STUDY OF VEHICLE-BRIDGE COUPLED VIBRATION USING MATLAB/ SIMULINK

A. Saidi
Archi PEL Laboratory, University Tahri Mohamed of Bechar

A. Hamouine
Archi PEL Laboratory, University Tahri Mohamed of Bechar

I. K. Bousserhane
Laboratory of smart-Grid and renewable energies, University Tahri Mohamed Bechar

L. Fali
FIMAS Laboratory, University Tahri Mohamed of Bechar

ABSTRACT

The vibration of a bridge caused by passage of vehicles is one of the most imperative considerations in the design of a bridge as a common sort of transportation structure. In the last few years, Used Simulink for modeling and simulating dynamic systems because it has become the most widely used software package in academia and industry. A major goal of this study is to create a simplified model of a vehicle bridge system in MATLAB and present the basic features of Simulink focusing on modeling of dynamic systems of the bridge vehicle interaction problem. To solve the problem of coupling, the bridge is modeled as Euler Bernoulli beam and the vehicle is considered as 4 D.O.F half car model has been investigated using the Runge-Kutta method applied in Simulink of Matlab. The model will then be used to study the influence of the velocity to vehicle-bridge vibrations.

Key words: Bridge, Vibration, Vehicle Passage, Coupled Vibration.

1. INTRODUCTION

The bridges vibrations caused by the passing of vehicles or trains has been a subject of researches since the nineteenth century (Timoshenko, S.P 1922) [1]. Owing to the construction of high-speed railways and the upgrading of existing railways world-wide, the problem of train–bridge interactions has received increasing attention (Richardson H.H & Wormley D.N 1974) [2]. Previously, vehicles were often approximated as moving loads, which in many cases allows the problem to be solved analytically. Aimed at consideration of the inertia effect of the moving vehicles, the moving mass model has been adopted instead (Stanišić M.M & Hardin J.C 1969) [3]. However, for the case where the riding comfort or vehicle response is of concern, it is necessary to consider the effect of the suspension systems of the vehicles. The simplest model in this regard is a lumped mass supported by a spring–dashpot unit, often referred to as the spring mass model (Tan C.P & Shore S 1968) [4]. Although still more sophisticated models can be devised for the vehicles, the efficiency of solution of the vehicle–bridge interaction (VBI) system becomes an issue of great concern, especially when there exists a number of vehicles, say, those constituting a train in railway engineering. The VBI problem in this regard is complicated by its time-dependent and a multiple contact points.

In the vehicle–bridge interaction systems analyzes, two sets of second order equations of motion must be written each for the vehicles and for the bridge. The interaction forces existing at the contact points makes the two subsystems coupled. As the contact points moves, the system matrices are, in general, time-dependent and must be updated and factored at each time step in an incremental analysis. To solve these two sets of equations, the iterative methods are often adopted (Hwang E.S & Nowak A.S 1991) [5].

For instance, by assuming the contact points displacements, the vehicle motion equations can be solved to obtain the interaction forces, then the bridge motion equations are solved by improved values of contact point’s displacements. This completes the first cycle of iteration. One drawback with methods of iteration is that the convergence rate is likely to be low with the more realistic case of a bridge carrying a large moving vehicles number. In the literature, Lagrange’s equation with multipliers and constraint equations has also been used in (Blejwas T.E et al 1979) [6]. However, the use of Lagrange multipliers increases the number of unknowns and the effort of computation, especially with problems involving a large number of moving vehicles. Still another category of methods exists for solving the VBI problems, e.g., those based on the condensation method. Garg and Dukkipati used the Guyan reduction technique to condense the vehicle degrees of freedom (DOFs) to the associated bridge DOFs (Garg V.K & Dukkipati R.V 1984) [7]. Yang and Lin (Yang Y.B & Lin B.H 1995) [8] used the dynamic condensation method to eliminate all vehicle's DOF on the element’s rang. These methods have been demonstrated to be efficient for computing the bridge responses. However, because of the approximations made in relating the vehicle (slave) DOFs to the bridge (master) DOFs, they are not adequate for computing the vehicle responses, which serves as an indicator of the riding comfort generally required in the design of high-speed rail bridges. By using the Newmark finite difference scheme to discretize the vehicle equations, rather accurate master–slave relations have been established and used in eliminating the vehicle DOFs from the bridge.

