DESIGN AND ANALYSIS OF SPUR GEAR

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ABSTRACT

Gear is one of the important machine element in the mechanical power transmission system. Spur gear is most basic gear used to transmit power between parallel shafts. Spur gear generally fails by bending failure or contact failure. This paper analyses the bending stresses characteristics of an involute spur gear tooth under static loading conditions. The tooth profile is generated using Catia and the analysis is carried out by Finite element method using ANSYS software. The stresses at the tooth root are evaluated analytically using existing theoretical models. The theoretical and FEM results are compared. The results obtained theoretically are in good agreement with those obtained from software. Also an attempt is made to introduce Stress and displacement characteristics of tooth under dynamic loading conditions.

Key words: Ansys, Bending stress & Deflection by FEA, Dynamic analysis, Static analysis, Spur gear.


1. INTRODUCTION

In the today’s world of industrialization Gears are the major means for the mechanical power transmission system, and in most industrial rotating machinery. Because of the high degree of reliability and compactness gears dominates the field of mechanical power transmission. Gearbox is used to convert the input provided by a prime mover into an output required by end application. Due to increasing demand for quiet and long-term power transmission in machines, vehicles, elevators and generators, people are looking for a more precise analysis method of the gear systems. Spur gear is the most basic gear used to transmit power between two parallel shafts with almost 99% efficiency. It requires the better analysis methods for designing highly loaded spur gears for power transmission systems that are both strong and quiet. Due to development of computers people are using numerical approach for the analysis purpose as it can give more accurate analysis results. The finite element method is capable of providing information on contact and bending stresses in gears, along with transmission errors, which can be done easily in ANSYS software. Gear analysis in the past was done by using analytical methods which requires complicated calculations. Now with the use of FEA we can calculate the bending stresses in the gear tooth for given loading condition and we can compare the FEA results with existing models to decide the accuracy. Also static as well as dynamic, both loading conditions of gear can be easily analyzed in Ansys which is not the case with Analytical method.
2. PROBLEM DEFINITION
For this problem we are doing our calculations analytically and compare results with software results. Any problem can be solved by following same procedure.

2.1. Question
The Following data is given for a spur gear pair made of steel and transmitting 5KW power from an electric motor running at 720 rpm to a machine:

No. of teeth on Pinion= 21, No. Of teeth on Gear= 40, Module= 5mm, Face width= 10m, Ultimate Tensile Strength for Pinion material= 600 N/mm², Ultimate Tensile Strength for Gear material= 400 N/mm², Tooth System =20 Degree Full-Depth Involute, Service Factor =1.25, Load Concentration Factor = 1.6, Tooth Form factor for pinion= .326, Tooth Form factor for gear= .389, Velocity factor = 6/ (6+v).

2.2. Solution
As, Strength of gear < Strength of pinion, gear is weaker than pinion in bending. Hence it is necessary to design the gear for bending.

Pitch Line Velocity (V) = \( \pi \times D_p \times N_p / (60000) \) = 3.9585 m/s

Theoretical Tangential Force \( F_t \) = \( P / V \) = 5000/3.9585=1263.1047 N (approx. 1200N)

3. STATIC ANALYSIS

3.1. The Lewis Formula (Stress Calculation)
The analysis of bending stress in gear tooth was done by Mr. Wilfred Lewis in his paper, ‘The investigation of the strength of gear tooth’ submitted at the Engineers club of Philadelphia in 1892. Even today, the Lewis equation is considered as the basic equation in the design of gears [1]. Wilfred Lewis was the first person to give the formula for bending stress in gear teeth using the bending of a cantilevered beam to simulate stresses acting on a gear tooth shown in Cross-section =b*t , height = h, Load=Ft uniform across the face.

![Lewis Equation](http://www.iaeme.com/IJMET/index.asp)
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Lewis considered gear tooth as a cantilever beam with static normal force F applied at the tip. He took the critical section as parabola through point ‘a’ and tangent to tooth curves at the root as shown in fig.1. This parabola shown by dotted line is a beam of uniform strength.

Assumptions made in the derivation are:
- The full load is applied to the tip of a single tooth in static condition,
- The radial component is negligible,
- The load is distributed uniformly across the full-face width,
- forces due to tooth sliding friction are negligible and
- Stress concentration in the tooth fillet is negligible.

In the current analysis of spur gear we follow the Lewis assumptions and equation.

3.2. Derivation of Deflection for Gear Tooth using Castigliano’s Theorem

Lewis has assumed parabola in the gear tooth. So we directly find the deflection of the parabolic teeth with minor errors with actual deflection [2, 3]. We use the castigliano’s theorem for the same

![Parabolic Gear Tooth Diagram](image)

3.2.1. Castigliano’s Theorem

Castigliano’s theorem is one of the energy methods (based on strain energy) and it can be used for solving a wide range of deflection problems.

