NUMERICAL SIMULATION OF WATER FLOW THROUGH SOIL FROM A TRICKLE IRRIGATION WITH WATER UPTAKE BY ROOTS

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ABSTRACT

A two-dimensional numerical model is developed to simulate water infiltration from a trickle irrigation system in unsaturated soils. The diffusive form of Richard’s equation incorporating evaporation and water extraction by roots is solved using fully implicit time scheme of the finite-volume method. Water uptake by roots is modeled as a continuous linear sink function. Numerical simulations were conducted with a silt loam soil at different values of water application rate from the trickle irrigation source that is inserted at a small circular pond area. A good agreement is obtained when comparing the predicted results of the wetting front advance with the values that was obtained from published empirical relationships.

Keywords: Finite volume method; Richard’s equation; root water uptake; trickle irrigation; unsaturated porous media.


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1. INTRODUCTION

Numerical simulation of unsaturated flow has a significant history in the field of soil irrigation science. It is accomplished by solving the unsaturated flow equation (Richard’s equation) using one of three methods: finite-difference method, finite-element method and finite-volume method. Analytical solutions have been developed to investigate the linearized form of the water flow equation, (Warrick, 1974; Warrick and Lomen, 1976; Ben-Asher et al., 1978; Abid, 2006; and Abid et al., 2012). On the other hand, numerical solutions have been developed to predict the water flow equation (Brandt et al., 1971; Taghavi et al., 1984; Lafolie et al., 1989; Elmaloglou and Grigorakis, 1997; and Abid, 2012 and 2015). Numerous one and two-dimensional models have been developed to simulate the transient infiltration.
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from a trickle irrigation source with respect to water uptake by the plant roots. Molz and Remson (1970 and 1971) developed a one-dimensional mathematical model which used a macroscopic extraction term to describe moisture removal from soil by the roots of transpiring plants. A sink term in soil moisture flow equation may depend on space, time, water potential, water content, or a combination of these variables. They used a numerical procedure based on Douglas-Jones predictor-corrector method to solve a mathematical model. The results obtained from a mathematical model indicate that extraction term models are computationally and physically feasible and give insight into mechanics of the overall moisture extraction process. Warrick et al. (1979) solved a linearized moisture flow equation for two-dimensional line sources with water extraction. Linearization is attained in the steady-state case by assuming the unsaturated hydraulic conductivity is exponentially related to the pressure head. The water extraction is one-dimensional. Warrick and Lomen (1981) developed steady-state solutions for two-dimensional sink-source combinations. They assumed the unsaturated hydraulic conductivity is exponentially related to the pressure head and plant uptake decrease exponentially with depth and with lateral distance. General solutions included three types of surface boundary conditions: specified flux, specified matrix flux potential and flux proportional to the matrix flux potential. Developments differ from those of Warrick et al. (1979) in that here the boundary conditions are much more general and the sink functions are two-dimensional. Elmaloglou and Malamos (2003) investigated the concept of two empirical relationships; the first describes vertical water movement during infiltration, while the second describes vertical water movement during the redistribution of the water in the soil profile. This method is compared to the mathematical model, which is used to simulate the water flow under a trickle surface line source, considering water uptake by roots. Elmaloglou and Malamos (2005 and 2006) used a cylindrical flow model that describes local infiltration from a surface point source, by incorporating evaporation and water extraction by roots to obtain numerical results that were the base for the development and testing of an empirical method for determining the surface and vertical components of the wetting front. The empirical methodology consisted of two simple, time-dependent empirical relationships: a power law for the stage of the infiltration, and a polynomial for the stage after the end of the irrigation. Verification of the proposed method took place against computed results from a cylindrical flow model that considers water uptake by roots and evaporation from the soil surface. The empirical model is useful tool in predicting the components of the wetting front throughout the soil profile under a surface point source, in order to reduce percolation losses. Yadav et al. (2009) developed a one-dimensional numerical model for simulating soil moisture flow in layered soil profile with plant growth. A dynamic root compensation mechanism is used for a nonuniform root distribution pattern to compute water uptake by plants in a moisture scarce environment. Results show that under favorable soil moisture conditions, plants extract water at the maximum rate according to the root distribution pattern and when the moisture stress is developed in the upper soil profile the diminished water uptake rate in the water scarce region is compensated for by an enhanced water uptake from the lower wetter layers.

