DEVELOPMENT OF A LAP-TIME SIMULATOR FOR A FSAE RACE CAR USING MULTI-BODY DYNAMIC SIMULATION APPROACH

Chitranjan Singh
School of Mechanical Engineering,
Vellore Institute of Technology, Vellore, India

Sakthivel Palanivelu
Automotive Research Center, School of Mechanical Engineering,
Vellore Institute of Technology, Vellore, India

ABSTRACT

There are many challenges in race car development process right from making of prototype to realize an actual car. The development cost is proportional to the number of prototypes used during the process. This brings in the necessity of a lap-time simulator that aid in data driven decision making during multiple lap configurations without an actual physical test of the vehicle. Development of lap-time simulator is a complex task. It requires a multi-body dynamic simulation approach. The simulator has to account for vehicle performance, handling and ride comfort. The vehicle performance is not only influenced by ideal power characteristics provided by the power unit and transmission system, but depends on the role of tire to maximize the tractive/braking effort realized at the contact patch during complex tire road interaction. It is mainly influenced by location of center of gravity, dynamic longitudinal load transfer, and the aerodynamic and rolling resistances. The vehicle response for handling inputs such as steering and environmental inputs decides the vehicle directional control and stability of the vehicle, which is influenced by dynamic load transfer, and cornering slip stiffness of the tires. The roll, pitch and bounce motion decides the ride comfort level of the vehicle. The main aim of this paper is to present a lap-time simulator developed for vehicle performance and handling characteristics for linear operational range. The modeling of the simulator has been divided into 3 segments. The first segment corresponds to tire modeling. The second segment has two sub-segments concerned with developing the vehicle modeling which includes the vehicle modeling for longitudinal dynamics as well as lateral dynamics [1]. The final segment concerned with creating an algorithm for tracking the vehicle on a predefined map and executing the prescribed motions. Hence, it is understood that there are many advantages of a lap-time simulator, importantly, it reduces number of prototypes and hence reduces the vehicle development cost.

Key words: FSAE race car, Lab-time simulator, longitudinal dynamics, lateral load transfer, map tracking, ride comfort.
1. INTRODUCTION

Every reputed Formula Student team around the world has developed a similar lap-time simulator for their car due to obvious advantages. In order to simulate all the parameters a Lap-time simulator is essential. Vehicle dynamics simulation tools is with the objective that it should be sufficiently accurate, and describe the performance of a vehicle well enough that they can be used to make decisions when designing or developing a vehicle and, thereby decreasing the testing time as well as cost of manufacturing. A linear model would cause huge deviation from actual data and would be proven inconsequential.

The same can be done by a professional simulation tool which is available in the market but it suffers from the disadvantage of taking a huge array of inputs to replicate reality to a high degree, but the insight provided that comes along with simplicity is lost, which is essential in early development stages. Finally, the advantage of taking batch runs is explored to be a significant aspect of custom built simulators. The method presented in the current work represents a step between these two extremes.

2. METHODOLOGY

The development of a lab time simulator is done in stages [2] and consisted of the following:

- Tire modelling
- Vehicle Modelling
- Map tracking

2.1. Tire Modelling

The popular magic formula version 5.2 is considered as the tire model for the vehicle. The formula gives the variation of longitudinal, lateral forces and aligning torque developed at the contact of the tire patch as a function of slip ratio and slip angle respectively. However the contact forces are function of vertical load felt at the contact patch. Software called Optimum T used to fit this curves using the data given by the TTC (Tire test consortium).

![Figure 1 Lateral force v/s slip angle for 1000, 1500 and 2000N](image)
These data are basically generated from a number of test runs on a specific tire to represent a semi-empirical tire model. The coefficient correction also carried out based on the curve fitting equations given in the reference. [3] The following Fig. 1 represents the variation of lateral force as a function of slip angle for different load conditions.

The linear variation range of lateral force is limited by very small range of slip angle for the normal load of 1000N, and it further reduces as the normal load increases. For a chosen, slip angle, the required lateral force increases as function of normal load. The tire load sensitivity appears to be a closer match of characteristics of tire model with that of the actual one. Which states that the lateral forces at given slip angle as a function of normal load but the variation is non-linear. So it necessitates the situation that the slip angle has to increase to draw the required lateral force. The fact that is to be observed is the tire model includes the nonlinearity of the cornering stiffness into its behaviour and causes the whole scenario to become closer to reality.

![Figure 2 Longitudinal force v/s slip ratio for 1000, 1500 and 2000N](image)

Similarly, the graph in Fig. 2 shows the variation of longitudinal forces as a function of slip ratio for varying normal loads.

