RELIABILITY ANALYSIS OF COUNTERFORT RETAINING WALL

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ABSTRACT

This paper presents stability analysis of typical counterfort retaining wall, accounting for uncertainties in the design input parameters in the framework of reliability theory. The factor of safety used for design of structure is unable to solve the uncertainties associated with the random variables. The First Order Reliability Method (FORM), Second Order Reliability Method (SORM) and Monte Carlo Simulation (MCS) Method are used to calculate the reliability index or probability of failure associated with various modes of failure (geotechnical and structural) of typical retaining wall. In addition, a sensitivity analysis is performed to evaluate the sensitivity of random variables involved in the stability analysis counterfort retaining wall. The results are presented in the form of reliability index and factor of safety for critical modes of failure. The results indicate that constant factor of safety is unable to get desired reliability index or probability of failure. In addition system reliability of counterfort retaining wall are also computed and partial correlation coefficient calculated for two modes of failure (sliding failure and overturning failure) between the random variables used in the counterfort retaining wall.

Key word: Counterfort Retaining Walls, Factor of Safety, FORM, SORM, MCS, Reliability Index, Probability of Failure.

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1. INTRODUCTION

Retaining walls are used to retain earth in a vertical position at locations where a sudden change in ground level occurs. The wall, hence, prevents the retained earth from considering its natural angle of repose. This causes the retained earth to exert a lateral pressure on the wall, thereby tending to bend, overturn and slide the retaining wall structure. Generally reinforced concrete cantilever walls are constructed for height up to 8 m and above this height, the bending moments generated becomes very large in the stem, heel slab and toe slab and hence, thickness required for the stem, heel and toe slab becomes larger. In order to reduce the bending moment, transverse supports called counterforts are placed at regular intervals and hence it is called a counterfort retaining wall.

Traditionally, in the deterministic design of retaining walls a ‘factor of safety’ approach is adopted to find the uncertainties in the input design parameters. The safety factor concept, however, has shortcomings as a measure of the relative reliability of structures and its performance is assigned single value (Fenton and Griffiths, 2008). Generally a constant factor of safety 1.5 is used for the design of structure (Retaining wall) against the instability of different modes of failure, in spite of considering various uncertainties involved in the various input design parameters. In this context, probabilistic approach provides more reliable approach for the analysis of geotechnical and structural engineering. These uncertainties are attributed to various factors such as complex geological variations, limited information of uncertain quality, poorer understanding of material behavior, material parameters such as modulus of concrete, steel stability of concrete and steel in different condition such as tension and flexure, soil unit weight, cohesion, angle of internal friction, pore water pressure, compressibility and permeability, mathematical modeling, methods of analysis, idealization of real situations, less controlled construction procedures, workmanship etc. Attempts have been made to account for these uncertainties in a more rational manner using probability theory by various researchers. Such as (Whitman, 1984; Christian et al. (1994), Chowdhury and Xu (1995), Liang et al. (1999), Phoon and Kulhawy, 1999; Duncan, 2000; Christian, 2004, Babu and Chouksey2013) and others have described excellent examples of use of reliability analysis in geotechnical engineering. Majority of the studies have focused on the reliability analysis of retaining walls for geotechnical failure modes (Hoeg and Muruga, 1974: Duncan, 2000; Fenton et al, 2005; Low 2005). However, few studies have focused on the reliability analysis of retaining walls for both geotechnical and structural failure modes (Sivakumar and Basha, 2008).

In the present study, the uncertainty associated with the input design parameters for a typical case of counterfort retaining wall is presented for geotechnical and structural modes of failure. The geotechnical failure modes, which need to be considered in the design of a counterfort retaining wall, are (i) horizontal sliding along the base of the wall, (ii) the rotation about the toe of the wall (overturning), and (iii) bearing capacity failure of the soil. The structural failure modes which are to be considered are the bending moment and shear failures of the toe, heel, stem and counterfort slabs. The results of the present study are presented in the form of reliability index or probability of failure vs. factor of safety based on FORM, SORM and MCS methods. The subsequent section presents the above discussion.