All the analysis methods proposed for the VBI analysis are step-by-step dynamic methods. At each time step, it is required to solve a set of second order differential equations of motion. This methods have been more frequently adopted in solving second-order differential equations in VBI problems including the direct integration methods such as, Newmark method, Wilson’s method, and fourth-order Runge–Kutta method (Chu K.H et al 1979) [9]. In addition to the direct integration schemes.
Simulink (Simulation and Link) is an extension of MATLAB by Mathworks Inc. It works with MATLAB to offer modeling, simulation, and analysis of mechanical systems under a graphical user interface (GUI) environment. It supports linear and nonlinear systems, modelled in continuous time, sampled time, or a hybrid of the two. It allows engineers to rapidly and accurately build computer models of mechanical systems using block diagram notation. Using Simulink we can easily build models from presentative schemes, or take an existing model and add to it. Simulations are interactive, so we can change parameters “on the fly” and immediately see the results. As Simulink is an integral part of MATLAB, it is easy to switch back and forth during the analysis process and thus, the user may take full advantage of features offered in both environments. So we can take the results from Simulink and analyze them in Matlab workspace.

In this paper we present the basic features of Simulink focusing on modeling of dynamic systems of the bridge vehicle interaction problem.

2. MATHEMATICAL MODEL

The suspension system of a 4-degree of freedom half car model moving on a bridge showed in Figure II-1, the vehicle moves with a constant velocity \( u(t) \). The bridge is contemplated initially free of any load or deflection. In order to generate the governing equations of motion of the coupled system of beam half car model, the Hamilton's principle is applied:

\[
\int_{t_i}^{t_f} \delta(T - U) \, dt + \int_{t_i}^{t_f} \delta W_{nc} \, dt = 0
\]

(1)

Where \( \delta T \), \( \delta U \) and \( \delta W_{nc} \) are respectively the virtual total kinetic energy, virtual potential energy and virtual works of non-conservative forces of the system in the interval between \( t_i \) and \( t_f \).

Figure 1 Suspension System of a 4 D.O.F Half Car Model Moving on a Bridge.

\( m_s \) is the mass of the body and the frame of the vehicle, \( M_{t_1}, m_{t_2} \) are the mass of the axle between the front and back wheel set and the tires, \( K_{s1}, K_{s2}, C_{s1}, C_{s2} \) are the stiffness and damping between wheel set and the body of the vehicle, \( K_{t1}, K_{t2}, C_{t2}, C_{t2} \) are the stiffness and damping of the tires, \( a_1, a_2 \) are the displacement from the center of gravity to the back wheel set or to the front wheel set. \( a = a_1 + a_2, Y_{c1}, Y_{c2} \) the displacement on the point which the bridge contacts with the front and back wheel set in the vertical and \( \dot{Y}_{c1}, \dot{Y}_{c2} \) are the velocity at the point which the bridge contacts with the front and back wheel set.