**2.2. Vehicle Modelling**

The vehicle model is developed for the following conditions.

**2.2.1. Model for Longitudinal Dynamics**

![Figure 3 Vehicle model](image)
Fig. 3 shows the model for straight line dynamics with the location of centre of gravity, and the dimension of wheel base. And the equation 1 and 2 are derived considering the dynamic equilibrium condition to calculate the dynamics load transfer in the front and rear axle during motion.

\[
W_f = W \left( \frac{c}{l} - \frac{ax}{g} \left( \frac{h}{l} \right) \right) = W_{fs} - \left( W \left( \frac{ax}{g} \left( \frac{h}{l} \right) \right) \right)
\]  

(1)

\[
W_r = W \left( \frac{b}{l} + \frac{ax}{g} \left( \frac{h}{l} \right) \right) = W_{rs} + \left( W \left( \frac{ax}{g} \left( \frac{h}{l} \right) \right) \right)
\]  

(2)

Where,

- \(W_f\)  , load on front wheels
- \(W_r\)  , load on rear wheels
- \(W\)  , total weight of the vehicle
- \(W_{fs}\)  , static load on front wheels
- \(W_{rs}\)  , static load on rear wheels
- \(b\)  , distance of front axle from the CG
- \(c\)  , distance of rear axle from the CG
- \(ax\)  , acceleration of the vehicle
- \(h\)  , height of CG from the ground
- \(l\)  , wheelbase

The pseudo force caused due to the acceleration acts on the centre of gravity of the vehicle that causes a moment about the lateral axis, due to its distance from the ground. Now the following scenario resembles a moment applied on a simply supported beam and therefore this takes care of the dynamic of transfer of the vehicle. But the acceleration is caused by the force generated at the wheels for that, the least force generated has to be identified. The force generated due to the tires’ friction (coefficient of road adhesion) or the force supplied due the output torque by the powertrain to the wheels through transmission system. Finally, the force which propels the car forward is limited by the least of these two forces.

![Figure 4 Tractive effort-speed characteristics [4]](image)

Hence, the transmission also needs to be modelled separately so as to identify the point where the gear shift needs to be done. The driveline modelling especially the gear shifting algorithm depends on the transmission ratios and the vehicle speeds. The basic function of

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transmission system is to bring the ideal power characteristics. The intersection points given in Fig.4 are the best point for a shift in the optimum region as the least amount of tractive force is wasted from the engine.

The transmission also needs to be modelled separately so as to identify the point where the gear shift needs to be done. The driveline modelling especially the gear shifting algorithm depends on the way the transmission ratios are present as well as the velocity the vehicle achieves. [4]

For the above mentioned objective the inputs required for the Simulink model were the following:

- The engine torque curve
- The gear ratios
- The vehicle parameters
- The CG position
- The wheel base
- The aerodynamic effect

The simulator can run a set of iteration for any one parameter being a variable and other parameters being constant this gives it an advantage of finding the best value for that parameter. For example the following Fig.5 is an iteration of the final drive ratio so as to find the ratio which results in the minimum value of time.
And the traction effect on the optimized ratio is obtained and is explained in Fig.6, this clearly indicates variation of longitudinal force as a function of velocity of the vehicle. In other words, it shows at what point of time during motion, what amount of traction to be developed to propel the vehicle forward. The sharp points in torque force curve marked by red depict the shifting points.

The following Fig.7 is an overview of the Simulink module which computes the traction for straight line dynamics of the vehicle.

![Figure 7: Simulink model for traction](image)

The next part of this segment was to find out about the braking capacity of the vehicle and for that the current model computes the deceleration value for a vehicle on a given set of tires. It takes in the static parameters and then using a closed loop keeps on iterating for the solution of the dynamic equation which computes the maximum braking force on a particular load, this provides a decelerating force which is then again used to find the load transfer. The new load transfer provides new loads on each wheel which provides a new set of maximum braking force required. This loop goes on for the front as well as the rear end until the solution converges to at least 5% difference in 2 consecutive iterations. Fig.8 shows the Simulink module which calculates the maximum braking force.

![Figure 8: Simulink Model for maximum Braking](image)
2.2.2. Vehicle model for lateral dynamics

The velocity that a race car can achieve during a turn is limited by the grip available for generating slip angles and yet remains a function of the lateral load transfer. The phenomenon is easily understood by the Milliken Moment diagram Fig.9 [1]. The diagram is a plot between normalized lateral acceleration and normalized yaw moments across a number of vehicle side slip angles and steer angles.