The aim of this paper is to present a two-dimensional finite-volume model to solve the water content -based form of the Richard’s equation with incorporating evaporation and water extraction by roots to predict the transient flow of water through homogeneous unsaturated porous media from a trickle source. Silt loam soil will be selected to represent the homogenous porous media. The fully implicit time scheme associated with arithmetic mean weighing formulas to estimate the approximate value of the nonlinear functions of soil water diffusivity and hydraulic conductivity at the interface between adjacent control volumes will be used. The predicted numerical results for the locus of wetting front advance will be
compared with those obtained from previously published data of the empirical method that was predicted by Elmaloglou and Malamos, (2006).

2. MATHEMATICAL MODEL

Fig. 1 illustrates the two-dimensional mathematical model. The trickle source is located at point A on the soil surface. However, for all practical purposes, ponded water is assumed from point A to B. The ponding radius $r_0$, is assumed to be constant in time.

![Figure 1 Mathematical model](image)

2.1. Governing Equation

The two-dimensional governing equation in cylindrical coordinates (the continuity and Darcy’s equations) for water flow in unsaturated soils is expressed as follows:

$$\frac{\partial \theta}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} [r D(\theta) \frac{\partial \theta}{\partial r}] + \frac{\partial}{\partial z} [D(\theta) \frac{\partial \theta}{\partial z}] = \frac{\partial}{\partial z} K(\theta) - S(\theta, z)$$  \hspace{1cm} (1)

Where: $\theta$ = volumetric water content of soil at a point in the soil ($L^3$ $L^{-3}$); $t =$ the time since the beginning of flow (T); $D(\theta)$ = soil water diffusivity ($L^2/T$); $K(\theta)$ = hydraulic conductivity of the soil (L/T); $r =$ radial (horizontal) coordinate; $z =$ vertical coordinate; and $S(\theta, z)$ = a distributed sink function representing the water uptake by roots ($L^3$ $L^{-3}$/$T$).

The soil water diffusivity defined as:

$$D(\theta) = K(\theta) \frac{dH}{d\theta}$$  \hspace{1cm} (2)

Where: $H =$ water pressure head (L).

2.2. Initial and boundary conditions

The initial condition is constant water content, $\theta_i$, throughout the domain:

$$\theta(r, z, 0) = \theta_i \quad at \ t = 0$$  \hspace{1cm} (3)

The boundary conditions are the following (Fig.1):

1. Along the soil upper surface AB:
2. Along the soil upper surface BC, a zero vertical flux boundary condition is maintained at all times:

\[-D(\theta) \frac{\partial \theta}{\partial z} + K(\theta) = -E_a \quad \text{at} \ t > 0, \ z = 0 \quad \text{and} \ (0 \leq r \leq r_o) \]  

(4)

3. Along the vertical symmetry line AE, the rate of water flow in the normal direction of the line is zero:

\[-D(\theta) \frac{\partial \theta}{\partial r} = 0 \quad \text{at} \ t > 0, \ (0 \leq z \leq Z) \quad \text{and} \ (r = 0) \]  

(6)

4. A no-flow condition applies to the vertical line CD:

\[-D(\theta) \frac{\partial \theta}{\partial r} = 0 \quad \text{at} \ t > 0, \ (0 \leq z \leq Z) \quad \text{and} \ (r = R). \]  

(7)

5. Along the soil lower surface ED:

\[-D(\theta) \frac{\partial \theta}{\partial z} + K(\theta) = 0 \quad \text{at} \ t > 0, \ z = Z \quad \text{and} \ (0 \leq r \leq R) \]  

(8)

Where: \( \theta_i \) = volumetric initial water content (L^3/L^3); \( Q \) = emitter flow rate (L^3/T); \( q = Q/(3,600\pi r_o^2) \) (L/T); \( r_o \) = radius of the pond (L); and \( E_a \) = actual evaporation rate (L^3/L^3/T).

