DESIGNING THREE-LAYER PLATES WITH VISCOELASTIC MIDDLE LAYER

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ABSTRACT

Growing demand in innovative efficient designs and materials with predetermined properties for various industries stipulated necessity of intensive development of theory of multilayer plates.

Design of multilayer plates in three-dimensional formulation is a complicated problem of structural mechanics. Nevertheless, some problems have been solved in three-dimensional theory of elasticity. However, in this work these procedures are not sufficiently efficient in terms of provision of minimum weight indices at required strength and rigidity.

In conducted studies it has been revealed that the existing designs not always meet all the requirements to modern designs and engineering elements. In such cases it is necessary to use multilayer designs.

This article stipulates the necessity to consider for specific peculiarities of elastic-plastic layers in multilayer designs and combined action of layers under external loads. Theoretical results of bending three-layer panels with foamed polyurethane filler are presented, criteria of rigidity provision are highlighted, a method of static design of three-layer panels with metallic shells is proposed.

This article also discusses the variant of multilayer plate where all layers have similar or close elastic modulus. This article stipulates necessity to calculate such designs as multilayer plates.

Key words: plate, stress and strain state, sandwich panel, door panel, multilayer plate, three-dimensional theory of elasticity, strength, rigidity


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1. INTRODUCTION

Modern building materials widely apply three-layer materials combined into a single panel, where the top and the bottom layers are thin sheets of strong material (for instance, steel),
joint by a layer of viscoelastic material (Fig. 1). Similar structures or their elements are some kind of matrix reinforced by high-strength and high-modulus fibers.

Such multilayer plates and shells become more and more popular in everyday life. They include various multilayer building materials, sandwich panels, leaves of metal entry doors filled with foamed polyurethane.

These designs are exactly multilayer plates.

High demand of various building industries for innovative efficient designs and building materials with predetermined properties stipulated necessity of intensive development of theory of multilayer plates.

2. METHODS
Aspects of design of multilayer, mainly three-layer, designs have been intensively developed in recent 15-20 years [1-4].

![Multilayer sandwich panels.](image)

In this case the design methods will be the same as for bending flat three-layer panels (including light transmitting), as well as for three-layer panels operating with compressing and bending stresses, and for three-layer spatial structures: shells.

Designing of such layered plates in three-dimensional formulation is a complicated problem of structural mechanics, nevertheless, some problems have been solved in three-dimensional theory of elasticity [5].

However, in this case the problem becomes more difficult due to existence of viscoelastic layer with the elastic modulus sufficiently different from other layers, thus, application of the classic theory of plates based on hypothesis of straight-normal hypothesis results in significant errors [2].

As a consequence, designing three-layer and multilayer plates with relatively soft fillers is possible due to application of precise methods of theory of elasticity, otherwise, by introducing certain hypotheses reflecting specificity of design operation and simplifying solution of the problem without significant errors [1].

Therefore, design of three-layer panels and shells is possible on the basis of precise methods of the theory of elasticity [6, 7].
Unfortunately, application of three-dimensional equations of elasticity is sufficiently complicated mathematical problem requiring for numerical methods in general case. Thus, various two-dimensional theories are widely applied in practice, which permits to reduce the time of problem solution using analytical approaches [4].

Since the middle layer resists relatively weakly against bending strain and in three-layer designs is accompanied by displacement with regard to shells, then the straight-normal hypothesis used in conventional theory of one-layer plates and shells cannot be applied [8, 9].

In working conditions, the theory of one-layer plates and shells is the most reasonable in terms of provision of minimum weight indices with required strength and rigidity. However, it not always meets all the requirements to structural designs and modern engineering elements, thus, it is required to apply multilayer designs. Theories of multilayer plates in addition to technical theory should take into account transversal strain and related factors. Consideration for specific features of soft layers in multilayer designs and combined operation of package layers under the impact of external loads, low transversal resistance of fillers result in multivariability of existing theories [10-13].

Let us consider a three-layer plate in the form of sandwich-panel of constant thickness. This plate is referred to Cartesian coordinates related with external surface of the first layer (Fig. 2). It is assumed that the contact between the layers excludes their separation and mutual slipping. The number of layers in the package is three, \( h_i \) is the thickness of the i-th layer.

Let us orient the plate in Cartesian coordinates \( x, y, z \) so that the plane \( z = 0 \) matches median plane of middle layer and axis \( y \) coincides with longitudinal direction of the plate [14-15].

**Figure 2** Design scheme of three-layered plate clamped at the edges and bending over cylindrical surface

Deformation of the given three-layer plate can be described using generalized theory of multilayer plates [3]. This approach makes it possible to select accuracy of description of stress-strain state as a function of composition of sandwich-panel (Fig. 3).
Figure 3 Determination of internal forces in a stressed point upon bending.

