HYDRODYNAMIC MARANGONI CONVECTION
IN A NANOFLUID FLOW WITH SORET AND DUFOUR EFFECTS

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ABSTRACT
In the present paper Soret and Dufour effects on MHD marangoni convection flow are studied. Further the flow is influenced by thermal radiation and first order homogeneous chemical reaction. Three metallic nanoparticles Copper, Silver and Alumina are suspended in Water to form a nanofluid. Similarity transformation is used to obtain dimensionless governing equations of motion. These coupled equations of motion are solved by Mat Lab ’bvp4c’ routine function. The physical properties of the fluid flow are analyzed graphically.

Keyword: Thermal radiation, nanofluid, heat and mass transfer, marangoni convection, Soret and Dufour effects.

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1. INTRODUCTION
Nanofluids are formed when the submicron solid particles are suspended in common fluids like water. These are very stable and possess high thermal conductivity than common fluids [1]. Convection in porous medium has various applications in the field of engineering. In particular, as a post accidental heat removal in nuclear reactors, drying process, geothermal and oil recovery, heat exchangers, building construction etc.,([2]-[3]). To develop a high thermal conductivity in the fluids Choi [4] first introduced nanofluids. Studies on heat transfer in nanofluids with natural convection are available in the literature ([5]-[9]). Effects of Brownian diffusion and thermophoresis are studied by Buongiorno[10]. Khan et al.[11]
extended this work by considering a constant heat flux. A comparative study is made by Kuznetsov et al.[12] between regular and nanofluid under the natural convection on an isothermal vertical plate. Recently, marangoni convection in nanofluids, play a vital role in the field of semiconductor processing and crystal growth melting. This is because of either thermal or solutal convections. Pop et al.[13] studied the nature of marangoni convection in forced convective regular fluid over a permeable surface. They noticed that marangoni number maximizes the velocity and minimizes the temperature profiles. Later Hamid [14] examined dual solution problem with suction and injection effects on marangoni forced convection. A numerical solution is obtained for a hydrodynamic marangoni convection nanofluid flow problem by Sastry et al.[15]. In this paper, it is proposed to study the combined Soret and Dufour effects with forced marangoni convection in a homogeneous fluid over a vertical plate. So far there is no such attempt made in the literature.

2. MATHEMATICAL FORMULATION

A steady incompressible two-dimensional boundary layer flow consisting of three different nano species Copper, Silver and Alumina which are submerged in Water forming nanofluid. We assume that fluid is in-viscid and thermal equilibrium is established among the fluid and nano species. A Cartesian coordinate system \((x, y)\), where \(x\) and \(y\) are the coordinates measured along the plate and normal to it, respectively, is considered. Assume that flow takes place over \(y \geq 0\). Further assume that \(T(x)\) and \(C(x)\) are the temperature and concentration of the fluid within the boundary layer. Let \(T_\infty\) and \(C_\infty\) are the corresponding quantities in free stream. The surface tension is approximated by the Boussinesq approximation

\[
\sigma = \sigma_0 \left[ 1 - \gamma (T - T_\infty) - \gamma' (C - C_\infty) \right]
\]

where \(\sigma_0\) is interface surface tension ratio and

\[
\gamma = -\frac{\partial \sigma}{\partial T} \quad \text{and} \quad \gamma' = -\frac{\partial \sigma}{\partial C}
\]

Along the normal to the surface, a uniform magnetic field \(H_0\) is imposed. The steady state boundary layer equations for a nanofluid in the Cartesian coordinates are

\[
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0
\]

\[
u \frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma^*}{\rho_{nf}} H_0^2 u
\]

\[
u \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} = \frac{\alpha_{nf}}{\rho_{nf} c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho_{nf} c_p} \frac{\partial q_f}{\partial y} + D_{TC} \frac{\partial^2 C}{\partial y^2}
\]

\[
u \frac{\partial C}{\partial x} + \nu \frac{\partial C}{\partial y} = D_S \frac{\partial^2 C}{\partial y^2} + D_{CT} \frac{\partial^2 T}{\partial y^2} - K (C - C_\infty)
\]
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The relevant boundary conditions may be taken as
\[
\frac{\partial u}{\partial y} = \gamma \frac{\partial T}{\partial x} + \gamma' \frac{\partial C}{\partial x} \quad \text{and} \quad u \to 0, T \to T_\infty, \text{and } C \to C_\infty \text{ as } y \to \infty
\]  

\(u\) and \(v\) are velocity components along \(x\) and \(y\) axes respectively. \(D_s\) is species diffusivity. \(D_{CT}\) is quantity of mass flux through temperature gradient. \(D_{TC}\) is thermo-phoretic diffusivity. \(K\) is chemical reaction parameter. \(\sigma^*\) is electric conductivity. \(a, b\) are the coefficients of temperature and concentration gradients respectively.