2.3. Soil physical characteristics

The unsaturated soil hydraulic functions in the eq. (1) are the soil water retention curve \( \theta(H) \), the hydraulic conductivity function \( K(\theta) \) and the soil diffusivity function \( D(\theta) \). To solve eq. (1) explicit expressions for the soil constitutive relationship between the dependent variable \( \theta \) and the nonlinear terms \( K(\theta) \) and \( D(\theta) \) are required. van Genuchten (1980) proposed an analytical function of \( \theta(H) \) a more flexible than the Brooks and Corey (1964) and combined with Mualem’s model (1976). The analytical \( \theta(H) \) function proposed by van Genuchten (1980):

\[ \theta(H) = \theta_r + \left( \frac{(\theta_s-\theta_r)}{[1+(\alpha H)]^{n/4}} \right)^{(n-1)/n} \]  

(9)

Where: \( \theta_r \) = residual water content (L^3/L^3); \( \theta_s \) = saturated water content (L^3/L^3); \( \alpha \) = shape factor related to the inverse of the air-entry pressure(1/T); and \( n \) = shape factor (dimensionless).

The soil hydraulic conductivity function is described by van Genuchten (1978 and 1980):

\[ K(\theta) = k_{sat} \left( \frac{\theta-\theta_r}{\theta_s-\theta_r} \right)^{1/2} \left( 1 - \left( \frac{\theta-\theta_r}{\theta_s-\theta_r} \right)^n \right)^{(n-1)/n} \]  

(10)

Where: \( k_{sat} \) = saturated hydraulic conductivity (L/T).

2.4. Root water uptake

Water uptake by plant roots (sink term) in eq. 1 describe a volume of water removed in a unit of time from a unit of soil. The sink term is a function of maximum water extraction rate at a depth, \( z \), and the water pressure head. Feddes et al. (1978) proposed a simplified semi-empirical root extraction term (Belmans et al., 1983):

\[ S(\theta, z) = S(H, z) = f(H)S_{max}(z) \]  

(11)
Where: $S(\theta, z) =$ distributed sink term (water uptake by roots) $(L^3 \text{L}^{-3}/\text{T})$; $f(H) =$ an uptake scaling parameter as a function of pressure head, dimensionless; $S_{\text{max}}(z) =$ the maximum water extraction rate $(1/\text{T})$; $H =$ water pressure head $(\text{L})$; and $z =$ depth $(\text{L})$.

A value $f(H)$ representing the function is calculated at each node using four input parameters $H_1$, $H_2$, $H_3$, and $H_4$ with $(H_4 \leq H_3 \leq H_2 \leq H_1 \leq 0)$. The value of $f(H)$ is taken to be zero (no uptake) if the current nodal soil water pressure head is greater than $H_1$ or less than $H_4$. It is unity for head values between $H_2$ and $H_3$. Value $f(H)$ is obtained from linear interpolation for values between $H_1$ and $H_2$ and between $H_3$ and $H_4$.

