamforming, the received signal is multiplied by the complex weights at each element to obtain the array output $y$ given by

$$y(k) = \sum_{m=1}^{M} w_m x_m(k)$$  \hspace{1cm} (4)

where $w_m$ = complex weight at the $m^{th}$ element, $x_m(k)$ is the signal received at the $m^{th}$ element.

3. BRIEF OVERVIEW OF THE ALGORITHMS/METHODS

Subspace-based DOA estimation methods decompose the array covariance matrix into signal and noise subspaces based on its eigenvalues. Peak searching follows the computation of pseudo spectrum. The dominant peaks correspond to the estimated DOAs. Subspace methods
that do not employ peak-searching are either based on polynomial rooting or overlapped subarray manipulations.

### 3.1 The Root-MUSIC Algorithm

The Root-MUSIC is an extension to the most well-known and widely used MUSIC algorithm. The MUSIC employs spectral peak-searching and its pseudo spectrum is given by

\[
P_{\text{music}}(\theta) = \frac{1}{|\bar{a}(\theta)^{H} \hat{E}_{N} \hat{E}_{N}^{H} \bar{a}(\theta)|} \tag{5}
\]

where the columns of the matrix $\hat{E}_{N}$ represent the noise eigenvectors.

Root-MUSIC was initially proposed to simplify the MUSIC for angle estimation in ULAs. This method involves forming a matrix $\hat{C} = \hat{E}_{N} \hat{E}_{N}^{H}$ from the denominator of the MUSIC pseudo spectrum. A new polynomial $D(z)$ is obtained such that its coefficients $c_{l}$ correspond to the sum of the elements along the $l^{th}$ diagonal of $\hat{C}$.

\[
D(z) = \sum_{l=-M+1}^{M-1} c_{l} z^{l} \tag{6}
\]

The roots $z_{i}$ of $D(z)$ that are close to the unit circle correspond to the poles of the MUSIC pseudo spectrum, and, hence give a measure of the estimated DOAs. The angles of arrival can be computed using

\[
\theta_{i} = -\sin^{-1}\left(\frac{\arg(z_{i})}{\beta d}\right); \quad i \in [1, D] \tag{7}
\]

### 3.2. The TLS-ESPRIT Algorithm

ESPRIT exploits the rotational invariance in the signal subspace created using linearly shifted sub-arrays (doublets) extracted from the main array. The subarrays are denoted by $A_{1}$ and $A_{2}$. $A_{1}$ lacks the last element of the main array while $A_{2}$ lacks the first element. $A_{1}$ and $A_{2}$ are translationally (linearly) apart from each other by a distance $d$. Since $d = \lambda/2$, the sub-arrays can also be thought to be displaced linearly by half wavelength. The array manifolds of the two sub-arrays are related by the formula

\[
\bar{A}_{2} = \bar{A}_{1} \bar{\Phi} \tag{8}
\]

where $\bar{\Phi}$ is a diagonal matrix with roots $e^{j\beta d \sin^{2}i}$, $i \in [1, D]$ on the main diagonal. The problem of estimating the DOAs boils down to knowing $\bar{\Phi}$. Since the arrays are linearly displaced, their respective eigenvectors $\bar{E}_{1}$ and $\bar{E}_{2}$, which form the signal subspace, are related by a rotational operator $\bar{\Psi}$ that maps $\bar{E}_{1}$ into the $\bar{E}_{2}$. The relation is given by

\[
\bar{E}_{1} \bar{\Psi} = \bar{E}_{2} \tag{9}
\]

The eigenvalues of $\bar{\Psi}$ will be the diagonal elements of $\bar{\Phi}$. The rest of the procedure involves few more formulations and manipulations that lead to the determination of the eigenvalues of $\bar{\Psi}$. Two approaches, namely, the Least Squares (LS), and the Total Least Squares (TLS) can be made use of. However, the TLS method has some advantages over the LS method. All instances of ESPRIT in the paper from here refer to the TLS-ESPRIT. The detailed steps can be found in [2] and [20].

