PERISTALTIC MOTION OF AN ELLIS FLUID MODEL IN A VERTICAL UNIFORM TUBE WITH WALL PROPERTIES

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ABSTRACT

The present article manages with the peristaltic flow of an Ellis fluid model in a vertical uniform tube with the wall properties utilizing long wavelength and low Reynolds number approximation. The analytical expressions have been obtained for stream function, velocity and temperature dispersion. The results are plotted and discussed in detail for the cases of shear thinning, shear thickening and viscous fluids. The impacts of different parameters are associated with the flow problem such as rigidity parameter $E_1$, stiffness parameter $E_2$, viscous damping force parameter $E_3$ and Brickman number $Br$. It is discovered that the velocity profile is increasing function of rigidity parameter, stiffness parameter and for viscous damping force parameter velocity decreasing and then increasing due to the less resistance offered by the walls but quite opposite behavior is depicted for shear thickening fluids. It is seen that Brickman number enhances the fluid temperature for all cases.

Key words: Peristaltic flow, uniform tube, Shear thickening, shear thinning fluids. Ellis fluid, exact solution.


1. INTRODUCTION

An assortment of complex rheological liquids can easily be transported from one place to another place with a special type of pumping known as Peristaltic Pumping. The pumping rule is called Peristalsis. The system incorporates involuntary periodic contraction followed by relaxation or expansion of the ducts. This leads to rise in pressure gradient that eventually pushed the fluid forward. This kind of pumping is seen in physiology where sustenance travels through the stomach related tract, pee transport from the kidney to the bladder through ureters, lymphatic liquids travels through lymphatic vessels, bile streams from the anony bladder into the duodenum, spermatozoa travel through the channels efferentes of the male regenerative tract and cervical trench, ovum travels through the fallopian tube, and blood circulates in small blood vessels. The engineering analysis of peristalsis was started.
substantially later than in physiological investigations. Applications in industrial fluid mechanics are resemble aggressive chemicals, high solid slurries, noxious fluid and other materials that are transported by peristaltic pumps. Roller pumps, hose pumps, tube pumps, ginger pumps, heart lung machines, blood pump machines, and dialysis machines are built on the premise of peristalsis.

The examination of peristalsis has received broad consideration over the recent couple of decades generally in light of its significance of designing and organic frameworks. A few investigations have been made examining both hypothetical and trial parts of the peristaltic movement of Newtonian and non-Newtonian fluids in various circumstances. With regards to such physiological and mechanical applications, the flow of peristaltic instrument has been discussed about in detail by different researchers [1-10], more over Kavitha et al. [11] studied the peristaltic transport of a Jeffrey fluid between porous walls with suction and injection. Eldabe et.al [12] analyzed the MHD peristaltic flow of a couple stress fluids with heat and mass transfer in a porous medium. Saravana et al. [13] discussed the MHD peristaltic flow of a Jeffrey fluid in a non-uniform porous medium channel with wall properties, slip condition, heat and mass transfer. Kavitha et al. [14] discussed the peristaltic transport of Jeffrey fluid in contact with Newtonian fluid in an inclined channel.

Peristaltic transport of two-layered power-law fluids in an axisymmetric tube is examined by Usha et al. [15]. Further over an extensive middle range of shear stress the logarithm of $\eta$ is linear in the logarithm of the shear stress. Taking the limitations on power-law model into consideration, Ellis suggested a three parameter model to describe the flow behavior even at low heat rate ranges by interchanging the roles of shear stress and strain rates. Ellis equation is given by Rathy [16].

$$\frac{dw}{dr} = \eta_0 \tau_{rz} + \eta_1 \tau_{rz}^{n-1} \tau_{rz}$$

Ellis fluid model is one of the fluid models where there exists a non linear relationship between the shear stress and strain rate. This fluid model has its significance as the three major categories for fluid are depicted for different values of nonlinear factor $\eta_1$. for $\eta_1 = 0$ this model represents Newtonian fluids, for $\eta_1 < 0$ it represents shear thickening fluids, and for $\eta_1 > 0$ it exhibits the behavior of shear thinning fluids.

The composition and rheology of gastric mucus have a major effect on the chime transport in the gastro intestinal tract. Further, the rate gastric mucus if observed to behave like a shear-thinning fluid Zahm et al. [17]. Narahari et al. [18] the peristaltic pumping flow of an Ellis fluid through a circular tube is studied and frictional force F is observed to have opposite behavior when compared to pressure difference. Vajravelu et al. [19] Peristaltic flow of Herschel-Bulkely fluid in an inclined tube is analyzed and the results for the flow characteristics reveal many interesting behaviors that warrant further study of the effects of Herschel–Bulkley fluid on the flow characteristics. Nadeem et al. [20] investigated the tangent hyperbolic fluid in a uniform inclined tube and the pressure rise, frictional force are found with help of numerical integration. Pandey et al. [21] illustrated the influence of magnetic field on the peristaltic flow of viscous fluid through a finite-length cylindrical tube and also studied the electrical conductivity on the mechanical efficiency of a peristaltic pumping characteristic.

