A FINITE DIFFERENCE APPROACH TO PRESSURE DISTRIBUTION ON FIXED PAD THRUST BEARING UNDER ISOTHERMAL CONDITION

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ABSTRACT

Hydrodynamic pad thrust bearings are widely used in high speed rotating machines such as pumps, compressors, turbines, turbo generators etc. because of their low friction, good load carrying capacity and high damping characteristics. The pressure distribution on the pad is given by solving the Reynolds’ equation. The solution has been obtained by Finite Difference Method. The results have been validated by solving the Reynolds’ equation for extreme cases.

The results obtained from the numerical method agrees well with those obtained by analytical method for the limiting cases of infinitely long radial dimension compared to circumferential dimension. The solution obtained by Finite Difference method has been validated with analytical results for the limiting cases.

Key words: Reynolds’ equation; Hydrodynamic pad thrust bearings; Finite differences; Hydrodynamic lubrication.


1. INTRODUCTION

Thrust pad bearings as shown in Fig. 1 are commonly used in turbines, ship’s propeller etc where thrust comes on the shaft in axial direction. Application of numerical computational methods to bearing design has been performed by several research worker. Gropper D., Wang L., and Harvey Terry J.[1] have presented a review paper where they have discussed the effects of different surface texturing and its effect on hydrodynamic lubrication. Similarly, Wasileczuk M., Rotta G. [2] have discussed as how to model the flow of lubricant in a hydrodynamic bearing. Bouyahia F., Hajjam M., Khlifi M El., and Souchet D. [3] have assumed the non-Newtonian nature of lubricants to model the flow. Sinha A. N., Athre K. and Biswas S.[4] and Ashour et al.[5] have discussed a mathematical model to estimate elastic distortion of a thrust pad on an elastic support by solving Reynolds’ and elasticity equations.
simultaneously. They compared results for cases where film thickness varies only along the
direction of flow. The objective of the present study is to numerically solve Reynolds’ equation
using FDM. While solving this equation the properties of the lubricant such as viscosity,
thermal conductivity etc. has been assumed independent of temperature. Moreover the film
thickness considered in the present work takes into account its variation both along the
direction of flow and across the direction of flow. The results obtained by FDM has been
validated with those obtained by the analytical solution of the Reynolds equation for the
limiting cases of infinite dimension across the direction of flow and infinite dimension along
the direction of flow.

2. METHODOLOGY

2.1. Film thickness equation
For sector pad geometry, under isothermal condition the film thickness is a function of both
circumferential and radial coordinates.

The compact film thickness expression reported by Etsion considers variation in
tangential and radial directions both and is given by:

\[ h = h_p + \gamma r \sin(\theta_p - \theta) \]  

The non-dimensional form of thickness given by Eq. (1) can be written by using \( H = h/h_p, R = r/R_2, \varepsilon = \gamma R_2/h_p \) in Eq. (1):

\[ H = 1 + \varepsilon R \sin(\theta_p - \theta) \]  

2.2. Reynolds equation:
The generalized Reynolds’ equation in cylindrical polar coordinates is given by:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( rh \frac{\partial p}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left( h^2 \frac{\partial p}{\partial \theta} \right) = 6\eta r \omega \frac{\partial h}{\partial \theta} \]  

We can write the non-dimensionalised Reynolds equation in the following form:
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\[
\frac{\partial}{\partial R}\left(RH^3 \frac{\partial P}{\partial R}\right) + \frac{1}{R} \frac{\partial}{\partial \theta} \left(H^3 \frac{\partial P}{\partial \theta}\right) = 12\pi R \left(\frac{R_i}{L}\right)^2 \frac{\partial H}{\partial \theta}
\]

(4)

Using central differencing the entire lubricant flow domain can be discretized as shown in Fig. III, therefore the above equation can be written as:

\[
P_{i,j} = \frac{(\Delta \theta)^2 (AA3 - AA4 + AA2)}{AA1} P_{i,j+1} + \frac{(\Delta \theta)^2 (AA4 - AA3 + AA2)}{AA1} P_{i,j-1} + \frac{(\Delta R)^2 (AA5 + AA6)}{AA1} P_{i+1,j} + \frac{(\Delta R)^2 (AA6 - AA5)}{AA1} P_{i-1,j} + 6\pi P_{i,j} \frac{(\Delta R)^2 (\Delta \theta)}{AA1} \left(\frac{R_i}{L}\right)^2 \left(H_{i,j-1} - H_{i+1,j}\right)
\]