The last step is to extract the DOAs from the eigenvalues $\lambda_{i}$ of $\bar{\Psi}$ using the following relation

\[
\theta_{i} = \sin^{-1}\left(\frac{\arg(\lambda_{i})}{\beta d}\right); \quad i \in [1, D] \tag{10}
\]
It is to be noted that Root-MUSIC involves the computational operations of EVD and polynomial root finding whereas ESPRIT needs multiple EVDs.

3.3. The Forward-Backward Spatial Smoothing

The source covariance matrix $\mathbf{R}_{ss}$ in (3) is a diagonal matrix when the sources are uncorrelated. However, it becomes non-diagonal, non-singular for partly correlated sources and non-diagonal, singular for coherent sources [6]. Presence of coherent sources renders the source covariance matrix to be rank-deficient. The aim of spatial smoothing is to revive its rank.

Owing to the non-diagonal structure of $\mathbf{R}_{ss}$, the array covariance matrix $\mathbf{R}_{xx}$ loses its Centro Hermitian property. Spatial smoothing restores this property through suitable array manipulations [21] but comes with the price of a reduced aperture in order to gain rank. Nevertheless, it is a preferred technique to deal with coherent arrivals.

The methods proposed in [6], [10] are known as the Forward Only Spatial Smoothing (FOSS), and the Forward-Backward Spatial Smoothing (FBSS), respectively. An $M$-element ULA can detect at most $M/2$ signal sources using FOSS, whereas it can detect $2M/3$ sources using FBSS [19]. FBSS is a very efficient way of spatial smoothing applicable to pair-wise coherent sources [5].

The following operations define the process of FBSS: -

- Let $\mathbf{J}$ denote an exchange matrix. It contains ones along the main anti-diagonal and zeros everywhere else. It is related to the identity matrix by $\mathbf{J}^2 = \mathbf{I}$.
- The backward array covariance matrix is defined by
  \[ \mathbf{R}_B = \mathbf{J} \mathbf{R}_{xx}^* \mathbf{J} \]  

- The Forward-backward (FB) averaged array covariance matrix is therefore given by
  \[ \mathbf{R}_{FB} = \frac{1}{2} [\mathbf{R}_{xx} + \mathbf{J} \mathbf{R}_{xx}^* \mathbf{J}] \]

- The FB averaged covariance satisfies the Centro Hermitian property given by
  \[ \mathbf{R}_{FB} = \mathbf{J} \mathbf{R}_{FB}^* \mathbf{J}. \]

Spatial smoothing can also be extended to more than two coherent sources. Several subarrays are formed from the main array. FB averaged covariance matrices corresponding to each subarray are averaged to obtain the final spatially smoothed covariance matrix. EVD is performed on this matrix. However, we do not need this method in the present work since only two coherent sources are considered.

3.4. The LMS Algorithm

The LMS algorithm is perhaps the most widespread adaptive filtering algorithm and one of the foremost gradient-based search algorithms that converges into the Optimal Wiener filter solution through iterative weight updation [22]. The Wiener-Hopf solution gives the optimal weights needed to achieve the minimum mean square error between the desired and obtained signals. The steepest-descent method uses an iterative search of the performance surface by updating the weight vector in successive iterations, thereby doing away with the need to invert the auto-correlation matrix. Initial weights are assumed to be zero and are updated till the bottom of the performance surface is reached, that is, till the optimum weights are obtained.

The LMS is a simple modification of the steepest-descent algorithm that replaces the expectation operator by a one-point sample mean (instantaneous approximation). The LMS weight vector is updated as follows

\[ \mathbf{w}(n + 1) = \mathbf{w}(n) + \mu e^*(n) x(n) \]  

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where $e(n)$ is the error signal between the desired and obtained response, $\mu$ is the step-size that determines how fast or how slow, the weights are updated (the algorithm converges), and $n$ denotes the time or the iteration number. The weight vector contains the set of weights needed for each element or filter tap. The choice of $\mu$ affects the convergence.

Convergence is guaranteed if

$$0 \leq \mu \leq \frac{1}{2 \times \text{trace}(R_{xx})}$$  (14)

LMS offers adaptive filtering functionality by adapting the weight vector to suit the changes in the signal environment. As mentioned previously, it is important to note that the beamforming or null steering algorithms are also susceptible to the presence of coherent sources and hence spatial smoothing needs to be performed before the array covariance matrix could be made use of in the LMS algorithm.