Castigliano’s theorem states that when a body is elastically deformed by a system of loads, the deflection at any point “P” in any direction “a” is equal to the partial derivative of the strain energy (U) with respect to a load at “F” in the direction “a”.

The theory applies to both linear and rotational deflections. It should be clear that Castigliano’s theorem finds the deflection at the point of application of the load in the direction of the load.

3.3. Analytical Results

3.3.1. Stress using flexure formula

The bending stress at the root of tooth can be given by flexure formula

\[ \sigma = \frac{6 \times F_t \times h}{b \times t^2} \]

Tangential load (F_t) = 1200N (approx.),
Height of tooth (h) =11.25mm,
Face Width (b) = 50mm,
Thickness (t) = 10.614mm (at root)
(Using basic values of various parameters).
Therefore, Stress (\( \sigma \)) = \( \frac{6 \times 1200 \times 11.25}{50 \times 10.614^2} \)
3.3.2. Deflection using Castigliano’s Theorem

Tangential load (Ft) = 1200N,
Face Width (b) = 50mm,
Thickness (t) = 10.614mm (at root),
Stiffness constant (E) = 206000 N/mm^2. (Using basic values for various parameters).

Height of tooth (h) calculated using Lewis constant, as we required height of parabola:-
Lewis constant Y = .389 (given for gear), Module m = 5mm (given), Thickness = 10.614mm (at root)
Therefore, 389*5 = 10.614^2/ (6*h)
Height of parabola tooth (h) = 9.6535mm
Therefore, Deflection δ = 16*1200*9.6535^3/ (206000*50*10.614^3)
δ = .0014mm

3.4. Analysis by using Ansys

3.4.1. Ansys Procedure

The entire analysis is done on the single gear tooth in Ansys 14.0. For that purpose we use the following procedure,

3.4.2. Discretization of Continuum

Draw gear tooth in catia of given dimensions → Divide the gear tooth in 25 sections → calculate mean thickness for each section. (Do not consider the fillet radius as assumed by Lewis.)

![Figure 3 Gear Tooth in Catia](image-url)
3.4.3. Ansys Modelling and Boundary Conditions
Import the IGS file of gear tooth in Ansys. Take element type as beam. Take 25 nodes equidistantly on length equals to tooth height and create different sections of calculated dimensions (length and breadth). Then create elements at nodes of corresponding element attribute. One end of tooth is fixed and at other end tangential load of 1200N (approx.) is applied at the tip. All boundary conditions are applied on nodes.

![Oblique View of Teeth](image)

**Figure 4** Oblique View of Teeth

3.4.4. Plotting Stress and Deflection Graphs
For end conditions.

![Von Mises Stress Plot (Top View)](image)

**Figure 5** Von Mises Stress Plot (Top View)
3.5. Ansys Results

**Stress:** As it can be seen from above images Maximum Von Mises Stress= 14.3018 N/mm$^2$

**Deflection:** From above images Maximum Deflection = .0014 mm

3.6. Comparison of Results

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Ansys Stress(N/mm$^2$)</th>
<th>Analytical Stress(N/mm$^2$)</th>
<th>% Accuracy</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.3018</td>
<td>14.3799</td>
<td>99.45%</td>
<td>.543%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Ansys Deflection(mm)</th>
<th>Analytical Deflection(mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.0014</td>
<td>.0014</td>
</tr>
</tbody>
</table>

As it can be seen from comparison that % error in stress is negligible and no error in deflection of gear tooth calculated using element type as beam. So these FEA models are good enough for stress analysis of spur gear teeth in static condition

4. DYNAMIC ANALYSIS

4.1. Introduction to Dynamic Analysis

Dynamic load is defined as the load which varies in magnitude, direction or point of application with respect to time. Dynamic load in the mechanical components causes the generation of fluctuating stresses. In case of spur gear the load acting on gear tooth is constant in magnitude as well as in direction but varies in point of application of load. Thus spur gear teeth are subjected to fluctuating stresses and lead to fatigue failure. Gears are the important power transmitting elements in mechanical system and hence sudden
failure of gear tooth may lead to danger. Therefore, it is necessary to perform the dynamic analysis of spur gear.

4.2. Load Distribution
Load distribution on gear tooth In order to conduct a dynamic stress analysis the loads have to be evaluated. The load on the tooth of the finite element model, which produces the largest bending stress, is the full load acting at the highest point of single tooth contact (HPSTC). The magnitude of load at any point of contact on profile of gear tooth as the load moves from root to tip of tooth depends on the contact ratio. Fig. shows the contact path, the contact ratio CR is defined as the ratio of length of path of contact AB to base circle pitch Pb [4, 5].