Hemadri Reddy et al. [22] the effect of induced magnetic field on peristaltic pumping of a Carreau fluid in an inclined symmetric channel filled with porous material. The effects of
various parameters on pumping characteristics and frictional forces are discussed. Hemadri Reddy et al. [23] made a study about the Peristaltic pumping of a non-Newtonian micropolar fluid in an inclined channel. The pressure rise over one wavelength and the pressure rise over one cycle of the wave and frictional force are obtained. Srinivas et al. [24] discussed the effect of thickness of the porous material on the peristaltic pumping when the inclined channel walls are provided with non-erodible porous lining. Hari Prabakaran et al. [25] analyzed the peristaltic transport of a Bingham fluid in contact with a Newtonian fluid in an inclined channel. This model is useful to understand the peristaltic pumping of blood in inclined small vessels.

2. MATHEMATICAL FORMULATION

Consider the peristaltic flow of an Ellis fluid through a vertical uniform tube of radius \(a\) (figure 1). The peristaltic waves is represented by

\[
R = H(Z,t) = a + b \sin \frac{2\pi}{\lambda}(Z - ct)
\]

(1)

Where \(a\) and \(b\) are the amplitude, \(\lambda\) is the wavelength and \(c\) is the wave speed.

![Physical Model](image)

**Figure 1 Physical Model**

Under the assumptions that the tube length is an integral multiple of the wavelength \(\lambda\) and the pressure difference across the ends of the tube is a constant, the flow is inherently unsteady in the laboratory frame \((R, \theta, Z)\) and becomes steady in the wave frame \((r, \theta, z)\) which is moving with velocity \(c\) along the wave. The transformation between these two frames is given by

\[
r = R, \quad \theta = \theta, \quad z = Z - ct, \quad \psi = \Psi - \frac{R^2}{2} \quad \text{and} \quad p(Z,t) = p(z)
\]

(2)

Where \(\psi\) and \(\Psi\) are stream functions in the wave and the laboratory frame respectively. We assume that the flow is inertia free and the wavelength is infinite. Using the non-dimensional quantities.
\[
\bar{r} = \frac{r}{a}, \quad \bar{z} = \frac{z}{\lambda}, \quad \bar{F} = \frac{F}{\lambda \mu c}, \quad \bar{w} = \frac{w}{c}, \quad \bar{\tau} = \frac{\tau}{\mu \left(\frac{c}{a}\right)^n}, \quad \bar{\tau}_e = -\frac{\tau_{rz}}{\mu \left(\frac{c}{a}\right)^n}
\]
\[
\bar{p} = \frac{pa^{n+1}}{\lambda c^* \mu}, \quad \bar{\rho} = \frac{\rho}{\alpha}, \quad \bar{q} = \frac{q}{\pi a^2 c}, \quad \phi = \frac{b}{a}, \quad \bar{p}_r = \frac{\mu c_{\rho}}{k}, \quad \bar{u} = \frac{uac}{\lambda}
\]
\[
\bar{Q} = \frac{Q}{\pi a^2 c}, \quad \bar{\psi} = \frac{\psi}{\pi a^2 c}, \quad u = \frac{1}{r} \frac{\partial \psi}{\partial z}, \quad w = \frac{1}{r} \frac{\partial \psi}{\partial r}, \quad \theta = \frac{T - T_i}{T_o - T_i}
\]
\[
E_c = \frac{c^2}{c_{\rho}(T_o - T_i)}, \quad \delta = \frac{a}{\lambda}, \quad R_e = \frac{\rho \sigma a^3}{\mu}, \quad E_1 = -\sigma a^3, \quad E_2 = \frac{ma^3 c}{\lambda^2 \mu}, \quad E_3 = \frac{Ca^3}{\lambda^2 \mu}, \quad B_r = \frac{\eta_m c^2}{T_o a^2}
\]

Where \( \bar{\sigma}, \bar{\nu} \) the radial and axial velocities in the wave are frame and \( \bar{\tau} \) is the yield stress.