2. RELIABILITY ANALYSIS

Reliability is defined as the relationship between loads on the system must carry and its capacity to carry. Reliability of the system is expressed in the form of reliability index (β) and it is related to the probability of failure of the system (Pf = 1 - Φ(β)). Risk and reliability are complementary terms. Risk is unsatisfactory performance or probability of failure. On the other hand reliability is satisfactory performance or probability of success.
The basic aim of structural reliability is to ascertain whether the desired strength (Resistance) $R$ is much larger than the actual load (Strength) $S$ throughout the useful life of the structure. Due to uncertainties in the determination of strength and loads, reliability can only be established in probabilistic terms, i.e., $P(R > S)$. The resultant variables (Resistance and Strength) are usually functions of several random variables $X_1, X_2, X_3, \ldots, X_n$.

The limit state function, associated with failure can be expressed as $M = R - S = g(X_1, X_2, X_3, \ldots, X_n)$ \hfill (1)

Where, $M$ is referred to as the safety margin, which is a function $g(X)$.
The condition $g(X) < 0$ implies failure, $g(X) > 0$ implies stable behavior.

The boundary, defined by $g(X) = 0$, separating the stable and unstable states is called the limit state boundary.

The basic principles implied in FORM are as follows:
The variables $X = (X_1, X_2, X_3, \ldots, X_n)$ are transformed into a vector $U = (U_1, U_2, U_3, \ldots, U_n)$ of standardized and independent normal variables.

The limit state surface $g(U) = 0$, formulated in this new space, is approximated by its tangent hyper plane at the point of smallest distance $\beta$ to the origin as shown in figure 1 for the case of two random variables. The distance $\beta$ is called the reliability (or safety) index.

![Figure 1](image)

First order Reliability method for two variables

The point of smallest distance to the origin $u^*$ is called the design point. The probability of failure is estimated by $P_f = \Phi(-\beta)$ where $\Phi(*)$ standard normal distribution function. The failure probability decreases with the increase of $\beta$.

For greater accuracy, when the curvature of the surface at the design point is significant, an improved method called SORM can be used (Breitung, 1984).

The above equation 1 can be solved by an exact probabilistic method such as MCS method, where the probability of failure is computed from the joint probability distribution of the random variables associated with the loads and resistances. The MCS method allows the determination of an estimate of the probability of failure, given by

$$P_f = \frac{1}{N} \sum_{i=1}^{N} I(X_1, X_2, X_3, \ldots, X_n)$$ \hfill (2)

Using MCS, an estimate of the probability of structural failure is obtained by

$$P_f = \frac{N}{N_f}$$ \hfill (3)

Where $N_f$ is number of simulation cycle when $g(X) < 0$

$N$ = is total number of simulation cycle
All the above methods have been discussed with an example of typical counterfort retaining wall. The following section presents the formulation reliability index or probability of failure vs. factor of safety.

3. COUNTERFORT RETAINING WALL

Guidelines for design of counterfort retaining walls (Pillai and Menon, 2009) are available at optimal proportions of typical counterfort retaining walls. The counterfort retaining wall adopted in the present study is based on such guidelines which will ensure the required deterministic factor of safety. The objective of this study is to check whether the proposed deterministic factor of safety is sufficient to find the uncertainties associated with the input design parameters. A suitable counterfort retaining wall to support a level backfill, 7.5 m high above the ground level on the toe side. By assuming good soil for foundation at a depth of 1.5 m below the ground level with a safe bearing capacity of 170 kN/m². Further assuming the backfill to comprise granular soil with a unit weight of 16 kN/m³ and an angle of shearing resistance of 30°. Assume the coefficient of friction between soil and concrete to be 0.5. Use M 25 and Fe 415 steel. The typical cross section of the selected counterfort retaining wall is shown in Figure 2. The backfill of the wall comprises of granular soil with a unit weight of 16 kN/m³ and an angle of internal friction of backfill soil as 30°. Good soil for foundation is available at a depth of 1.5 m below the ground level with a safe bearing capacity of 170 kN/m². The thickness of the stem is linearly tapered from 300 mm at top to 600 mm at bottom. The coefficient of friction between the soil and concrete is 0.5. A shear key of size 0.30 m ×0.40 m is provided at a distance of 2.4 m from the toe as shown in Fig 2. The material considered are 25 MPa concrete and reinforcing bars having yield strength of 415 MPa. The counterforts having a thickness of 0.5 m are placed at a clear spacing of 3m.