2.1. Vehicle Model

The total kinetic energy of the system can be defined as:

\[
T = \frac{1}{2} \left\{ \int_0^L \rho \dot{y}^2 (x, t) \, dx + \int_0^L \rho l (x) \dot{\psi}^2 (x, t) \, dx + m_s \dot{y}_s^2 (t) + J \dot{\theta}^2 (t) \right\} + m_{t1} \ddot{y}_{t1}(t) + m_{t2} \ddot{y}_{t2}(t)
\]

(2)

Thus the virtual kinetic energy of the system can be described as:

\[
\delta T = \int_0^L \rho \dot{y} (x, t) \frac{\delta \dot{y}}{\delta t} \, dx + \int_0^L \rho l (x) \dot{\psi} (x, t) \frac{\delta \dot{w}}{\delta t} \, dx + m_s \dot{y}(t) \frac{\delta y_s}{\delta t} + J \dot{\theta}(t) \frac{\delta \theta}{\delta t} + m_{t1} \ddot{y}_{t1}(t) + m_{t2} \ddot{y}_{t2}(t)
\]

(3)
Consequently,

\[ \int_{t_i}^{t_f} \delta T dt = - \int_{t_i}^{t_f} \left( \int_0^L \rho \ddot{y}(x, t) dx \delta y + \int_0^L \rho I \dddot{\psi}(x, t) dx \delta \psi + m_3 \ddot{y}_2 \delta y_5 + m_4 \delta \theta + m_{t1} \dot{y}_{t1} \delta y_{t1} + m_{t2} \dot{y}_{t2} \delta y_{t2} \right) dt \]

The total potential energy of the system can be written as:

\[ U = \frac{1}{2} \left( \int_0^L EI \left( \frac{\partial^2 y}{\partial x^2} \right)^2 dx + \int_0^L k_2 A G \left( \frac{\partial y}{\partial x} + \psi \right)^2 dx + k_1 \dot{y}_5 + a_1 \theta - y_{t1} \right)^2 + \right. \]

\[ \left. k_2 \left[ y_5 - a_2 \theta - y_{t2} \right]^2 + k_{t1} \left[ y_{t1} - y(\alpha_1, t) \right]^2 H(x - \alpha_1) + k_{t2} \left[ y_{t2} - y(\alpha_2, t) \right]^2 H(x - \alpha_2) \right) \]

Where \( EI \) represents the flexural rigidity of the beam, and \( H(.) \) is the Heaviside function.

The total potential energy of the system can be written as:

\[ U = \frac{1}{2} \left( \int_0^L EI \left( \frac{\partial^2 y}{\partial x^2} \right)^2 dx + \int_0^L k_2 A G \left( \frac{\partial y}{\partial x} + \psi \right)^2 dx + k_1 \dot{y}_5 + a_1 \theta - y_{t1} \right)^2 + \right. \]

\[ \left. k_2 \left[ y_5 - a_2 \theta - y_{t2} \right]^2 + k_{t1} \left[ y_{t1} - y(\alpha_1, t) \right]^2 H(x - \alpha_1) + k_{t2} \left[ y_{t2} - y(\alpha_2, t) \right]^2 H(x - \alpha_2) \right) \]

The locations of the contact point of the front and rear tires with the bridge surface are given by the expressions:

\[ a_1(t) = u(t) + a_1 \]

\[ a_2(t) = u(t) - a_2 \]

The non-conservative virtual work of the system is:

\[ \int_{t_i}^{t_f} \delta W_{nc} dt = \int_{t_i}^{t_f} \left( - \int_0^L c \dot{y}(x, t) dx \delta y - C_1 \dot{S}_3 \delta S_3 - C_2 \dot{S}_4 \delta S_4 - \right. \]

\[ \left. C_1 \dot{S}_5 \delta S_5 H(x - \alpha_1) - C_2 \dot{S}_6 \delta S_6 H(x - \alpha_2) + \int_0^L f_g(x, t) dx \delta y \right) dt \]

The equation of the vertical motion (bounce) for the sprung mass (vehicle body) can be written as:

\[ m_3 \ddot{y}_5(t) + C_1 \left[ \ddot{y}_5(t) + a_1 \dot{\theta}(t) - \dot{y}_{t1}(t) \right] + C_2 \left[ \ddot{y}_5(t) - a_2 \dot{\theta}(t) - \dot{y}_{t2}(t) \right] = 0 \]