Effective dynamic viscosity
\[
\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2\frac{s}{c}}}
\]

Effective density
\[
\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s
\]

Effective Heat capacitance
\[
(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s
\]

Effective thermal diffusivity
\[
\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}
\]

Effective thermal conductivity
\[
k_{nf} = k_f \left( \frac{k_s + 2k_f - 2\phi (k_f - k_s)}{k_s + 2k_f + \phi (k_f - k_s)} \right)
\]

Further \(\phi\) quantifies species volume fraction and the subscripts \(nf, f, s\) denote the thermo physical properties of the nanofluid, base fluid and nano-solid particles respectively.

Flow equations given by equations (3) – (6) are transformed to a set of coupled nonlinear ordinary differential equations by a similarity transformation. Let \(\eta = C_2y\) be similarity variable and \(\psi = C_1x f(\eta)\) be stream function which satisfies \(u = \frac{\partial \psi}{\partial x}\) and \(v = -\frac{\partial \psi}{\partial y}\)

where
\[
C_1 = \left( \frac{d\sigma}{dT_c} \frac{a \mu_f}{\rho_f^2} \right)^{\frac{1}{3}} \quad \text{and} \quad C_2 = \left( \frac{d\sigma}{dT_c} \frac{a \mu_f}{\mu_f^2} \right)^{\frac{1}{3}}
\]

In terms of similarity variable, Temperature and Concentration may be defined as
\[
T = T_\infty + \theta(\eta) ax^2 \quad \text{and} \quad C = C_\infty + g(\eta) bx^2
\]

Under Rosseland approximation, \(q_r = \frac{16}{3} \frac{\sigma}{k^*} T_\infty^3 \frac{\partial T}{\partial y}\)

where \(\sigma\) is Stefan-Boltzmann constant. \(k^*\) is mean absorption coefficient. Expanding \(T^4\) in Taylor’s series about \(T = T_\infty\) the energy equation (5) is modified as
where \( N_r = \frac{16 \sigma T_x^3}{3 k' K_m} \) is radiation parameter. Applying similarity transformation on equations (4), (12) and (6), one can derive the following non linear dimensionless ordinary differential equations:

\[
\begin{align*}
    f''' &= (1-\phi)^{2.5} \left\{ \left( 1-\phi + \phi \frac{\rho_u}{\rho_f} \right) \left( f''^2 - f'' \right) + M^2 f'' \right\} \\
    \theta'' &= \frac{P_r}{\lambda(1+N_r)} \left( 1-\phi + \phi \frac{(\rho C_p)_s}{(\rho C_p)_f} \right) \left( 2 f' \theta - f \theta' \right) - \frac{D_f}{1+N_r} g'' \\
    g'' &= S_r \left( 2gf' - fg + K^s g \right) - S_r \theta''
\end{align*}
\]

Further given boundary conditions (7) transform to

\[
\begin{align*}
    f(0) &= 0, \theta(0) = g(0) = 1, \left( \frac{1}{(1-\phi)^{2.5}} \right) f''(0) = -2(1+\varepsilon) \text{ at } y = 0 \\
    \text{and } f'(\infty) &= 0, \theta(\infty) = 0, g(\infty) = 0 \text{ as } y \to \infty
\end{align*}
\]

The non-dimensional constants such as Magnetic number \( (M) \), Reaction parameter \( (K^s) \), Schmidt number \( Sc \), Soret number \( Sr \), marangoni parameter \( \varepsilon \), Prandtl number \( Pr \) and Dufour number \( D_f \) which appear in above equations are defined as follows:

\[
M = \left[ \frac{1}{\sigma^{3/2} H_0 \mu_f^{1/6}} \right], \quad K^s = k_0 \left[ \frac{\mu_f \rho_f}{\left( \frac{d\sigma}{dT}_c \right)^2 \alpha^2} \right]^{1/3}, \quad S_r = \frac{V_f}{D_s}, S_r = \frac{D_{CT} (T - T_\infty)}{D_s (C - C_\infty)},
\]

\[
\varepsilon = \frac{\Delta C}{\Delta T} \left( \frac{d\sigma}{dT}_c \right), \quad P_r = \frac{V_f (\rho C_p)_f}{k_f}, \quad D_f = \Delta C \frac{D_{rC}}{\alpha_d \Delta T}.
\]

