STRUCTURAL ANALYSIS OF SPUR GEAR USING ANSYS WORKBENCH 14.5

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ABSTRACT

Gear is basically a toothed wheel. But, it is one of the most important machine elements in mechanical power transmission system. It has a wide range of applications starting from wrist watch to heavy industries due to its high degree of reliability and compactness. Reckoning the bending stress and total deformation of gear tooth is considered to be the paramount objective for modern gear design industry. The present work is an attempt to estimate the root-bending stress and total deformation of a spur gear tooth. Dimensions of the gear tooth are taken from practical understanding and is analysed for different torque specifications of the three existing vehicle-models of Maruti Suzuki using ANSYS (Workbench R14.5) software considering structural steel as tooth material. The present study also concentrates on the manufacturing of gear based on composite materials at the critically stressed section with the help of software based analysis. We suggest that, generative manufacturing or any other forms of layer-by-layer manufacturing methods could be employed for the generation of precision gears in this regard. Validation of the cantilever beam concept and Castigliano’s theorem associated with the analytical calculation of gear design, has also been contemplated in the present work.

Key words: Bending stress, Total deformation, Cantilever Beam, Castigliano’s Theorem, CATIA V5R20, ANSYS (Workbench R14.5)
1. INTRODUCTION

Gear is the most common means of power transmission system in modern mechanical engineering world. It is a positive drive maintaining constant velocity ratio and possesses high efficiency in transmission. Among the various types gears spur gear with involute profile is the most simplest considering the design and manufacturing cost [1]. In this paper static analysis of the spur gear in the existing automobile gear box has been discussed with various torque specifications. The modern power transmission system requires highly efficient gear, for that the evaluation of stress and deflection of the tooth is vital [2]. Research work on this field revealed that for determining the practical strength the compressive and shear stresses on the tooth are neglected as their effects are marginal as compared to the bending stress [3]. This paper also establishes the above concept for stress on the gear tooth under mating and rotating condition. Apart from that, the study of stress on the tooth is important to predict and estimate the critical value beyond which the gear would fail. A pair of teeth in action generally fail by two types of stresses; namely bending fatigue due to bending stress at the root of the tooth and the another one is the contact fatigue which is a surface failure. Highest stress occur at two locations i.e. the point at which the force acts and the root of the tooth [4]. A lot of research works have been carried out on this gearing technology considering its material, geometrical parameters, manufacturing processes and some are going on for the betterment. S. Kumar, K. Mishra, J. Madan [5] presented the FEA method to analysis the stress on tooth and established that the bending stress and contact stress are major sources of gear failure. Kailash C.Bhosale [6] in his paper a comparison study has been presented between the lewis equation and the FEA method for calculating the bending strength in case of helical gear with straight tooth. Shubham et al., [7] have worked out the static and dynamic analysis of a single tooth of spur gear using ANSYS 14 considering the velocity factor approximation. Extensive research has been carried out for reduction of root stress and for that attention has taken to the tooth fillet, the location where the fracture initiates [2]. This work has been extended to evaluate the tooth strength of involute profile considering the asymmetric [8] and symmetric [9] gear with static and dynamic loading [10]. A reduction of bending stress has also been established by introducing the circular root fillet in comparison to the standard root fillet [11]. A number of research studies have been carried out in this context to replace the material of the gear to enhance the loading capacity by reducing the stress and total deformation induced on the tooth along with the reduction the overall weight of an automobile Anuj Nath et al.,[12] in their work, have shown the design and analysis of a composite spur gear made of 50% carbon fibers in epoxy resin matrix and compared to steel gear, 0.5324% lesser stress and 55.619% reduction in total deformation have been found for the composite gear.