The equation of the angular motion (pitch) of the sprung mass has the form of:

\[ j\ddot{\theta}(t) + C_1 a_1 \left[ \ddot{y}_5(t) + a_1 \dot{\theta}(t) - \dot{y}_{t1}(t) \right] - C_2 a_2 \left[ \ddot{y}_5(t) - a_2 \dot{\theta}(t) - \dot{y}_{t2}(t) \right] + K_4 a_1 \left[ \ddot{y}_5(t) + a_4 \dot{\theta}(t) - \dot{y}_{t1}(t) \right] + K_2 a_2 \left[ \ddot{y}_5(t) - a_2 \dot{\theta}(t) - \dot{y}_{t2}(t) \right] = 0 \]

The equation of the vertical motion (bounce) for the front axle is:

\[ m_{t1} \ddot{y}_{t1}(t) + C_1 \left[ \ddot{y}_{t1}(t) - \ddot{y}_5(t) - a_1 \dot{\theta}(t) \right] + \]

\[ C_1 \left[ \ddot{y}_{t1}(t) - \ddot{y}_5(t) - a_1 \dot{\theta}(t) \right] + K_1 \left[ \ddot{y}_{t1}(t) - \ddot{y}_5(t) - a_1 \dot{\theta}(t) \right] + K_{t1} \left[ \ddot{y}_{t1}(t) - \ddot{y}_5(t) - a_1 \dot{\theta}(t) \right] = 0 \]

And the vertical motion (bounce) of the rear axle is governed by

\[ m_{t2} \ddot{y}_{t2}(t) + C_2 \left[ \ddot{y}_{t2}(t) - \ddot{y}_5(t) + a_2 \dot{\theta}(t) \right] + C_{t2} \left[ \ddot{y}_{t2}(t) - \ddot{y}(\alpha_2(t), t) \right] H(\alpha_2(t) - x) + \]

\[ K_2 \left[ \ddot{y}_{t2}(t) - \ddot{y}_5(t) + a_2 \dot{\theta}(t) \right] + K_{t2} \left[ \ddot{y}_{t2}(t) - \ddot{y}(\alpha_2(t), t) \right] H(\alpha_2(t) - x) = 0 \]

The dynamics of the bridge is described by two coupled equations. The first equation governs the traversed deflection of the beam(\( y \)) as:

\[ \int_0^L \rho \ddot{y}(x, t) dx - \left[ \int_0^L k_2 A G \left( \frac{\partial^2 y}{\partial x^2} + \frac{\partial \psi}{\partial x} \right) dx \right] \]

\[ + \int_0^L k_1 \dot{y}_{t1} \delta^*(x - \alpha_1) dx \]

\[ + \int_0^L \dot{y}_{t2} \delta^*(x - \alpha_2) dx + \int_0^L \dot{y}(\alpha_1(t), t) \delta^*(x - \alpha_1) dx \]

\[ + \int_0^L \dot{y}(\alpha_2(t), t) \delta^*(x - \alpha_2) dx + \int_0^L \dot{y}(x, t) dx - \int_0^L f_g(x, t) dx = 0 \]
And the second equation describes the orientation of the beam cross-section \((y)\) around \(z\) axis as:

\[
\int_0^L \frac{E}{I} \frac{\partial^2 y}{\partial x^2} \, dx = 0
\]

Transforming the equations into matrix we can get:

\[
[M_v] \{\dot{y}_v\} + [C_v] \{\dot{y}_v\} + [K_v] \{y_v\} = \{F_v\}
\]

Where \([M_v]\), \([C_v]\), \([K_v]\), \([y_v]\) and \([F_v]\) are respectively the mass matrix, the damping matrix, the stiffness matrix separately of the model, the vector of the degree of freedom of the vehicle and the vector of exciting force of vibration of the vehicle.