In transversal cross sections of plate normal and shearing stresses occur which can be determined as follows [3]:

\[
\sigma_x = \frac{M}{I_x} z; \quad \sigma_y = \frac{M}{I_y} z \\
\tau_{xz} = \frac{Q_x}{2l} \left( \frac{h^2}{4} - z^2 \right); \quad \tau_{yz} = \frac{Q_y}{2l} \left( \frac{h^2}{4} - z^2 \right)
\]

(1)

(2)

where \( I_x \) is the inertia moment of transversal cross section with regard to neutral axis; \( z \) is the distance between neutral axis to the considered fiber.

Internal forces \( M_x, M_y, Q_x, Q_y \) are determined by the bending function \( f = (x, y) \):

\[
M_x = -D \left( \frac{\partial^2 f}{\partial x^2} + \nu \frac{\partial^2 f}{\partial y^2} \right) \\
Q_x = -D \frac{\partial^2 f}{\partial x^2} (\nabla^2 f); Q_y = -D \frac{\partial^2 f}{\partial y^2} (\nabla^2 f)
\]

(3)

(4)

Partial differential equation for rectangular plate, where variables are taken from Cartesian coordinates (along \( x, y \)), can be presented as follows:

\[
\nabla^2 \nabla^2 f = \frac{\partial^4 f}{\partial x^4} + 2 \frac{\partial^4 f}{\partial x^2 \partial y^2} + \frac{\partial^4 f}{\partial y^4} - \frac{1}{D} q,
\]

(5)

where \( D = \frac{Eh^3}{12(1-\nu^2)} \) is the plate cylindrical rigidity; \( E \) is the elastic modulus of the 1-st kind (coefficient of proportionality) of the plate; \( \nu \) is the Poisson coefficient; \( q \) is the intensity of distributed load.

Using calculation procedure of three-layer plates [2] let us determine maximum normal stresses in the top layer of three-layer plate:

\[
\sigma_{\text{max}} = \frac{qB_2}{2Bfr(h_2-h_1)^2} \left[ \frac{t_2^2}{24} + \frac{cBfr}{G} \right] - \frac{q}{t_2(h_2-h_1)} - \frac{cBfr}{G}
\]

(6)

plate bending in arbitrary cross section:
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\[ f = \frac{q x (l-x)}{2 B f r (h_2-h_1)^2} \left[ \frac{c B f r}{G} + \frac{x(l-x)}{12} \right] \]  
(7)

and maximum bending in the plate middle:

\[ f = \frac{q l^2}{8 B f r (h_2-h_1)^2} \left[ \frac{c B f r}{G} + \frac{l^2}{48} \right] \]  
(8)

where \( f \) is the bending in arbitrary cross section; \( q \) is the intensity of distributed load; \( G \) is the elastic modulus of the second kind (shear modulus), \( B f r \) is the coefficient taking into account interaction between the layers.

The presented above procedure (8) can be applied for three-layer plates with two carrying layers and middle layer: filler (usually polymer).

In the case when all plates are made of similar material with one and the same elastic modulus, then on the basis of Eq. (1) and principle of independence of forces the equation for bending of multilayer plate can be written as follows:

\[ f_{max} = \frac{q a^4}{64 (D_1+D_2+D_3+\cdots+D_n)} \]  
(9)

3. RESULTS

Let us consider designing features of three-layer plates exemplified by door panel.

Nowadays the market of building materials in Russia is sufficiently saturated with multilayer sandwich panels [16].

In addition to high amount of imported items, significant market portion is presented by Russian building materials. Metal entry doors occupy significant portion of the market. Due to increasing competitiveness between the manufacturers of steel doors, this design is continuously modified on the basis of new technological solutions. One of such engineering approaches is filling internal space of door panel with foamed polyurethane as heat and sound insulator (Fig. 4). In addition to increased heat and sound properties, rigidity characteristics of such doors also alter.

In order to determine possible strains (bending) of such door panels, let us apply the presented above procedure.

**Figure 4** Panel door with internal layer of foamed polyurethane.
The internal layer strain can be presented in finite element formulation using software (Fig. 5).

Figure 5 Strain distribution in intermediate plate in the middle of span.

4. CONCLUSIONS

This work presents partial solution of three-layer plates with thin shells.

Capabilities of this method provide sufficiently precise determination of strains of three-layer plates (roof element in the form of sandwich-panel or door panel with polyurethane filler) both in the middle of span and in arbitrary cross section.

In addition, this article considers the case when all plates in the package have identical elastic modulus. As demonstrated by calculations based on the proposed theory, the strain patterns of solid and combined plates are significantly different.

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