The trapezoidal form of an uptake scaling parameter, $f(H)$, as used by Feddes et al. (1978) can be expressed mathematically by the equation:

$$f(H) = \begin{cases} 
0 & \text{for } H \geq H_1 \text{ or } H \leq H_4 \\
\frac{H - H_1}{H_2 - H_1} & \text{for } H_2 < H < H_1 \\
\frac{H_4 - H}{H_4 - H_3} & \text{for } H_4 < H < H_3 \\
1 & \text{for } H_3 \leq H \leq H_2 
\end{cases} \quad (12)$$

Where: $H_1 =$ pressure head below which the roots start to extract water from the soil; $H_2 =$ pressure head below which the roots start to extract water optimally from the soil; $H_3 =$ pressure head below which the roots cannot extract water optimally from the soil; and $H_4 =$ pressure head below which the roots cannot extract soil water (pressure head at wilting point). Parameters on root water uptake are given in Table 1. (Elmaloglou and Malamos, 2006).

### Table 1 Parameters on Root Water Uptake

<table>
<thead>
<tr>
<th>H (cm)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>-25</td>
</tr>
</tbody>
</table>

The relationship of a maximum water extraction rate, $S_{\text{max}}(z)$ to depth, $z$, assumes a linear variation given by formula (Hoogland et al., 1981):

$$S_{\text{max}}(z) = a \cdot b \mid z \mid \quad (13)$$

Where: $a =$ maximum extraction rate at the surface $(L^3 \text{L}^{-3}/\text{T})$, ($a = 0.15*10^{-2} \text{ cm}^3 \text{ cm}^{-3}/\text{hr}$); and $b =$ reduction coefficient $(L^{-1}/\text{T})$, ($b = 0.2222 \times 10^{-4} \text{ cm}^{-1}/\text{hr}$). The values of $a$ and $b$ based on information from Elmaloglou and Malamos, 2006.

The model considers the actual soil evaporation rate from the soil surface, as an exponential function of the potential soil evaporation rate and the pressure head:

$$E_a = E_p e^{\delta H} \quad (14)$$

Where: $E_a =$ actual soil evaporation rate $(1/\text{T})$; and $E_p =$ potential soil evaporation rate $(\text{L}/\text{T})$. The values of $E_p = 0.01 \text{ cm/hr}$ and $\delta =0.001$ (Elmaloglou and Malamos, 2006).

### 3. FINITE-VOLUME MODEL

#### 3.1. Grid generation

In the finite-volume method (FVM), the first step is to divide the domain (the computational space) into discrete control volumes; $(N_r$ and $N_z$). A general nodal point is identified by $P$. 

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The nodes to the west, east, south, north, top, and bottom of P are identified by W, E, S, N, T, and B, respectively. The side boundaries of the control volume are referred by w, e, s, n, top, and b for the west, east, south, north, top, and bottom sides, respectively (Fig. 2-a). Time domain is divided into a number of time steps of size $\Delta t$. Variables at the previous time level are indicated by the superscript, o (old). In contrast, the variables at the new time level are not superscripted (Versteeg and Malalasekera, 1995; Abdulkareem, 2014).

![Figure 2](http://www.iaeme.com/IJCIET/index.asp)

3.2. Discretization of the Governing Equation

Since the computational domain is subdivided into a collection of finite volumes, the node located inside the center of each finite volume is the locus of computational values. Discretization equations are obtained by integrating the governing equation of water flow through porous media [Eq. (1)] over the span of each finite volume and multiplying it by (dV/dt). The infinitesimal volume in the two-dimensional model using cylindrical coordinates is given by (dV = r d$\phi$ dr dz) (Fig. 2-b) and then integrates Eq. (1) over the control volume faces, yielding the following form:

$$\int_{t}^{t+\Delta t} \int_{S}^{e} \int_{S}^{n} \int_{b}^{top} \frac{\partial \theta}{\partial t} (r d\phi dr dz dt) = \int_{t}^{t+\Delta t} \int_{S}^{e} \int_{S}^{n} \int_{b}^{top} \frac{1}{r} \frac{\partial}{\partial r} (r D(\theta) \frac{\partial \theta}{\partial r}) (r d\phi dr dz dt) + \int_{t}^{t+\Delta t} \int_{S}^{e} \int_{S}^{n} \int_{b}^{top} \frac{\partial}{\partial z} (D(\theta) \frac{\partial \theta}{\partial z} - K(\theta)) (r d\phi dr dz dt) - \int_{t}^{t+\Delta t} \int_{S}^{e} \int_{S}^{n} \int_{b}^{top} \frac{\partial}{\partial z} (S(\theta,z)) (r d\phi dr dz dt)$$