These matrices are given by:

\[
[M_v] = \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & J & 0 & 0 \\ 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & m_2 \end{bmatrix}, \quad [C_v] = \begin{bmatrix} c_{i1} + c_{i2} & c_{i1}a_1 - c_{i2}a_2 & -c_{i1} & -c_{i2} \\ c_{i1}a_1 - c_{i2}a_2 & c_{i2}a_1^2 + c_{i2}a_2^2 & -c_{i1}a_1 & c_{i2}a_2 \\ -c_{i1} & -c_{i1}a_1 & c_{i1} + c_{i2} & 0 \\ -c_{i2} & c_{i2}a_2 & 0 & c_{i2} + c_{i2} \end{bmatrix}
\]

\[
[K_v] = \begin{bmatrix} k_{i1} + k_{i2} & k_{i1}a_1 - k_{i2}a_2 & -k_{i1} & -k_{i2} \\ k_{i1}a_1 - k_{i2}a_2 & k_{i2}a_1^2 + k_{i2}a_2^2 & -k_{i1}a_1 & k_{i2}a_2 \\ -k_{i1} & -k_{i1}a_1 & k_{i1} + k_{i2} & 0 \\ -k_{i2} & k_{i2}a_2 & 0 & k_{i2} + k_{i2} \end{bmatrix}
\]

\[
[\theta] = \begin{bmatrix} y_s \\ \theta \end{bmatrix}, \quad \{F_v\} = \begin{bmatrix} y_s \\ y_t \end{bmatrix}, \quad \{F_v\} = \begin{bmatrix} k_{i1}y_{i1} + c_{i1}\dot{y}_{i1} \\ k_{i2}y_{i2} + c_{i2}\dot{y}_{i2} \end{bmatrix}
\]

### 2.2. Bridge Model

According to the Euler-Bernoulli beam theory:

\[
EI \frac{\partial^4 y_b (x, t)}{\partial x^4} + \rho \frac{\partial^2 y_b (x, t)}{\partial t^2} + \mu \frac{\partial y_b (x, t)}{\partial t} = -F(x, t) \delta (x - vt)
\]

\(E\) is Young’s modulus of the beam, \(I\) is the moment of inertia of the cross-section; \(EI\) is the flexural rigidity of the beam, \(\rho\) is the mass of the beam per unit length, \(\mu\) is the damping coefficient per unit length, \(F(x, t)\) is the coupled force on the beam, \(y_b (x, t)\) is the beam’s displacement on the contact \(l\) point in the vertical at time \(t\).

\(\delta\) is the function of Dirac has the following characteristics: \(f(x)\) is continuous in the closed interval \([a, b]\).

\[
\int_a^b f(x) \delta(x - t) \, dx = \begin{cases} 0, & (t < a) \\ f(t), & (a \leq t \leq b) \\ 0, & (t > b) \end{cases}
\]

### 2.3. The Vehicle-Bridge Coupled System

The function of mode of vibration:

\[
\phi_i(x) = \frac{1}{\sqrt{\rho l}} \sin \frac{inx}{l}
\]

The absolute coordinates can be written as a function of modal coordinates with
\[ y_b(x, t) = \sum_{i=1}^{N} \varphi_i (x) \eta_i(t) \]  

Then we can get:

\[
\frac{\partial^2 y_b(x, t)}{\partial x^4} = \sum_{i=1}^{N} \varphi_i^{(4)} (x) \eta_i(t) ; \quad \frac{\partial^2 y_b(x, t)}{\partial t^2} = \sum_{i=1}^{N} \varphi_i (x) \ddot{\eta}_i(t) ; \quad \frac{\partial y_b(x, t)}{\partial t} = \sum_{i=1}^{N} \varphi_i (x) \dot{\eta}_i(t)
\]  

\[
\int_0^l \varphi_i(x) EI \sum_{j=1}^{N} \varphi_i^{(4)} (x) \eta_j(t)\,dx + \int_0^l \varphi_i(x) \rho \sum_{j=1}^{N} \varphi_i (x) \ddot{\eta}_j(t)\,dx + \int_0^l \varphi_i(x) \mu \sum_{j=1}^{N} \varphi_i (x) \dot{\eta}_j(t)\,dx = - \int_0^l \varphi_i(x) F(x, t) \delta(x-vt)\,dx
\]  