(5)

where,

\[
AA1 = 2H^3 i,j \left(\frac{R_{i,j}(\Delta \theta)^2 + (\Delta R)^2}{R_{i,j}}\right), \quad AA2 = \frac{(RH^3)_{i,j}}{4}, \quad AA3 = \frac{(RH^3)_{i,j}}{4}, \quad AA4 = \frac{(RH^3)_{i,j-1}}{4}, \quad AA5 = \frac{H^3}{R_{i,j}}, \quad AA6 = \frac{H^3}{4R_{i,j}}
\]

Algorithm for calculation of pressure profile:

Step1: Initial values of pressure at all nodal points are assumed to be zero.

Step2: Viscosity is assumed to be constant as isothermal condition has been considered.

Step3: The film thickness values are obtained from the standard expression proposed by Etsion.

Step4: The pressure values at all grid points are obtained from Reynolds equation using convergence criteria.

Convergence Criteria for Pressure:

\[
\sum_{i=0}^{n} \sum_{j=1}^{m} \left|\frac{P_{i,j}^{k+1} - P_{i,j}^{k}}{P_{i,j}^{k}}\right| < \varepsilon_p
\]

Figure 3 Grid for a Sector Pad
where, \( k \) = no. of iteration

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>SYMBOL</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pad extent angle in radians</td>
<td>( \beta )</td>
<td>( \pi/6 )</td>
</tr>
<tr>
<td>Pad thickness in m.</td>
<td>( t_p )</td>
<td>0.097</td>
</tr>
<tr>
<td>Inner radius of the pad in meter</td>
<td>( R_1 )</td>
<td>0.7</td>
</tr>
<tr>
<td>Outer radius of the pad in meter</td>
<td>( R_2 )</td>
<td>1.2</td>
</tr>
<tr>
<td>Nodes in theta direction (tangential direction)</td>
<td>( n )</td>
<td>10</td>
</tr>
<tr>
<td>Nodes in radial direction</td>
<td>( m )</td>
<td>10</td>
</tr>
<tr>
<td>Min. film thickness in meter</td>
<td>( a =R_2/2.38 )</td>
<td>0.5042</td>
</tr>
<tr>
<td>Max. film thickness/min. film thickness</td>
<td>( (a+b)/a )</td>
<td>2.19</td>
</tr>
<tr>
<td>Tilt parameter</td>
<td>( \epsilon =\gamma R_2/ h_P )</td>
<td>0.74732</td>
</tr>
<tr>
<td>Tilt about pitch line in radians</td>
<td>( \gamma )</td>
<td>2</td>
</tr>
<tr>
<td>Dynamic co-efficient of viscosity of SAE 30 lubricant at 50(^{\circ})C (NS/ m(^2))</td>
<td>( \eta )</td>
<td>0.04305</td>
</tr>
<tr>
<td>Specific gravity of SAE 30 lubricant at 50(^{\circ})C</td>
<td>( S )</td>
<td>0.898</td>
</tr>
</tbody>
</table>

3. RESULTS

**Figure 4** Pressure Distribution At Mid-radial Plane

ANALYTICAL SOLUTION OF THE REYNOLDS EQUATION FOR THE LIMITING CASE OF INFINITE DIMENSION ACROSS THE DIRECTION OF FLOW [Table 1]. [Fig. II].

Assuming, dimension across the direction of flow >> dimension along the direction of flow
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Dimension across the lubricant flow is very much greater than the dimension along the lubricant flow as shown in Fig 2. Hence variation of pressure across the flow of lubricant may be neglected. That is,
\[ \frac{\partial p}{\partial r} = 0; \]
We can assume:
\[ \frac{L}{(R_2 + R_1)\beta} = k \]

But here, \( L = R_2 - R_1 \)  Therefore, \( \frac{R_2 - R_1}{(R_2 + R_1)\beta} = k \)

Or, \( R_2 = \frac{(2 + k\beta)R_1}{(2 - k\beta)} \)

Clearly, \( k\beta < 2 \)  \( \Rightarrow \beta < \frac{2}{k} \)  (6)

For a given inner radius of the pad, \( R_1 \)
If we take, \( k = 20 \)  then \( \beta < \frac{2}{20} \)  \( \Rightarrow \beta < 0.1 \)