3.3. Fully Implicit Scheme

Applying the fully implicit scheme on Eq. (8) and rearranging yields:

$$\left(\theta_p - \theta_p^o\right) \frac{r_p \Delta r \Delta z}{\Delta t} = \left[r_e D_e \Delta z \left(\frac{\theta_e - \theta_p^o}{\Delta r}\right) - r_w D_w \Delta z \left(\frac{\theta_p - \theta_p^o}{\Delta r}\right) + \left[r_p D_r \Delta r \left(\frac{\theta_p - \theta_p^o}{\Delta z}\right) - r_p K_s \Delta r\right] - \left[r_p D_s \Delta r \left(\frac{\theta_p - \theta_p^o}{\Delta z}\right) - r_p K_s \Delta r\right] - S \Delta r \Delta z \right]$$

Where: $\theta_p^o$ denotes the value of $\theta_p$ at time (t), and $\theta_p$ denotes the value of $\theta_p$ at time ($t + \Delta t$).

Each of the boundary conditions is substituted into Eq. (16), and rearranging yields:
\begin{align*}
\left\{ \begin{array}{l}
a_p \theta_p = a_E \theta_E + a_W \theta_W + a_N \theta_N + a_S \theta_S + a_p^0 \theta_p^0 + S_u \\
a_p = a_E + a_W + a_N + a_S + a_p^0 - S_p \\
a_p^0 = r_p \Delta r \Delta z / \Delta t
\end{array} \right.
\end{align*}

Where: \( \Delta r_w = \Delta r_e = \Delta r = R / N_r \) and \( \Delta z_n = \Delta z_s = \Delta z = Z / N_z \). The values of \( (a_E, a_W, a_N, a_S, a_p^0) \) are listed in Table 2 and \( (S_p = 0) \).