\[
EI \int_0^l [\ddot{\varphi}_i(x)]^2 \,dx \eta_n(t) + \mu \int_0^l \varphi_i^2(x) \,dx \dot{\eta}_n(t) + \rho \int_0^l \varphi_i^2(x) \,dx \ddot{\eta}_n(t) = - \varphi_n(vt) F(vt, t)
\]  

\[ 2 \zeta_n \omega_n = \frac{\mu}{\rho} ; \quad \omega_n^2 = EI \int_0^l [\ddot{\varphi}_i(x)]^2 \,dx = EI \left( \frac{\pi n}{l} \right)^4 \]

So, the mode equation for the bridge is:

\[ \ddot{\eta}_n + 2 \zeta_n \omega_n \dot{\eta}_n + \omega_n^2 \eta_n = -F(vt, t) \varphi_n(vt) \]

Because of the different number of contact points, the force will be different. For the model in the figure 1, there will be two forces on the contact points. So the vibration equation of the bridge can be written as:

\[ EI \frac{\partial^2 y_b(x, t)}{\partial x^4} + \rho \frac{\partial^2 y_b(x, t)}{\partial t^2} + \mu \frac{\partial y_b(x, t)}{\partial t} = -\sum_{i=1}^{N} F_i(x_i, t) \delta_i(x_i - vt) \]

Then, the mode equation for the bridge will be:

\[ \ddot{\eta}_n + 2 \zeta_n \omega_n \dot{\eta}_n + \omega_n^2 \eta_n = -F_1(t) \varphi_{1n} \delta_1 - F_2(t) \varphi_{2n} \delta_2 \]

\[ \varphi_{1n} = \varphi_n(vt) = \frac{1}{\sqrt{\omega_n^2}} \sin \frac{\pi v t}{l} ; \quad \varphi_{2n} = \varphi_n(vt - a) = -\frac{1}{\sqrt{\omega_n^2}} \sin \frac{\pi v (vt - a)}{l} \]

\[ \delta_1(t) = \begin{cases} 1 & \text{if } 0 \leq t \leq \frac{l}{v} \\ 0 & \text{else} \end{cases} \quad \text{and} \quad \delta_2(t) = \begin{cases} 1 & \text{if } \frac{a}{v} \leq t \leq \frac{l+a}{v} \\ 0 & \text{else} \end{cases} \]

2.4. Road Surface Condition

The compatibility condition of the displacement is as follows:

\[ y_{ci} = y_b(x, t) + r(x, t) \]

Where \( y_{ci} \) is the displacement of the \( i \)th wheel on the contact point in the vertical direction, \( y_b(x, t) \) is the displacement of bridge on the \( i \)th contact point in the vertical direction, \( r(x, t) \) is the irregularity from the bridge’s surface on the \( i \)th point.

\[ y_{c1|x_1=vt} = y_b(x_1, t)|_{x_1=vt} + r(x_1)|_{x_1=vt} = \sum_{i=1}^{N} \varphi_i(vt) \eta_i(t) + r_1 \]

\[ y_{c2|x_2=vta} = y_b(x_2, t)|_{x_2=vta} + r(x_2)|_{x_2=vta} = \sum_{i=1}^{N} \varphi_i(vt - a) \eta_i(t) + r_2 \]

\[ r_1 = (vt) ; \quad r_2 = (vt - a) \]