It is also essential to calculate local Nusselt and Sherwood numbers which measure the heat and mass transfer in the vicinity of the boundary layer are defined as

\[
Nu_x = \frac{x q_w(x)}{k_f (T - T_\infty)}
\]
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\[ Sh_t = \frac{x m_w(x)}{D_s (c - c_w)} \]  
(18)

where \( q_w(x) \) and \( m_w(x) \) represent heat and mass fluxes at wall are stated as

\[ q_w(x) = -k_{nf} \left( \frac{\partial T}{\partial y} \right)_{y=0} \quad \text{and} \quad m_w(x) = -D_s \left( \frac{\partial C}{\partial y} \right)_{y=0} \]  
(19)

Using non dimensional quantities one can get Local Nusselt and Sherwood numbers in dimensionless form as follows:

\[ \left( \frac{k_f}{k_{nf}} \right) \frac{Nu_s}{c_2 x} = -\theta'(0) \quad \text{and} \quad \frac{Sh_t}{c_2 x} = -g'(0) \]  
(20)

where \( C_2 x \) is a dimensionless quantity.

3. RESULT ANALYSIS

The coupled set of non-linear boundary layer equations (13)-(15) with the relevant boundary conditions (16) does not possess a closed form solution. Hence, it has been solved numerically, using MATLAB ‘bvp4c’ Routine Programme. The value of \( \eta \) is found to each iteration loop by the assigned statement \( \eta = \eta + \Delta \eta \). The step size \( \Delta \eta = 0.05 \) is used while obtaining the numerical solution with \( \eta_{\text{max}} = 10 \) and by considering a six decimal place for convergence criterion. Results are carried out by taking Prandtl number of water as 6.785. All the results are carried out for the following parametric values: \( M = Nr = Sr = D_f = 2; Sc = 0.6; \varepsilon = \phi = K^{*} = 0.2 \)

The analysis is mainly focused on the effects of Soret and Dufour on flow characteristics. Figure 1 depicts the influence of the Dufour number on temperature profile. Dufour number enhances the thermal boundary layer. Alumina particles exhibit low temperature profile. Soret number enhances both species concentration and concentration boundary layer of the nanoparticles. This can be viewed from the figure 2. Silver particles diffuse more rapidly than others. Schmidt number is inversely proportional to mass diffusion rate. So increase in Schmidt number will reduce the concentration level of the species, the same is noticed in figure 3. Nanoparticles temperature and concentration profiles are significantly affected by the marangoni parameter. A decrease in the thermosolutal surface tension ratio built up an increase in both profiles. This can be addressed like this: Increase in marangoni parameter will produce more induced flows within the boundary layer which enhances the velocity of the fluid flow, as a result both temperature and concentration profiles are suppressed. This is witnessed from the figure 4 and figure 5. The effect of Dufour number on heat and mass transfer can be seen from the figure 8 and figure 9. Increase of Dufour number enhances heat transfer rate and reduces mass transfer rate of the fluid particles. Further it is noticed that the rate of heat transfer in Alumina particles is more than that of Silver particles. A complete reversal is observed in the case of mass transfer. Figure 10 illustrates the behaviour of heat transfer on chemical reaction parameter. Heat transfer decreases with increase in the reaction parameter.
Figure 1 Dufour number vs Temperature

Figure 2 Soret number vs Concentration

Figure 3 Schmidt number vs Concentration

Figure 4 Marangoni number vs Temperature

Figure 5 Marangoni number vs Concentration

Figure 6 Soret number vs Nusselt number

Figure 7 Soret number vs Sherwood number

Figure 8 Dufour number vs Nusselt number
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Figures 9 and 10 illustrate the Dufour number vs Sherwood number and Chemical reaction vs Nusselt number, respectively.

<table>
<thead>
<tr>
<th>Physical property</th>
<th>Pure Water</th>
<th>Cu</th>
<th>Ag</th>
<th>Al₂O₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$ (kg/m³)</td>
<td>997.1</td>
<td>8933</td>
<td>10500</td>
<td>3970</td>
</tr>
<tr>
<td>$C_p$ (J/kg K)</td>
<td>4179</td>
<td>385</td>
<td>235</td>
<td>765</td>
</tr>
<tr>
<td>$k$ (W/m K)</td>
<td>0.613</td>
<td>401</td>
<td>429</td>
<td>40</td>
</tr>
</tbody>
</table>

Table 1: Thermo physical properties of nano particles.

4. CONCLUSIONS

The effect of heat and mass transfer in marangoni convective nanofluid over a flat plate is discussed. The base fluid contains Copper, Silver and Alumina nanoparticles. The influence of Dufour and Soret numbers are illustrated through graphs. Governing equations of motion are solved numerically using MATLAB ‘bvp4c’ routine. Soret number enhances both species concentration and concentration boundary layer of the nanoparticles. Dufour number enhances heat transfer rate and reduces mass transfer rate of the fluid particles. Further it is noticed that the rate of heat transfer is predominant in Alumina particles.

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