![Figure 9 Milliken Moment Diagram (Cn-Ay) [1]](image)

As can be seen from Fig.9, each and every slip angle has to be addressed in a form of steer angle and a body side slip angle. That would then help us in finding the exact point on the graph at any moment of the vehicle’s maneuver. In other words, for a given steering input, the side slip generates accordingly. So, as the vehicle enters a curve it travels along the constant beta line ($\beta=0$) which is the vehicle side slip angle as illustrated in Fig.10. Vehicle side slip angle is defined by the angle included between the vehicle heading direction and its actual direction of travel which is dictated by the path it follows. When the vehicle just enters a turn there is no side slip because right now only the front wheels have been steered. As the front wheels are steered a moment as well as acceleration is generated. Then increasing the steer gives rise to the moment start giving an effect, the side slip is generated which varies across a constant steer line to reach a steady-state value where there is no yaw moment. Thus, the grip available and the corresponding force generated on each end depend on the way the vehicle is modelled. The whole modelling process revolves around the modified bicycle model.

![Figure 10 Bicycle model [1]](image)
Figure 10 shows the bicycle model that represents the collapsed wheels in the front and the rear with basic assumption neglecting the effect of suspension compliance and hence the roll. But this model is good enough to address the directional control and stability. With this longitudinal weight transfer model, a lateral load transfer model shown in Fig.11 is included additionally in the current work.

![Figure 11 Single mass weight transfer model [5]](image)

The calculation of lateral load transfer involves the whole vehicle roll stiffness, which is dictated by the stiffness of springs and anti-roll bar. The anti-roll bar is split into two torsional springs working in parallel. And therefore, the lateral load transfer calculated is directly proportional to the roll angle subtended by the sprung mass about the roll axis. This assumption also has to take into account that the two torsional springs are connected by a chassis which has a finite torsional stiffness of its own. The following equation 3 and 4 taken from [5] illustrates front and rear lateral load transfer.

\[
\nabla F_{zf} = \left( \frac{m}{l_f} \right) \left( \left( \frac{k_f}{k_f + \left( \frac{k_c}{k_f+k_c} \right)} \right) \frac{b}{l} d_f \right) + \left( \left( \frac{k_f \left( \frac{k_c}{k_f+k_c} \right)}{k_f + \left( \frac{k_c}{k_f+k_c} \right)} \right) \frac{a}{l} d_r \right) + \frac{b}{l} z_f
\]

\[
\nabla F_{zr} = \left( \frac{m}{l_f} \right) \left( \left( \frac{k_r}{k_r + \left( \frac{k_c}{k_r+k_c} \right)} \right) \frac{a}{l} d_r \right) + \left( \left( \frac{k_r \left( \frac{k_c}{k_r+k_c} \right)}{k_r + \left( \frac{k_c}{k_r+k_c} \right)} \right) \frac{b}{l} d_f \right) + \frac{a}{l} z_r
\]

where,

- \(m\), mass of vehicle
- \(l\), wheelbase
- \(a\), distance of front axle from the CG
- \(b\), distance of rear axle from the CG
- \(k_f\), front roll stiffness
- \(k_r\), rear roll stiffness
- \(k_c\), chassis roll stiffness
- \(d_f\), distance between front mass height and front roll center height
- \(d_r\), distance between rear mass height and rear roll center height
- \(z_f\), front roll center height
- \(z_r\), rear roll center height
- \(a_y\), lateral acceleration
- \(\Delta F_{zf}\), front lateral load transfer
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ΔF_{sr}, rear lateral load transfer

The lateral acceleration model was built using the same equations 3 and 4 for weight transfer and extensively analyzed by [5] and in order to relate the force generated directly to the body side slip angle as well as steer angle equation 5 and 6 were used:

\[ \alpha_f = \beta + \frac{a}{R} - \delta \]  \hspace{1cm} (5)

\[ \alpha_r = \beta - \frac{b}{R} \]  \hspace{1cm} (6)

Where,

\( \alpha_f \), front slip angle
\( \alpha_r \), rear slip angle
\( R \), radius of turn
\( \delta \), steer angle
\( \beta \), body side slip angle

We also assume that the left and right wheels are making the same slip angles. The Milliken moment diagram is plotted to represents the variation of yaw moment as a function of lateral acceleration for various runs varying body slip and steering angle. On 10m radius curvature for two different biased normal load on rear axle simulations were performed and the observation are given below the figures 12 and 13 respectively.

![Figure 12 Cn-Ay graph for 53% rear load](image)

![Figure 13 Cn-Ay graph for 30% rear load](image)
The maximum lateral acceleration obtained was 1.62 g. The normalized yaw moment at this maximum lateral acceleration was +0.01 and maximum lateral acceleration at 0 yaw moment was 1.60 g. The above was the performance for a 10m radius on 53% biased normal load on rear. The time taken is 4.9876 seconds.