The governing equations of motion and energy in simplified form can be written as (lubrication approach)

\[
\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \quad (4)
\]
\[
R_e \delta \left( \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \frac{1}{r} \frac{\partial (r \tau_{rz})}{\partial r} + \delta \frac{\partial (r \tau_{rz})}{\partial z} + \rho g \quad (5)
\]
\[
R_e \delta^3 \left( \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial r} + \delta^2 \frac{\partial (r \tau_{rz})}{\partial z} + \delta \frac{\partial (r \tau_{rz})}{\partial r} - \rho g \quad (6)
\]
\[
R_e \delta P_t \left( \frac{\partial \theta}{\partial r} + \frac{\partial \theta}{\partial z} \right) = \left[ \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial r^2} + \delta^2 \left( \frac{\partial^2 \theta}{\partial z^2} \right) \right] + Br \left( \frac{\partial \tau_{rr}}{\partial r} + \frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\partial \tau_{zz}}{\partial r} \right) \quad (7)
\]

Under the assumption of long wavelength and low Reynolds number Equations (5) - (7) takes the following form

\[
\frac{1}{r} \frac{\partial}{\partial r} \left[ r \tau_{rz} \right] = \frac{\partial p}{\partial z} - \frac{1}{F} \quad (8)
\]
\[
\frac{dw}{dr} = \tau_{rz} + \eta_1 |F|^{-1} \tau_{rz} \quad (9)
\]
\[
\left( \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial r^2} \right) + Br \tau_{rz} \left( \frac{\partial w}{\partial r} \right) = 0 \quad (10)
\]

Where \( B_r = E_r P_t \) and \( F = \frac{\mu c^n}{\rho g a^{n+1}} \)

\[
w = 0, \quad \text{at} \quad r = h(z) = a + b \sin \frac{2\pi}{\lambda} (z - t) \quad (11)
\]
\[
\tau_{rz} = 0 \text{at} \quad r = 0 \quad (12)
\]
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\[ \Psi = 0, \quad \text{at} \quad r = 0 \quad (13) \]

\[ \theta = 0, \quad \text{at} \quad r = h \quad (14) \]

\[ \frac{\partial \theta}{\partial r} = 0, \quad \text{at} \quad r = 0 \quad (15) \]

The governing equation of motion of the flexible wall is given by

\[ L(H) = p - p_0 \quad (16) \]

Where \( L \) is the operator that is used to characterize the motion of the stretched membrane with damping forces and \( p_0 \) is the pressure on the outside surface of the wall due to tension in the muscle, which is assumed to be zero, and \( L \) can be written as

\[ L = -\sigma \frac{\partial^2}{\partial z^2} + m \frac{\partial^2}{\partial t^2} + C' \frac{\partial}{\partial t} \quad (17) \]

After dimensionless it becomes

\[ \frac{\partial p}{\partial z} - \frac{1}{F} = \frac{\partial L(H)}{\partial z} = \left( E_1 \frac{\partial^3 h}{\partial z^3} + E_2 \frac{\partial^3 h}{\partial z \partial t^2} + E_3 \frac{\partial^3 h}{\partial z \partial t} \right) = A \quad \text{at} \quad r = h \quad (18) \]

3. SOLUTION OF THE PROBLEM

Equation (8) after simplification with boundary condition (12) can be written as

\[ \tau_{\varepsilon z} = \frac{rA}{2}, \quad (19) \]

\[ A = p - k, \quad p = \frac{\partial p}{\partial z}, \quad k = \frac{1}{F} \]

Substituting (19) in to (9) and using boundary condition (11) we get

\[ w = \left( r^2 - h^2 \right) \frac{A}{4} + \left( r^{n+1} - h^{n+1} \right) \frac{\eta A^n}{2^n (n+1)} \quad (20) \]

Now solving equation (10) using (14) and (15) we get

\[ \theta = \frac{BrA^2}{64} \left( h^4 - r^4 \right) + \left( h^{n+3} - r^{n+3} \right) \frac{Br\eta A^{n+1}}{2^{n+1} (n+3)^2} \quad (21) \]

Velocities in terms of stream function relation can be defined as

\[ u = -\frac{1}{r} \frac{\partial \psi}{\partial z}, \quad w = \frac{1}{r} \frac{\partial \psi}{\partial r} \quad \text{at} \quad r = h \quad \text{and} \quad \psi = 0 \quad \text{at} \quad r = 0 \quad (22) \]

By solving equation (18) we get

\[ A(z,t) = 8\pi^3 \phi \left[ -\cos 2\pi(z-t)(E_1 + E_2) + E_3 \frac{\sin 2\pi(z-t)}{2\pi} \right] \quad (23) \]
4. RESULTS AND DISCUSSION

Velocity Profiles

Equation (20) gives the expression for velocity as a function of w. Velocity profiles are plotted from Figure (2) - (7). Figure (2), (3), (4) is drawn to study the effect of rigidity parameter, stiffness parameter and viscous damping force parameter $E_1, E_2, E_3$ on the velocity distribution $w$. It is observed that the velocity profiles are parabolic and the velocity increases with increasing $E_1, E_2$ and whereas for $E_3$ velocity increases in small variations.