Study has been carried out on the choice of gear tooth profile and is found that only involute and cycloidal curves satisfy the law of gearing [13]. Involute gears have certain advantages over the cycloidal gears like varying centre distance with constant velocity ratio during matting, constant pressure angle with less wear and finally ease of manufacturing. The only problem with the involute profile is the interference. Although the cycloidal gear is not totally obsolete. It is used in spring driven watches, in some instruments [1]. The most common gear-cutting processes include hobbing, broaching, milling, and grinding. Such cutting operations may occur either after or instead of forming processes such as forging, extruding, investment casting, or sand casting. The present course of study concentrates on the manufacturing of gear based on composite materials at the critically stressed section with the help of software based analysis. Generative manufacturing or any other forms of layer-by-layer manufacturing methods could be employed for the generation of gear in this regard.
2. MATHEMATICAL FORMULATION

2.1. Assumption of Lewis Equation

The analysis of bending stress in gear tooth was done by Mr. Wilfred Lewis in his paper, ‘The investigation of the strength of gear tooth’ submitted at the Engineers club of Philadelphia in 1892. Even today, the Lewis equation is considered as the basic equation in the design of gears [1]. Lewis considered gear tooth as a cantilever beam with static normal force F applied at the tip. Assumptions made in the derivation are [16]:

1. The full load is applied to the tip of a single tooth in static condition.
2. The radial component is negligible.
3. The load is distributed uniformly across the full face width.
4. Forces due to tooth sliding friction are negligible.
5. Stress concentration in the tooth fillet is negligible.

The Figure 1 shows clearly that the gear tooth is stronger throughout than the inscribed constant strength parabola, except for the section at ‘a’ where parabola and tooth profile are tangential to each other [16].

![Figure 1](image1.png)
![Figure 2](image2.png)

Figure 1 Gear tooth as a cantilever beam [16].

Figure 2 Parabolic gear tooth [7].

In the above Figure 2 the following notations are used: F is the Full load, F_r and F_t are the Radial and Tangential component of the full load. h, b and t are the height, face-width and thickness of the tooth at critical section respectively.

2.2. Design Specification

For calculating bending stress and total deformation we have taken a standard model for designing the spur gear tooth [17] and different torque specification from the existing vehicle-models of Maruti Suzuki [18, 19, 20]. The following data is given for the design of 20° full depth spur gear made of structural steel transmitting torque at different rpm:

http://www.iaeme.com/IJMET/index.asp 134 editorial@iaeme.com
The designing parameters of the gear are taken from existing automobile gear box model and others parameters can be found from the standard module (m). Structural steel was considered as the tooth material with an elastic modulus $E = 2.1 \times 10^5$ MPa, tensile yield strength $S_{yt} = 250$ MPa and Poisson’s ratio=0.3 while performing the analysis in ANSYS (Workbench 14.5) software.

### Calculation of Bending Stress [1]

In the current analysis of bending stress of tooth we consider the Lewis assumption as discussed above in 2.2. From the Figure 2 at point ‘a’

Bending moment $M_b = F_t \times h$  

Area moment of inertia $I = \frac{b \times t^3}{12}$  

Then the bending stress is given by $\sigma_b = \frac{6 \times F_t \times h}{b \times t^2}$ ; from the Eq. (1) and Eq. (2). After the rearranging we have

$$F_t = b \times \sigma_b \times \left(\frac{t^2}{6 \times h}\right)$$  

Multiplying numerator and denominator by module (m) from the Eq. (3) we have the tangential component of the force is given by

$$F_t = m \times b \times \sigma_b \times \left(\frac{t^2}{6 \times h + m}\right)$$  

$Y = \left(\frac{t^2}{6 \times h + m}\right)$ is known as Lewis form factor. Equation. (4) Can be rewritten as

$$F_t = m \times b \times \sigma_b \times Y$$  

When the tangential force increased the stress also increases. When the stress reaches the permissible magnitude of bending stress the corresponding force $F_t$ is known as Beam strength and denoted by $S_b$. So replacing $F_t$ in the Eq. (5) we have

$$S_b = m \times b \times \sigma_b \times Y.$$  

### Calculation of Total Deformation [7]

It is observed that the cross section of the gear tooth varies from free end to the fixed end. Lewis has assumed it as a constant strength parabola. Using Castiglione’s Theorem total deformation of the tooth can be found with minor error. For linearly elastic structure, where external forces only cause deformations, the complementary energy is equal to the strain energy. For such structures, the Castigliano’s first theorem may be stated as the first partial derivative of the strain energy of the structure with respect to any particular force gives the displacement of the point of application of that force in the direction of its line of action [21]. The theory applies to both linear and rotational deflection $\delta = \frac{SU}{\delta F}$. It should be clear that Castiglione’s theorem finds the deflection at the point of application of the load in the direction of the load.