The maximum acceleration was 1.61 g, normalized yaw moment at maximum lateral acceleration was -0.03 and maximum lateral acceleration at 0 yaw moment was 1.55 g. The above was the performance for a 10m radius on 30% biased normal load on rear. The vehicle has now become understeered with negative yaw moment and a time of 5.0896 seconds. So as is evident a change of 20% static load distribution caused the car to change its characteristics.

![Figure 14](image1.png)

**Figure 14** Maximum lateral acceleration as a function of front stiffness ratio

Then the batch runs were done to estimate maximum lateral and yaw moment responses. This is a significant advantage of the proposed simulator. Fig.14 represents a batch run for different values of stiffness of the front end for its maximum lateral acceleration which can be attained. Fig.15 gives the variation of normalized yaw moment variation at the maximum lateral acceleration as a function of front end stiffness ratio.

![Figure 15](image2.png)

**Figure 15** Variation of Yaw moment at maximum lateral acceleration as a function of front end stiffness ratio

### 2.3. Map Tracking

Map tracking refers to a vehicle’s motion geometry. It requires kinematics of the vehicle. The final aspect of the simulation development is to create an algorithm where a vehicle is assumed to be a point mass as it is important to compute the lap trajectory. It can move on a predefined map and predict ahead the maximum velocity which can be reached on turns. It computes the least time to brake and the earliest time to accelerate on any segment of the map. The Logic given in [6] is used, where it runs initially with the maximum possible speed
and does a comparative analysis on the curved segment between the speed of the vehicle and
the maximum allowable speed and chooses the lesser of the two to be the actual speed. Then
considering this velocity as final velocity it computes the shortest braking distance required to
reach this velocity.

The algorithm to find the radius of curvature from a set of three points is illustrated in
Fig.16 and is calculated using the equation 7. As already mentioned the simulator would
compute the maximum entry velocity of every segment and as soon as the exit velocity of the
previous segment exceeds the entry velocity of the next segment. The simulation would
backtrack by then assuming the velocity to be reached after braking being already known.

\[
\cos A = \frac{(b^2 + c^2 - a^2)/2bc}{\cos A}
\]

\[
P = \frac{(360 - 2A)}{2} = 180 - A
\]

\[
\sin P = \frac{(\frac{1}{2}) a}{R}
\]

\[
R = \frac{a}{(2 \sin P)} = \frac{a}{(2 \sin (180 - A))}
\]

This establishes a procedure for an arbitrary lap on which batch runs were conducted. The
following section presents the results obtained from these batch runs by the proposed
simulator.

3. RESULTS
To understand the influence of wheel base, the incremental variation in wheelbase is given for
the first batch run of straight motion of the car. The variation of acceleration time as a
function of wheelbase is plotted. The graph in Fig.17 refers to these acceleration runs of 75m
long path. As expected acceleration is faster with shorter wheelbase as more load transfer
helps in gaining more tractive force but the important point is that the trend is not linear that
means a variation in wheelbase on the shorter end gives more return in acceleration
performance of a rear wheel drive car than a change of wheelbase on the longer wheelbase
end.
The second batch of iterations is carried out for the normal load distribution from 10% to 90% front biased for the different maps and the variations are shown in Fig.18.

In acceleration scenario having all the weight on the rear is the best condition which is evident. Yet here also the same effect of diminishing returns can be seen.

Third batch run is to determine the lap time and its variation as a function of rear weight fraction. For this study lap run on a 15m radius circle was considered. It is observed that the a weight bias of 50-50 is the best for turns and is observed in Fig.19. Fourth batch run was conducted to predict the influence of chassis stiffness on the 15m radius circle. It is evident from Fig.20 that the chassis effect is not significant hence the effect is negated. The result
clearly shows that the torsional stiffness directly affects the lateral load transfer and hence increasing the torsional stiffness decreases the lap timings.

Figure 20 Lap time as a function of chassis torsional stiffness

4. CONCLUSIONS
The proposed lap-time simulator includes tire modelling, vehicle modelling and map tracking. Tire modelling was done using Magic Formula version 5.2, combined bi-cycle and roll models were used in vehicle modelling and an algorithm developed for map tracking considering the vehicle to be the point mass. There were four batch runs were conducted to predict the lap time variation due to the wheelbase, the roll stiffness distribution, the normal load distribution, the final drive ratio and the chassis torsional stiffness. Though there are established techniques of higher degree of freedom models to represent the whole vehicle and availability of commercial software to simulate the racing laps, the current attempt given the good experience to understand the hot spots of intended objective through reading and understanding the classical text books.

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