3.5. Smart Antenna Systems

Adaptive antenna arrays are also known as smart antennas in the context of wireless communications. They keep track of the changing radio conditions and continuously adapt their response or radiation pattern to match the changing signal and interference environment. The smartness to these antenna arrays comes from the digital signal processing blocks which bestow flexibility to the array to adapt its response to changing radio conditions [23]. Figure 2 shows the basic functions expected of a smart antenna system.

![Figure 2 Functions of a smart antenna system](image)

DOA estimation algorithms terminate once the angles are computed. Contrarily, the beamforming algorithms begin with an assumption that the DOAs are known beforehand and focus only on obtaining the optimal weight vector needed to obtain the desired response. It is during the design of smart antennas that the above two procedures need to be tied together to obtain the functionality of a complete system.

4. SIMULATION METHODOLOGY

A large-scale ULA of 60 elements with $d = 0.5\lambda$ operating at 30 GHz was considered. Five uniformly separated narrowband sources were considered. The first two sources were assumed to be pair-wise coherent. An angular separation of $2^\circ$ was assumed between adjacent sources, keeping in mind, the angular resolution of the 60-element array which is same as its half-power beamwidth ($\theta_{\text{hpbw}} = \frac{100^\circ}{M} = 1.7^\circ$). After spatial smoothing, the array would be left with 58 elements (a resolution of $1.76^\circ$) and should be able to detect the DOAs. Though super-resolution algorithms such as the Root-MUSIC and the ESPRIT can resolve sources that lie within the main beam of the array (sources that lie closer than $\theta_{\text{hpbw}}$), it is during the process of LMS null steering that the array resolution assumes importance. Null steering does
not give accurate results if the interference lies within the main lobe of the array [2]. Hence, the $2^\circ$ separation between sources is justified.

The Root Mean Square Error (RMSE) between the true and estimated DOAs was used as a performance measure to evaluate the estimation accuracy of the algorithms. The following formula given in [24] was used to compute the average RMSE (ARMSE) or simply the RMSE

$$ARMSE = \overline{RMSE} = \frac{1}{D} \sum_{d=1}^{D} \sqrt{\frac{1}{MC} \sum_{n=1}^{MC} (\hat{\theta}_{d,n} - \theta_d)^2}$$  \hspace{1cm} (15)$$

where $MC$ is the total number of Monte Carlo runs, $\hat{\theta}_{d,n}$ is the estimate of the $d^{th}$ source at the $n^{th}$ run.

The parameters such as the inter-element spacing and the number of snapshots were not tweaked since it is known that optimum array response is obtained when $d = 0.5\lambda$, and when the number of snapshots is high [16]. Also, we did not consider any variants of the LMS algorithm, nor did we vary the step size. Table 1 shows the simulation parameters.

**Table 1 Simulation Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of sources</td>
<td>$D = 5$</td>
</tr>
<tr>
<td>No. of snapshots</td>
<td>$K = 200$</td>
</tr>
<tr>
<td>No. of Monte-Carlo runs</td>
<td>$MC = 200$</td>
</tr>
<tr>
<td>Operating frequency and wavelength</td>
<td>30 GHz and $\lambda = 0.01m$</td>
</tr>
<tr>
<td>Inter-element spacing</td>
<td>$\lambda/2$</td>
</tr>
<tr>
<td>Source DOAs (S1 to S10) for all simulations except simulation III</td>
<td>$2^\circ$, $4^\circ$, $6^\circ$, $8^\circ$, and $10^\circ$</td>
</tr>
<tr>
<td>Coherent sources – Pair-wise</td>
<td>[S1, S2]</td>
</tr>
<tr>
<td>Simulation I – The effect of spatial smoothing on the RMSE</td>
<td>The true and estimated DOAs of Root-MUSIC and ESPRIT before and after FBSS were used to compute the RMSE</td>
</tr>
<tr>
<td>Simulation II – The effect of SNR on the RMSE</td>
<td>SNR was varied from -4 dB to 20 dB in steps of 4 dB. All other parameters remain unchanged.</td>
</tr>
<tr>
<td>Simulation III – The effect of DOA separation $h$ on the RMSE</td>
<td>$h$ was taken to be $0.25^\circ$, $0.5^\circ$, $1.0^\circ$, $1.5^\circ$, $2.0^\circ$, $2.5^\circ$, and $3.0^\circ$. Source DOAs were assumed to start from $2^\circ$ and were incremented in steps of $h$. For example, when $h=0.5^\circ$, the DOAs were $2^\circ$, $2.5^\circ$, $3.0^\circ$, $3.5^\circ$, and $4.0^\circ$</td>
</tr>
<tr>
<td>Simulation IV – The effect of number of sensors on the RMSE</td>
<td>$M$ was varied from 30 to 100, in steps of 10.</td>
</tr>
<tr>
<td>Simulation V – Speed of execution of Root-MUSIC and ESPRIT</td>
<td>The number of array elements (sensors) is varied from 10 to 100, in steps of 10.</td>
</tr>
<tr>
<td>LMS step-size</td>
<td>$\mu = \frac{1}{4} Re[\text{trace}(R)]$</td>
</tr>
<tr>
<td>Computational setup</td>
<td>64-bit Windows10 operating system and Intel i7-7500U processor with a clock frequency of 2.70GHz and 16GB main memory (RAM)</td>
</tr>
</tbody>
</table>
5. RESULTS AND DISCUSSION