Here $U$ is the strain energy given by $U = \int_0^L \frac{M^2}{2 \times E \times I} \, dx$, where $M$ is the moment due to the load. Consider the parabolic tooth of height $h$ and tooth thickness $t$. The equation of parabola $y^2 = 4a \times x$. Consider the Fig 2.
We have the following boundary condition at x = h, y = t/2. After substituting the equation of the parabolic tooth is $y^2 = \frac{t^2}{4h^2}$ and $y^3 = \left(\frac{t}{2}\right)^3 \ast \left(\frac{x}{h}\right)^{1.5}$.

Putting the value of $M = F_t \ast x$, $I = \left(\frac{2}{3}\right) \ast b \ast y^3$, the strain energy $U = \int_0^h \frac{M^2}{2E\ast l} \, dx$ will be

$$U = \int_0^h \frac{(F_t \ast x)^2}{2E\ast \left(\frac{2}{3}b\ast y^3\right)} \, dx$$

from this we have $U = \frac{8\ast F_t \ast h^3}{E\ast b\ast t^3}$  \(\text{(7)}\)

Again we know deflection is given by $\frac{\delta U}{\delta F}$. From Eq. (7) we have the $\delta = \frac{16\ast F_t \ast h^3}{E\ast b\ast t^3}$  \(\text{(8)}\)

This Eq. (8) is the equation of tooth deflection of spur gear when tangential load $F_t$ is applied at the tip of the tooth.

2.3. Analytical Calculation

From the relation of maximum bending stress ($\sigma_b$) and deflection($\delta$) based on the above design specifications from the Table 1, and considering steel as a tooth material the analytical calculation is carried out for the torque of 132 N-m at 3000 rpm is carried out.

\underline{2.3.1. Calculation for Bending Stress}

Consider torque $T = 132$ N-m at 3000 rpm. The tangential load $F_t$ can be found from the below:

$F_t = \frac{2\ast T}{d}$ ; where d is the pitch circle diameter. $F_t = \frac{2\ast 132}{180\ast 10^{-3}} = 1466.67$ N \text{-} m. Number of teeth in both gear and pinion is 18, thickness $t = 15.71$ mm. face-width $b = 54$ mm. Lewis form factor $Y = 0.308$ [22]. The value of bending stress is given by from Eq. (5) is $\sigma_b = \frac{F_t}{(m\ast b\ast Y)}$ ; Then the theoretical bending stress is given by $\sigma_b = \frac{1466.67}{(10\ast 54\ast 0.308)} = 8.818$ MPa. Ultimate tensile strength of gear material is 460 MPa. Considering factor of safety as 3, then the allowable bending stress is 153.33 MPa $> 8.818$ MPa. So, the design is safe. As the gear and pinion are identical so there is no question of checking the following relation i.e. strength of gear $<$ strength of pinion.

\underline{2.3.2. Calculation of Deflection}

Consider the Eq. (8) and the value of $h$ can be found from the relation $Y = \frac{t^2}{(6\ast h \ast m)}$, putting the required value we have $h = 13.33$ mm. Now the value of deflection $\delta = \frac{16\ast 1466.67\ast 13.33^3}{200000\ast 54\ast 15.73} = 0.0013$ mm (approx.)

Subsequently the bending stresses and deflections are calculated for the torque of 192 N-m at 2000 rpm and 225 N-m at 4000 rpm and all are presented in the following Table 2.