### Table 2 Discretization Parameters of the Two-Dimensional Model

<table>
<thead>
<tr>
<th>Zone</th>
<th>( a_E )</th>
<th>( a_W )</th>
<th>( a_N )</th>
<th>( a_S )</th>
<th>( S_u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal nodes</td>
<td>( r_e D_e \Delta z / \Delta r )</td>
<td>( r_w D_w \Delta z / \Delta r )</td>
<td>( r_p D_n \Delta r / \Delta z )</td>
<td>( r_p D_s \Delta r / \Delta z )</td>
<td>( (K_s - K_n - S \Delta z) r_p \Delta r )</td>
</tr>
<tr>
<td>AE</td>
<td>( r_e D_e \Delta z / \Delta r )</td>
<td>0</td>
<td>( r_p D_n \Delta r / \Delta z )</td>
<td>( r_p D_s \Delta r / \Delta z )</td>
<td>( (K_s - K_n - S \Delta z) r_p \Delta r )</td>
</tr>
<tr>
<td>CD</td>
<td>0</td>
<td>( r_w D_w \Delta z / \Delta r )</td>
<td>( r_p D_n \Delta r / \Delta z )</td>
<td>( r_p D_s \Delta r / \Delta z )</td>
<td>( (K_s - K_n - S \Delta z) r_p \Delta r )</td>
</tr>
<tr>
<td>DE</td>
<td>( r_e D_e \Delta z / \Delta r )</td>
<td>( r_w D_w \Delta z / \Delta r )</td>
<td>( r_p D_n \Delta r / \Delta z )</td>
<td>0</td>
<td>( (-K_n - S \Delta z) r_p \Delta r )</td>
</tr>
<tr>
<td>AB</td>
<td>( r_e D_e \Delta z / \Delta r )</td>
<td>( r_w D_w \Delta z / \Delta r )</td>
<td>0</td>
<td>( r_p D_s \Delta r / \Delta z )</td>
<td>( (-q + E_a + K_s - S \Delta z) r_p \Delta r )</td>
</tr>
<tr>
<td>BC</td>
<td>( r_e D_e \Delta z / \Delta r )</td>
<td>( r_w D_w \Delta z / \Delta r )</td>
<td>0</td>
<td>( r_p D_s \Delta r / \Delta z )</td>
<td>( (E_a + K_s - S \Delta z) r_p \Delta r )</td>
</tr>
<tr>
<td>Corner A</td>
<td>( r_e D_e \Delta z / \Delta r )</td>
<td>0</td>
<td>0</td>
<td>( r_p D_s \Delta r / \Delta z )</td>
<td>( (-q + E_a + K_s - S \Delta z) r_p \Delta r )</td>
</tr>
<tr>
<td>Corner C</td>
<td>0</td>
<td>( r_w D_w \Delta z / \Delta r )</td>
<td>0</td>
<td>( r_p D_s \Delta r / \Delta z )</td>
<td>( (E_a + K_s - S \Delta z) r_p \Delta r )</td>
</tr>
<tr>
<td>Corner D</td>
<td>0</td>
<td>( r_w D_w \Delta z / \Delta r )</td>
<td>( r_p D_n \Delta r / \Delta z )</td>
<td>0</td>
<td>( (-K_n - S \Delta z) r_p \Delta r )</td>
</tr>
<tr>
<td>Corner E</td>
<td>( r_e D_e \Delta z / \Delta r )</td>
<td>0</td>
<td>( r_p D_n \Delta r / \Delta z )</td>
<td>0</td>
<td>( (-K_n - S \Delta z) r_p \Delta r )</td>
</tr>
</tbody>
</table>

### 3.4. Solver

The system of Eq. (17) is solved efficiently using the Thomas algorithm, the tridiagonal matrix algorithm (TDMA) (Versteeg and Malalasekera 1995).

### 3.5. Interblock Nonlinear Functions

The interblock nonlinear functions of soil water diffusivity and hydraulic conductivity is estimated as arithmetic mean of the water diffusivities and hydraulic conductivities at the neighboring nodes. The interblock nonlinear functions of soil water diffusivity and hydraulic conductivity using arithmetic means are given in Table 3.
3.6. Evaluation criteria
To validate the FVM accuracy, several measures were used: average error (AE), root mean square error (RMSE), root mean square (RMS), modeling efficiency (EF) and coefficient of residual mass (CRM). The mathematical expressions which describe these measures of analysis are shown in Table 4 (Loague and Green, 1991). The AE, RMSE, RMS statistics have as a lower limit the value of zero, which is the optimum value for them, as it is for CRM. Positive values of CRM indicate that the model underestimate the measurements and negative values indicate the model’s tendency to overestimate (Hack-ten Broeke and Hegmans, 1996). The maximum and simultaneously the optimum value for EF is 1.

**Table 4 Equations of FVM Performance Criteria**

<table>
<thead>
<tr>
<th>Index</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>average error</td>
<td>$AE = \frac{\sum_{i=1}^{n}(P_i - O_i)}{n}$</td>
</tr>
<tr>
<td>root mean square error</td>
<td>$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(P_i - O_i)^2}{n} \times 100}$</td>
</tr>
<tr>
<td>root mean square</td>
<td>$RMS = \sqrt{\frac{\sum_{i=1}^{n}(P_i - O_i)^2}{n}}$</td>
</tr>
<tr>
<td>modeling efficiency</td>
<td>$EF = \frac{\sum_{i=1}^{n}(O_i - O_{av})^2 - \sum_{i=1}^{n}(P_i - O_i)^2}{\sum_{i=1}^{n}(O_i - O_{av})^2}$</td>
</tr>
<tr>
<td>coefficient of residual mass</td>
<td>$CRM = \frac{\sum_{i=1}^{n}O_i - \sum_{i=1}^{n}P_i}{\sum_{i=1}^{n}O_i}$</td>
</tr>
</tbody>
</table>