Simulations were carried out using MATLAB 2016a. To get a visual feel of what spatial smoothing has to offer, the MUSIC pseudo spectrum was plotted before and after applying FBSS. Figure 3 shows that the array’s ability to detect the DOAs of coherent sources $S_1$ and $S_2$ is improved after applying spatial smoothing.

![Figure 3 MUSIC pseudospectrum with and without spatial smoothing](image)

5.1. Root-MUSIC and ESPRIT before and after FBSS

The estimated DOAs and the RMSE for both the algorithms, before and after spatial smoothing, considering 200 snapshots and 200 Monte-Carlo runs, are shown in Table 2 below.

<table>
<thead>
<tr>
<th>True DOAs</th>
<th>Estimated DOAs before FBSS</th>
<th>Estimated DOAs after FBSS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RM</td>
<td>ESPRIT</td>
</tr>
<tr>
<td>2</td>
<td>1.6773</td>
<td>-35.26</td>
</tr>
<tr>
<td>4</td>
<td>4.3214</td>
<td>3.0009</td>
</tr>
<tr>
<td>6</td>
<td>6.0002</td>
<td>6.0003</td>
</tr>
<tr>
<td>8</td>
<td>7.9997</td>
<td>8.0002</td>
</tr>
<tr>
<td>10</td>
<td>9.9999</td>
<td>10.0003</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.128928</td>
<td>7.651989</td>
</tr>
</tbody>
</table>

Figure 4 shows the mismatch between the true and estimated DOAs in the absence of spatial smoothing. Figure 5 shows the estimated DOAs after spatial smoothing. Spatial smoothing enables the subspace-based DOA methods to detect coherent sources accurately.
DOA Estimation and Adaptive Nulling in 5G Smart Antenna Arrays for Coherent Arrivals Using Spatial Smoothing

Figure 4 DOAs estimated by Root-MUSIC and ESPRIT before Spatial Smoothing

Figure 5 DOAs estimated by Root-MUSIC and ESPRIT after Spatial Smoothing closely match the true DOAs

5.2. RMSE versus SNR
The change in RMSE with respect to the SNR is shown in Figure 6. It is observed that the RMSE reduces with increasing SNR as the estimation accuracy gets better at high SNR.

Figure 6 Effect of SNR on RMSE
5.3. RMSE versus the DOA Separation
The angular separation \( h \) between adjacent sources was varied and the results are shown in Figure 7. The RMSE decreased as the angular spacing between the sources increased.

![Figure 7 RMSE versus the DOA separation angle](image)

5.4. RMSE versus the Number of Array Elements
The number of array elements was varied from 30 to 100. It can be seen from Figure 8 that the RMSE decreases as the number of sensors increase.