\underline{Table 2 Theoretical stress and deflection for various vehicle model at specified torque condition.}

<table>
<thead>
<tr>
<th>Vehicle model of Maruti Suzuki</th>
<th>Torque (N-m)</th>
<th>Tangential load (N)</th>
<th>Theoretical stress (MPa)</th>
<th>Theoretical Deflection(mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baleno 132 @ 3000 rpm</td>
<td>1466.67</td>
<td>8.81</td>
<td>0.0013</td>
<td></td>
</tr>
<tr>
<td>Swift Dzire 190 @ 2000 rpm</td>
<td>2111.11</td>
<td>12.69</td>
<td>0.0019</td>
<td></td>
</tr>
<tr>
<td>Grand Vitara 2.4. 225 @ 4000 rpm</td>
<td>2500.00</td>
<td>15.03</td>
<td>0.0022</td>
<td></td>
</tr>
</tbody>
</table>

It is to be noted here that all the above theoretical values are compared with the software based result for the validation of Lewis equation.
3. STATIC ANALYSIS
The aim of this analysis is to investigate the bending stresses and the deflection on the tooth of a spur gear within the desirable limits to obtain a practical validation for the theoretical results. After geometric modeling in CATIA V5R18 software the gear is subjected to static analysis, performed in ANSYS (Workbench 14.5) software. The computer compatible mathematical description of the geometry of the object is called geometric modeling. CATIA is basically CAD (computer-aided design) software that allows the mathematical description of the object to be displayed and manipulated as the image on the monitor of the computer [23], whereas, ANSYS is an engineering simulation software that predicts with confidence about the performance of the product under the real-world environments incorporating all the existing physical phenomena [24]. The layout of static analysis involves meshing, boundary conditions and loading. The imported geometry is shown below in the Figure 3.

3.1. Meshing
Meshing is basically the division of the entire model into small cell so that at each and every cell the equations are solved. It gives the accurate solution and also improves the quality of solution [25]. Here the element size of 1 mm with medium smoothing is considered for mesh generation. Minimum edge length of the elements is 2.886 mm. Within the solution domain under the Adaptive Mesh Refinement segment, the Max. Refinement Loops is taken as 1 and Refinement Depth as 2. Within the Patch Confirming Method domain the method is taken as Tetrahedrons. For the convergence plot, the maximum allowable change was considered as 4%. The whole geometry is selected for mesh generation and total number of nodes and elements are observed as 51818 and 31133 respectively. Fig.4 (a).and Fig.4 (b) show the meshed geometry and boundary conditions.

3.2. Boundary Conditions
Based on the assumptions of Lewis equation, the boundary conditions are set in ANSYS Workbench. The fixed support is used at the root end of the tooth and the force is applied on the face having components in Y and Z directions. In the following Fig. 5(b) the tangential force ($F_t$) having magnitude 1466.67 N has been introduced with component at Y and Z direction as 1378.2 N and 501.6 N respectively.

4. RESULT AND DISCUSSION
Considering the actual physics to the CAD generated model in ANSYS the Bending stress (known as Equivalent Von Mises stress) and the deflection (known as Total Deformation) has been evaluated. All these are presented in the following figures.
Here the theoretical and ANSYS based results are compared for three distinct transmission specifications in connection with the different vehicle models of Maruti Suzuki in the following Table 3. A good agreement has been found between the theoretical bending stress and the ANSYS based Von Mises stress. Hence it validates the concept of cantilever lever beam drawn from Lewis’ assumption during the stress analysis of spur gear tooth. We further conclude that the maximum stress developed on the tooth root is well within the safe limits considering the factor of safety of 3. It has been observed that with the higher torque conditions the % of error becomes lesser and lesser.

Table 3 Comparison between Theoretical stress and ANSYS based Von Mises stress for various vehicle models at specified torque conditions.

<table>
<thead>
<tr>
<th>Vehicle model of Maruti Suzuki</th>
<th>Torque (N m)</th>
<th>Theoretical Bending stress(MPa)</th>
<th>ANSYS based Von Mises stress(MPa)</th>
<th>% Accuracy</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baleno</td>
<td>132 @ 3000 rpm</td>
<td>8.81</td>
<td>8.80</td>
<td>99.89</td>
<td>0.114</td>
</tr>
<tr>
<td>Swift Dzire</td>
<td>190 @ 2000 rpm</td>
<td>12.69</td>
<td>12.68</td>
<td>99.92</td>
<td>0.078</td>
</tr>
<tr>
<td>Grand Vitara 2.4.</td>
<td>225 @ 4000 rpm</td>
<td>15.03</td>
<td>15.02</td>
<td>99.93</td>
<td>0.067</td>
</tr>
</tbody>
</table>

In the similar way the theoretical deflection and ANSYS based total deformation has been depicted through the following figures at various torque specifications.