Hence, \( \beta = 0.099 \) radian
(assumed)

Putting, \( R_1 = 0.7 \) m, \( \beta = 0.099 \) rad, and \( k = 20 \) in eq. (6) we have \( R_2 = 139.3 \) m

The non-dimensional form of the Reynolds’ equation reduces to:

\[ \frac{1}{R} \frac{\partial}{\partial \theta} \left( H^2 \frac{\partial p}{\partial \theta} \right) = 12\pi R \left( \frac{R_2}{L} \right)^2 \frac{\partial H}{\partial \theta} \]

Where,
\[ H = \frac{a}{b} + (1 - \theta / \beta) \]

Or, \( \frac{\partial}{\partial \theta} \left( H^2 \frac{\partial p}{\partial \theta} \right) = 12\pi R^2 \left( \frac{R_2}{L} \right)^2 \frac{d\left\{ \frac{a}{b} + (1 - \theta / \beta) \right\}}{d\theta} \)  \( \theta \) (7)

Integrating both sides of eq. (7):

\[ P = 12\pi R^2 \left( \frac{R_2}{L} \right)^2 \left( \frac{1}{\beta} \right) \left( \frac{\beta \theta}{2H^2} - \frac{\beta^2}{2H^2} \left( 1 + \frac{a}{b} \right) + \frac{\beta^2}{H} + C_2 \right) \]

Now, imposing the below boundary condition we can write:

Boundary conditions: (i) At \( \theta = 0 \) \( \Rightarrow H = \frac{a}{b} + 1 \)  \( P=0; \)

(ii) At \( \theta = \beta \) \( \Rightarrow H = \frac{a}{b} \)  \( P=0; \)

Imposing the above boundary conditions we get

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\[ C_2 = -\frac{\beta^2}{\left(1 + \frac{2a}{b}\right)} \]

and

\[ \beta = \frac{\beta \left(1 + \frac{a}{b}\right)}{\left(1 + \frac{2a}{b}\right)} \]

**Figure 5** Comparison of Pressure Distribution at mid-radial distance of a Pad with Very Large Radial Extent (numerical and analytical solutions)

### 4. CONCLUSION

Fig. IV shows the plot of pressure distribution on the thrust pad along the direction of flow at mid radial plane. Fig. V shows that the numerical result agrees well with the analytical results of an infinite bearing in radial direction.

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Definition</th>
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</thead>
<tbody>
<tr>
<td>a</td>
<td>minimum film thickness, m</td>
</tr>
<tr>
<td>b</td>
<td>amount of taper, m</td>
</tr>
<tr>
<td>h₁</td>
<td>inlet film thickness, m</td>
</tr>
<tr>
<td>h₂</td>
<td>outlet film thickness, m</td>
</tr>
<tr>
<td>bₚ</td>
<td>film thickness along the pitch line, m</td>
</tr>
<tr>
<td>h</td>
<td>film thickness at a radius r, m</td>
</tr>
<tr>
<td>H</td>
<td>non-dimensional thickness at a radius r</td>
</tr>
<tr>
<td>P</td>
<td>dimensionless pressure</td>
</tr>
<tr>
<td>p</td>
<td>hydrodynamic pressure, N/m²</td>
</tr>
<tr>
<td>R</td>
<td>coordinate in the radial direction, m</td>
</tr>
<tr>
<td>R₁</td>
<td>inner radius of the pad, m</td>
</tr>
<tr>
<td>R₂</td>
<td>outer radius, m</td>
</tr>
<tr>
<td>β</td>
<td>pad extent angle, radians</td>
</tr>
<tr>
<td>ω</td>
<td>angular velocity of runner and shaft, radians/s</td>
</tr>
<tr>
<td>ρ</td>
<td>density of the lubricant at p, kg/m³</td>
</tr>
<tr>
<td>γ</td>
<td>tilt about pitch line</td>
</tr>
<tr>
<td>θₚ</td>
<td>angular extent of the pitch line of the pad, radians</td>
</tr>
<tr>
<td>x</td>
<td>coordinate in X axis direction</td>
</tr>
<tr>
<td>y</td>
<td>coordinate across the oil film, m</td>
</tr>
<tr>
<td>z</td>
<td>coordinate along the axial direction, m</td>
</tr>
<tr>
<td>Θ</td>
<td>polar coordinate across the rotation of the runner, radians</td>
</tr>
</tbody>
</table>
REFERENCES