![Figure 8 RMSE versus the number of array elements](image)

Figures 7 and 8 are two ways of looking at the basic fact that the array aperture, resolution, and the estimation accuracy increase with increasing number of sensors. It was proved in [25] that the mean square error (MSE) is related to the number of sensors by \( O(M^{-3}) \). That is the MSE reduces asymptotically with increasing number of sensors according to a cubic relation. It is obvious that the RMSE varies as per \( O(M^{-3/2}) \).

5.5. Execution Time for Root-MUSIC & ESPRIT
The execution time needed for the Root-MUSIC and ESPRIT routines was observed using the \( tic \) and \( toc \) functions available in MATLAB. The number of sensor elements was varied from 10 to 100, in steps of 10. Figure 9 shows the comparison between the execution time of the two methods for 200 independent runs.
Root-MUSIC takes more time for execution than the ESPRIT. This can be attributed to the fact that the complexity of Root-MUSIC increases drastically with the number of array elements. It involves solving a polynomial rooting problem of a \((2M - 2)\) order polynomial, the complexity of which is of the order \((2M - 2)^3\) (e.g., the eigenvalue method used for realizing the function \textit{roots} in MATLAB). A fast realization of Root-MUSIC using a multi-taper polynomial was proposed in [26]. Another reduced-order Root-MUSIC proposed in [27] removes the computational redundancy in the polynomial rooting step by bringing down the polynomial order to \((M - 1)\) using Schur spectral factorization technique. Irrespective of the array size, such a method would provide an eight-fold speedup in the computations. The computational burden for the spectrally factorized polynomial would be still of the order \((M - 1)^3\).

Another evidence about Root-MUSIC needing more Floating Point Operations (FLOPS) than ESPRIT even for a modest 25 or lesser sensors can be found in [28]. It is obvious that the difference between the FLOPS needed for the two algorithms grows bigger with the number of sensors.

### 5.6. Adaptive Null Steering using the LMS Algorithm

The LMS algorithm was used for null-steering considering a pair of coherent signals. The first source in the pair was assumed to be the desired signal and the second was assumed to be the interfering signal. The LMS null steering response for sources \(S_1\) and \(S_2\) is shown in Figure 10. It is observed that spatial smoothing ensures correct placement of main lobe and nulls.

![Figure 10 LMS null-steering before and after spatial smoothing](image-url)
5.7. Loss in Array Aperture due to Spatial Smoothing

The amount of aperture lost during the process of spatial smoothing is given by the expression

\[
\text{Loss} \% = \frac{D_c}{M_a} \times 100\%
\]  

(16)

where \( M_a \) is the array aperture in terms of the inter-element spacing \( d \), and \( D_c \) is the number of coherent sources in the received signal. Table 3 below provides the percentage aperture lost for various arrays.

<table>
<thead>
<tr>
<th>( M )</th>
<th>( \text{Original aperture} )</th>
<th>( \text{New aperture with one coherent pair} )</th>
<th>( \text{Loss in aperture} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>48</td>
<td>47</td>
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<td>3.1</td>
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<td>69</td>
<td>2.8</td>
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<td>93</td>
<td>2.1</td>
</tr>
<tr>
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<tr>
<td>128</td>
<td>127</td>
<td>125</td>
<td>1.6</td>
</tr>
</tbody>
</table>

We summarize our observations as follows:

- ESPRIT is faster in execution than Root-MUSIC for large-scale arrays and, therefore, must be preferred. If the speed of execution is not a criterion, then Root-MUSIC is preferable as it has many valuable properties.
- RMSE decreases with an increase in SNR, angular separation between sources, and the number of array elements.
- Spatial smoothing restores the ability of subspace DOA estimation methods and beamforming algorithms to function accurately in the presence of coherent signals.
- The aperture lost during spatial smoothing is a small fraction of the total aperture available in large-scale 5G linear arrays.

6. CONCLUSIONS

Spatial smoothing is a mandatory prerequisite to perform DOA estimation (using subspace methods) and beamforming in the presence of coherent signals. The array aperture lost due to spatial smoothing is not so significant in 5G arrays. ESPRIT was found to be faster in execution than Root-MUSIC. It remains as a future extension to compare the performance of unitary Root-MUSIC and unitary ESPRIT for large arrays.

REFERENCES


DOA Estimation and Adaptive Nulling in 5G Smart Antenna Arrays for Coherent Arrivals Using Spatial Smoothing