Then we can get

\[ \dot{y}_{c1|x_1=vt} = \left[ \sum_{i=1}^{N} \varphi_i(vt) \dot{\eta}_i(t) + \varphi_i(vt) \dot{\eta}_i(t) \right] + \ddot{r}_1 \]

\[ \dot{y}_{c2|x_2=vta} = \left[ \sum_{i=1}^{N} \varphi_i(vt - a) \eta_i(t) + \varphi_i(vt - a) \dot{\eta}_i(t) \right] + \ddot{r}_2 \]
When the vehicle is running on the bridge, the force to the bridge from the \(i^{th}\) wheel is made up of two parts. One is the static load of the wheel and the vehicle body gravity that wheel undertakes; the other one is the elasticity from the transformation of the wheel and the damping force generate from the viscous damping. So the function is:

\[
F_i(x, t) = W_i - K_i (y_i - y_{ci}) - C_i (\dot{y}_i - \dot{y}_{ci})
\]

\(W_i\) is the static load of the wheel and the vehicle body gravity of the wheel undertakes

\[
F_1(x_1, t) = W_1 - k_{i1} (y_{i1} - y_{c1}) - c_{i1} (\dot{y}_{i1} - \dot{y}_{c1}) \quad W_1 = \left(m_i \frac{a_2}{a} + m_{i1}\right) g \tag{37}
\]

\[
F_2(x_2, t) = W_2 - k_{i2} (y_{i2} - y_{c2}) - c_{i2} (\dot{y}_{i2} - \dot{y}_{c2}) \quad W_2 = \left(m_i \frac{a_1}{a} + m_{i2}\right) g \tag{38}
\]

\[
c_{i1} (\dot{y}_{i1} - \dot{y}_{c1}) + k_{i1} (y_{i1} - y_{c1}) = -m_{i1} \ddot{y}_{i1} - \frac{a_2 m_i \ddot{y}_1}{a} - \frac{J \ddot{\theta}}{a} \tag{39}
\]

\[
c_{i2} (\dot{y}_{i2} - \dot{y}_{c2}) + k_{i2} (y_{i2} - y_{c2}) = -m_{i2} \ddot{y}_{i2} - \frac{a_1 m_i \ddot{y}_2}{a} + \frac{J \ddot{\theta}}{a} \tag{40}
\]

\[
\{ F(x, t) \} = \begin{cases} F_1(t) & \text{if } x_1 < a \\ F_2(t) & \text{if } x_1 > a \end{cases} \begin{cases} m_i \frac{a_2}{a} + m_{i1} \frac{a}{a} & g + m_{i1} \ddot{y}_{i1} - \frac{a_2 m_i \ddot{y}_1}{a} - \frac{J \ddot{\theta}}{a} \\ m_i \frac{a_1}{a} + m_{i2} \frac{a}{a} & g + m_{i2} \ddot{y}_{i2} - \frac{a_1 m_i \ddot{y}_2}{a} + \frac{J \ddot{\theta}}{a} \end{cases} \tag{41}
\]

Figure 2 The model of double axle’s vehicle-bridge.

3. NUMERICAL EXAMPLES

Using MATLAB Simulink to solve numerically the equation based with the Runge-Kutta method. We define the mass, stiffness and damping \([M], [K] \text{ and } [C]\) matrices at \(t_n=(t_{n-1}+\Delta t)\) time; And we Calculate effective stiffness matrix; after we calculate effective force \(\{F(t)\}\) and deflections at \(t_n\) time. Finally for each time step we calculate the accelerations and velocities at \(t_n\) time.

The properties of the Euler Bernoulli Beam and the Vehicle are given in Tables 1 and Table 2. The figure 3a shown the vertical displacements of the oscillator’s mass for an integration time step size \(\Delta t=0.00001\) and for moving velocities of \(v=50, 90,\) and \(120 \text{ km/h}\). The maximum
displacement of the oscillator occurs after it has passed the mid-point the velocity of $v=90$ km/h. Depending on the velocity and the interaction with the beam, the time and amplitude of the maximums are changed.