4. RESULTS AND DISCUSSIONS
A numerical two-dimensional finite-volume model is developed to solve Richard’s equation in water content-based form to simulate the wetting front of the transient flow of water through unsaturated homogeneous porous media from a trickle irrigation source in a cylindrical flow region with a radius and depth of 50 and 60 cm, respectively. Silt loam soil is selected to represent this porous media. The soil water characteristic curves are approximated
by the relationships given by van Genuchten (1980) and are listed in Table 5. This cylindrical flow domain of soil is initially at uniform water content, \( \theta_i = 0.1908 \text{ cm}^3\text{ cm}^{-3} \). Numerical runs were conducted with a silt loam soil at two water application rates of \( Q = 2,000 \) and \( 3,000 \text{ cm}^3\text{ hr}^{-1} \) at the trickle irrigation source with pond radii \( r_o = 8 \) and \( 10 \text{ cm} \), respectively. A spatial step size of \( \Delta r = 1 \text{ cm} \) and \( \Delta z = 4 \text{ cm} \) is used in all simulations. The time step size for the fully implicit time scheme is selected, \( \Delta t = 6 \text{ sec} \).

**Table 5** Soil Hydraulic Parameter for the Analytical Functions of van Genuchten (1980) for Silt Loam Textural Classes of the USDA Textural Triangle (Schaap and Leij, 1998)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>water content, ( \theta )</td>
<td>( \theta_r + \frac{(\theta_s - \theta_r)}{[1 + (\alpha H)]^{(n-1)/n}} )</td>
</tr>
<tr>
<td>hydraulic conductivity, ( K(\theta) )</td>
<td>( k_{sat} \left( \frac{\theta - \theta_r}{\theta_s - \theta_r} \right)^{1/2} \left( 1 - \left[ 1 - \left( \frac{\theta - \theta_r}{\theta_s - \theta_r} \right)^n \right]^{(n-1)/n} \right)^2 )</td>
</tr>
<tr>
<td>soil water diffusivity, ( D(\theta) )</td>
<td>( \frac{-K(\theta)}{\alpha (1-n)(\theta_s - \theta_r)} \left( \frac{\theta - \theta_r}{\theta_s - \theta_r} \right)^{n+1} \left[ \frac{\theta - \theta_r}{\theta_s - \theta_r} \right]^{1-n} )</td>
</tr>
<tr>
<td>residual water content, ( \theta_r )</td>
<td>0.065 cm$^3$ cm$^{-3}$</td>
</tr>
<tr>
<td>initial water content, ( \theta_i )</td>
<td>0.1908 cm$^3$ cm$^{-3}$</td>
</tr>
<tr>
<td>saturated water content, ( \theta_s )</td>
<td>0.439 cm$^3$ cm$^{-3}$</td>
</tr>
<tr>
<td>saturated hydraulic conductivity, ( K_{sat} )</td>
<td>0.760 cm/hr</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.00506 cm$^{-1}$</td>
</tr>
<tr>
<td>( n )</td>
<td>1.66341</td>
</tr>
</tbody>
</table>