![Figure 7 Path of Contact](image)

Contact ratio calculated as 1.64 which is approximately equal to 2. Now while applying load we have considered 39 nodes on left edge of tooth numbered from 6 to 44. Bottom nodes left to consider the clearance and the load variation is assumed symmetric variation from bottom to top.

4.3. Time Step
The time of contact T of gear tooth depends on the rotational speed of the gear. If the gear is assumed to run at a speed of n-rpm, the time taken for one revolution of the gear will be (60/n) sec. In one revolution, Z number of teeth will get engaged and disengaged.

\[ T = 0.00397 \text{ sec.} \]
Thus, time step for 40 nodes is T/40= 0.0000925 sec.

The Initial conditions for displacements and velocities for the gear tooth in the proposed model are taken as zero for all degrees of freedom. We have assumed equal time is required to transfer load from one node to other. For nodes 32 to 44 load=600 N (Half load), for nodes 19 to 31 load= 1200N (Full load) and for nodes 6 to 18 load= 600 N (Half load) is applied [5].

4.4. Ansys Procedure

- Draw gear tooth in Catia→ Import IGS file in Ansys→ Choose element type as solid-plane 182(Quad 4 node 182) with element behaviour of plane stress with thickness input as Lewis has assumed the load acting on tooth as uniformly distributed load and this will generate plain stress condition at each cross section in off teeth of shown orientation→ Give its real constant (thickness) and material properties (density, young’s modulus and poisson ratio) →Create area→ Mesh (Fine meshing).. 2. Loads→ new analysis→ transient analyse→ reduced methods→ Master degree of freedom→ user defined method→ select all nodes except nodes which are on zero displacement base line→ apply boundary conditions on last line.

- Load Step opts→ Time/frequency→ Time/Time-step option→ Put Initial conditions and load type-stepped→ Write this LS file as 1→ start applying forces at various nodes for a particular time→ follow same procedure as mentioned above→ write this file as 2→ Similarly apply forces at various nodes for different time and write there file as 3, 4, 5 etc. → Solve- From LS files- LS file 1-41.

Figure 9 Meshed Tooth
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**Figure 10** Master DOF with Applied Force

- Time Hist Post Pro → adds data → DOF Solution (X-component of displacement) → Choose Node 1 (Upper most node) → Graph Data to get following solution.
- Time Hist Post Pro → Add data → DOF Solution (X-component of displacement) → Choose Node 1 (Upper most node) to get expanded solution for displacement.

![Master DOF with Applied Force](image)

**Figure 11** X-Component of Displacement for Node 1

4.5. Results

- Displacement at any node at particular time during engagement can be calculated. For example, at Time = 0.42734E-02 sec displacement plot of teeth is

![Displacement Plot of Teeth](image)

**Figure 12** Displacement Plot of Teeth
• Displacement of particular node can be calculated for any time. For example, for tip node displacement v/s time graph is

![Graph](image1)

**Figure 13** X-Component of Displacement for Node 1 (Expanded Solution)

• Stress at any node at particular time during engagement can be calculated. For example, At Time= 0.42734E-02 sec stress plot of teeth is

![Graph](image2)

**Figure 14** Stress Plot of Teeth

• Stress of particular node can be calculated for any time. For example, For root node stress v/s time graph is:
5. CONCLUSION

In this project an attempt is made to co-relate the bending stresses and displacement of a spur gear tooth which are obtained analytically as well as by FEM. Recent developments in the field of mechanical engineering are demanding refined gear teeth in terms of loading capacities and speed with which they can operate. The static and dynamic analysis of spur gear tooth helps to determine maximum displacement, maximum induced stress and effect of stress variation with respect to time. The loading capacity and operating speed of geared system can be increased by reducing the maximum induced stresses.

The following conclusions can be drawn from the results obtained:

- The maximum stresses for gear teeth occur in the root region of gear tooth.
- Static analysis of spur gear tooth with element type as Beam give more accurate results for maximum displacement and maximum induced stress. The accuracy of stress calculated using FEA model and analytical calculations is 99.45% and that of displacement is almost 100%. Thus we can conclude that beam element is more suitable for static analysis of spur gear. This is in accordance with Lewis bending stress calculation formula who has considered the gear tooth as cantilever beam.
- With the help of dynamic analysis the stress and displacement variation at any node with respect to time and stress and displacement variation at any time instant for all nodes can be known and corresponding results can be represented graphically. Nature of graph can be used to predict force variation along tooth.
- Maximum value of stresses and displacement of dynamic analysis are obtained from the graph are comparable with static analysis results. Hence FEM model is good enough for dynamic stress consideration.

REFERENCE


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