**Table 1** Properties of the Vehicle

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body mass ($m_s$)</td>
<td>1794 kg</td>
</tr>
<tr>
<td>Body Rotational Mass Moment of Inertia ($I$)</td>
<td>443.05 kgm²</td>
</tr>
<tr>
<td>First axle mass ($m_{i1}$)</td>
<td>87.15 kg</td>
</tr>
<tr>
<td>Second axle mass ($m_{i2}$)</td>
<td>140.4 kg</td>
</tr>
<tr>
<td>First Axle Damping Ratio ($C_{i1}$)</td>
<td>1190 Ns/m</td>
</tr>
<tr>
<td>Second Axle Damping Ratio ($C_{i2}$)</td>
<td>1000 Ns/m</td>
</tr>
<tr>
<td>First Tire Damping Ratio ($C_{t1}$)</td>
<td>14.6 Ns/m</td>
</tr>
<tr>
<td>Second Tire Damping Ratio ($C_{t2}$)</td>
<td>14.6 Ns/m</td>
</tr>
<tr>
<td>First Axle Stiffness ($K_{i1}$)</td>
<td>66824.4 N/m</td>
</tr>
<tr>
<td>Second Axle Stiffness ($K_{i2}$)</td>
<td>18615.0 N/m</td>
</tr>
<tr>
<td>First Tire Stiffness ($K_{t1}$)</td>
<td>01115.0 N/m</td>
</tr>
<tr>
<td>Second Tire Stiffness ($K_{t2}$)</td>
<td>01115.0 N/m</td>
</tr>
<tr>
<td>$b_1$</td>
<td>1.271 m</td>
</tr>
<tr>
<td>$b_2$</td>
<td>1.716 m</td>
</tr>
</tbody>
</table>

**Table 2** Properties of the Euler Bernoulli Beam

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Modulus</td>
<td>207 GPa</td>
</tr>
<tr>
<td>Mass per Unit Length ($\rho$)</td>
<td>20000 Kg/m</td>
</tr>
<tr>
<td>Second Moment of Inertia ($I$)</td>
<td>0.174 m⁴</td>
</tr>
<tr>
<td>Beam Structural Damping ($c$)</td>
<td>1750 Ns/m</td>
</tr>
<tr>
<td>Cross Sectional Area</td>
<td>4.94 m²</td>
</tr>
<tr>
<td>Beam Length ($L$)</td>
<td>100 m</td>
</tr>
</tbody>
</table>

**Figure 3** A simplified block diagram of the vehicle bridge interaction implemented in Matlab Simulink
Figure 4 The vertical displacement of vehicle in different speeds.

From figure 4, we can see that the maximum of the mid-span displacement will increase with the increasing speed of the vehicle, although the relationship is not linear. We also can note that the maximum of the mid-span displacement coming out is not when the vehicle runs on the middle of the bridge, which happens on the left or right of the mid-span. Figure 4 shows us also that the maximum displacement of the bridge happens when the vehicle locates in the front or back of the mid-span. With increasing speed, the maximum displacement will increase. But, when the speed is at some value, the value of the maximum displacement will be smaller than that in the front speeds. This is related to the resonance of the vehicle-bridge structures in combination with the speed of the vehicle.

4. CONCLUSION

In this study, the effects of different parameter of dynamic analysis of bridges under moving vehicle are investigated by constructing the models of the bridge and vehicle separately based on the theory of vehicle-bridge coupled vibration. The models were then connected by suing geometric relationships force equilibrium conditions to construct the relation between the equation of the vehicle vibration and the equation of bridge vibration. Finally, using MATLAB to handle the coupled problem based on the Runge-Kutta method. The MATLAB code in Simulink was then used to study the influence of various parameters especially the velocity’s one. As the simulation results shown, the speed of the vehicle has a significant effect on the dynamic response of the bridge. In general, the maximum dynamic displacement is remarked when the faster speed of vehicle is applied, also when the bridge span is bigger, the maximum of displacement will become bigger with the constant vehicle speed.
REFERENCES