The execution time of an Intel Core i7- 4810MQ CPU@2.80 GHz–8 GB RAM personal computer to achieve a convergence value of 0.0001, (which is estimated on the basis of the difference between the predicted water content values in two successive iterations) for the time of wetting front after 8 hrs is 8.77 and 10.0 sec for water application rates of \( Q = 2,000 \) and \( 3,000 \text{ cm}^3\text{ hr}^{-1} \), respectively. To validate the predicted results, the simulated vertical water front movement, along \( r=0 \), and the corresponding empirical estimation are plotted with time as shown in Fig.3 for the silt loam soil applying two values of water application rates. The simulated water front movement at the surface, along \( z=0 \), and the corresponding empirical estimation are plotted with time as shown in Fig.4 for the silt loam soil applying two values of water application rates. In general, good agreement is obtained when comparing the predicted results positions of the wetting front advance with the empirical estimation when using a fully implicit numerical model associated with the arithmetic mean value of the interblock nonlinear soil water diffusivity and hydraulic conductivity, as shown in Figs. 3 and 4. The accuracy of the predicted results when compared with the empirical horizontal and vertical water front movement is shown in Table 6, indicating a slight deviation between the numerical and empirical values. Average error (AE) values were positive in the case of negative coefficient of residual mass (CRM) values. Table 6 summarizes the statistical criteria used for the evaluation of the numerical method. Figs. 5 and 6 shows the contour lines describing the predicted water content distribution at the end of 1, 3, 4 and 7 hrs for \( Q = 2,000 \) and \( 3,000 \text{ cm}^3\text{ hr}^{-1} \), respectively. Fig. 7 and 8 shows the predicted contour line of water content value (\( \theta = 0.2 \text{ cm}^3\text{ cm}^{-3} \)) for silt loam soil for different values of time frame for \( Q = 2,000 \) and \( 3,000 \text{ cm}^3\text{ hr}^{-1} \), respectively.
Numerical Simulation of Water Flow through Soil from a Trickle Irrigation with Water Uptake by Roots

Table 6 Values of Statistical Criteria used for the Evaluation

<table>
<thead>
<tr>
<th>Water application rate (cm³/hr)</th>
<th>Wetted radius</th>
<th>Wetted depth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AE cm</td>
<td>RMSE %</td>
</tr>
<tr>
<td>2</td>
<td>0.462</td>
<td>2.720</td>
</tr>
<tr>
<td>3</td>
<td>0.397</td>
<td>1.567</td>
</tr>
</tbody>
</table>

Figure 3 Numerical and estimated values of the surface water advance versus time, for the silt loam soil

Figure 4 Numerical and estimated values of the vertical water advance versus time, for the silt loam soil
Figure 5 Water content (cm$^3$ cm$^-3$) distribution predicted by the present fully implicit FVM model at different values of infiltration time frames from a trickle source in the silt loam soil (Q = 2,000 cm$^3$/hr)
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Figure 6 Water content ($cm^3/cm^3$) distribution predicted by the present fully implicit FVM model at different values of infiltration time frames from a trickle source in the silt loam soil ($Q = 3,000 cm^3/hr$)

Figure 7 Wetting front advances during an infiltration run from a trickle source in the silt loam soil under an application rate ($Q= 2,000 cm^3/hr$) and water content ($\theta=0.2 cm^3/cm^3$) at different times

Figure 8 Wetting front advances during an infiltration run from a trickle source in the silt loam soil under an application rate ($Q=3,000 cm^3/hr$) and water content ($\theta=0.2 cm^3/cm^3$) at different times
5. SUMMARY AND CONCLUSIONS

The nonlinear Richard’s equation that describes the transient, two-dimensional water infiltration through unsaturated porous media incorporating evaporation and water extraction by roots is predicted numerically using the fully implicit finite-volume method. The arithmetic means is used to estimate the inter block nonlinear soil water diffusivity and hydraulic conductivity. The agreement between the numerical simulation results and empirical data is good. The results show that the finite-volume method is mass conservative, and it is a recommended method for solving Richard’s equation. The advantage of the proposed finite-volume model is its ability to deal with soil water movements through a soil environment in homogeneous porous media with water uptake by roots and its ability to treat irregularly shaped flow regions. The results provide support for using finite-volume method as a tool for investigating and designing trickle irrigation management practices.

REFERENCES

Numerical Simulation of Water Flow through Soil from a Trickle Irrigation with Water Uptake by Roots


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