Figure (5) and (6) drawn to study the effect of Ellis parameter $\eta_1$ for shear thinning and shear thickening on velocity distribution $w$. It is observed velocity increases for shear thinning ($\eta_1 > 0$) and opposite behavior is depicted for shear thickening case ($\eta_1 < 0$). Figure (7) is drawn to study the effect of frictional force $F$ and on the velocity distribution $w$. It is observed that the velocity increases with increasing $F$.

![Figure 2](image1.png)  
**Figure 2** The variation of $w$ with $r$ for different values of $E_1$ for fixed $E_2=1, E_3=1.5$, $z=0.22$, $t=0.25$, $n=2$, $\phi=0.01$, $\eta_1=0.5$, $F=0.5$ 

![Figure 3](image2.png)  
**Figure 3** The variation of $w$ with $r$ for different values of $E_2$ for fixed $E_1=0.5$ $E_3=1.5$, $z=0.22$, $t=0.25$, $n=2$, $\phi=0.01$, $\eta_1=0.5$, $F=0.5$ 

![Figure 4](image3.png)  
**Figure 4** The variation of $w$ with $r$ for different values of $E_3$ for fixed $E_1=0.5$, $E_2=1$, $z=0.22$, $t=0.25$, $n=2$, $\phi=0.01$, $\eta_1=0.5$, $F=0.5$ 

![Figure 5](image4.png)  
**Figure 5** The variation of $w$ with $r$ varying $E_1, E_2, E_3$ with $\eta_1=0.5$ and for fixed $z=0.22$, $t=0.25$, $n=2$, $\phi=0.01$, $F=0.5$
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Figure 6 The variation of $w$ with $r$ varying $E_1, E_2, E_3$ with $\eta_1 = -0.5$ and for fixed $z=0.22$,  
$t=0.25, n=2, \phi =0.01, F=0.5$  
Figure 7 The variation of $w$ with $r$ for different values of $F$ for fixed $E_1=0.5, E_2=1, E_3=1.5$,  
$z=0.22, t=0.25, n=2, \phi =0.01, \eta_1 =0$.

Temperature Profiles

Equation (21) gives the expression for temperature as a function of $r$, the temperature profiles are almost parabolic are plotted from Figure (8) to (13). In Figure (8) - (12), we observed that temperature increases with increasing $Br, E_1, E_2, E_3$. Figure (13) and (14) is drawn to study the effect of temperature for shear thinning and shear thickening case. It is observed that temperature increases in shear thinning case and opposite behavior is depicted for shear thickening case.

Figure 8 The variation of $\theta$ with $r$ for different values of $Br$, for fixed $E_1=0.5, E_2=1, E_3=1.5$,  
$z=0.22, t=0.25, n=2, \phi =0.01, \eta_1 =0.5$  
Figure 9 The variation of $\theta$ with $r$ for different values of $E_1$, for fixed $E_2=1, E_3=1.5, Br=1$  
$z=0.22, t=0.25, n=2, \phi =0.01, \eta_1 =0.5$

Figure 10 The variation of $\theta$ with $r$ for different values of $E_2$, for fixed $E_1=0.5, E_3=1.5, Br=1$  
$z=0.22, t=0.25, n=2, \phi =0.01, \eta_1 =0.5$  
Figure 11 The variation of $\theta$ with $r$ for different values of $E_3$, for fixed $E_1=0.5, E_2=1, Br=1$  
$z=0.22, t=0.25, n=2, \phi =0.01, \eta_1 =0.5$
5. CONCLUSIONS

In the current study, we have discussed the peristaltic flow of an Ellis fluid in a vertical uniform tube with wall properties. Exact solution is calculated for velocity and temperature profiles.

- Velocity profiles are increasing for increasing rigidity parameter, stiffness parameter whereas velocity increases in small variations for damping force parameter.
- Velocity profiles are increasing for Ellis parameter in shear thinning case and opposite behavior depicted for shear thickening case.
- Velocity profiles are increasing for increasing friction force parameter $F$.
- Temperature profiles are increasing for increasing Brickman number, rigidity parameter, stiffness parameter and viscous damping force parameter.
- Temperature profiles are increasing in shear thinning case and opposite behavior is observed in shear thickening case.

REFERENCES

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