Figure 5 Distribution of equivalent (Von Mises) stress of spur gear tooth at a torque of (a) 132 N-m, 3000 rpm, (b) 190 N-m, 2000 rpm, (c) 225 N-m, 4000 rpm

Figure 6 Distribution of total deformation of spur gear tooth at a torque of (a) 132 N-m, 3000 rpm, (b) 190 N-m, 2000 rpm, (c) 225 N-m, 4000 rpm
Here the theoretical and ANSYS based results are compared with different transmission specification of the three different vehicle models of Maruti Suzuki in the following Table 4. It has been noticed that, the theoretical deflections by Castigliano’s theorem and the ANSYS based total deformations are close to each other.

**Table 4** Comparison between Theoretical deflection and ANSYS based Total deformation for various vehicle models at specified torque conditions.

<table>
<thead>
<tr>
<th>Vehicle model of Maruti Suzuki</th>
<th>Torque (N m)</th>
<th>Theoretical Deflection(mm)</th>
<th>ANSYS based Total Deformation(mm)</th>
<th>% Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baleno</td>
<td>132 @ 3000 rpm</td>
<td>0.0013</td>
<td>0.0011</td>
<td>84.62</td>
</tr>
<tr>
<td>Swift Dzire</td>
<td>190 @ 2000 rpm</td>
<td>0.0019</td>
<td>0.0016</td>
<td>84.21</td>
</tr>
<tr>
<td>Grand Vitara 2.4.</td>
<td>225 @ 4000 rpm</td>
<td>0.0022</td>
<td>0.0019</td>
<td>86.36</td>
</tr>
</tbody>
</table>

Now the graphs are plotted between the theoretical and ANSYS based results both for the stress and deflection at various torque specifications. In the figure 8 (a) torque on X-axis, stress values on Y-axis and figure 8 (b) torque on X-axis, deflection values on Y-axis are shown. It is seen, that the stress values increases almost linearly with the increase in load. Also, we see that the theoretical and ANSYS based results are close.

![Graph](image)

**Figure 7** variations of stresses (a) and deflections (b) at different torque conditions.

Similarly the total deformation on the tooth follows almost a linear increment with the increase of torque. The theoretical and ANSYS based results are close enough

5. **CONCLUSION AND FUTURE SCOPE OF WORK**

In the present work, the spur gear tooth is modelled in CATIA V5R18 and same is analyzed in the Static structural domain of ANSYS software. The results are discussed in the preceding section and it is concluded that, for the given design specifications, the maximum root bending stresses for three distinct torque conditions are well within the safe limits, considering the factor of safety as 3. For the torque specification of 132 N-m at 3000 rpm the root bending stress (Von Mises stress) is found to be 8.80 MPa. It has been observed that with the increasing torque condition the % of error becomes lesser and lesser. We further conclude that, the deflection of the tooth using Castigliano’s theorem and the ANSYS based total deformation of the tooth tip are close to each other. With this for the torque specification of 132 N-m at 3000 rpm the tooth deflection (Total deformation) is found to be 0.0011 mm.
A number of research studies have been carried out in the context of spur gear using composites as the gear material. In the present study, we would like to propose a distinct construction technique by using the composite materials for manufacturing of the gear tooth only, while general materials for the core of the gear. It has been noticed in above analysis that the root of the gear tooth is critically stressed. So instead of using composites for the entire structure of the gear, focus should be given in these critical parts keeping the core same. It will result in comparatively lower cost of the gear as the advanced materials like composites are pretty expensive. However, this kind of structure will be particularly suitable for precision applications. Developing such a manufacturing technique which will endorse the use of different materials at the same structure is a difficult task, but cannot be neglected. An extended version of the above work based on the same software can also be carried out for the analysis of contact stress between two mating gears.

**REFERENCES**