Figure 2 Cross Section of the Counterfort Retaining Wall (Pillai and Menon, 2009)

4. RELIABILITY ANALYSIS OF COUNTERFORT RETAINING WALL

The reliability analysis of the counterfort retaining wall is carried out with the statistical distribution of the variables and the values of coefficient of variation (COV) as shown in Table1.
Table 1 Statistical properties of random variables for geotechnical failure modes.

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Random Variable</th>
<th>Mean</th>
<th>COV</th>
<th>Distribution</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Unit weight of Soil, ( \gamma_s ) (X1) KN/m³</td>
<td>16</td>
<td>0.7</td>
<td>Normal</td>
<td>Castillo et. al (2004)</td>
</tr>
<tr>
<td>2.</td>
<td>Angle of friction, ( \Phi ) (X2) degree</td>
<td>30</td>
<td>0.1</td>
<td>Normal</td>
<td>Castillo et. al (2004)</td>
</tr>
<tr>
<td>3.</td>
<td>Coefficient of friction, ( \mu ) (X3)</td>
<td>0.5</td>
<td>0.15</td>
<td>Normal</td>
<td>Castillo et. al (2004)</td>
</tr>
<tr>
<td>4.</td>
<td>Unit weight of Concrete, ( \gamma_c ) (X4) KN/m³</td>
<td>25</td>
<td>0.04</td>
<td>Normal</td>
<td>R. Ranganathan(1999)</td>
</tr>
<tr>
<td>5.</td>
<td>Cohesion of soil at base, ( C ) (X5) KN/m²</td>
<td>17</td>
<td>0.2</td>
<td>Normal</td>
<td>Shivkumar Babu and Basha(2008)</td>
</tr>
<tr>
<td>6.</td>
<td>Angle of friction between soil and base, ( \varphi ) (X6)</td>
<td>14</td>
<td>0.1</td>
<td>Normal</td>
<td>Shivkumar Babu and Basha(2008)</td>
</tr>
</tbody>
</table>

The following section presents different modes of failures (Geotechnical and Structural) and generation of performance functions for the estimation of reliability index or probability of failure using the following methods: (i) FORM, (ii) SORM, (iii) MCS method.

Geotechnical modes of failure

The various modes of geotechnical failures that can occur on the counterfort retaining walls are (i) Sliding, (ii) Overturning and (iii) Bearing failures.

Sliding Failure Mode

The factor of safety against sliding failure (FOS), can be computed as

\[ SF_s = \mu \frac{W}{P_a} \]  

Where, \( W \) is the weight of the retaining wall and backfill over the heel, \( \mu \) is the coefficient of friction at the interface of the concrete base slab and underlying soil and \( P_a \) is the total active earth pressure on the wall.

Expressing all the input design parameters such as \( \mu \), \( W \) and \( P_a \) in terms of the random variables \( X_1, X_2, X_3, X_4, X_5, \) and \( X_6 \) present in Table 1, the performance function \( g(X) \) (Equation 1) can be written as

\[ g(X) = R - S = \mu W - P_a \]

\[ g(X) = 21.675X_1X_3 + 6.405X_4X_3 - 40.5X_1\tan^2(45 - \frac{X_2}{2}) \]  

Overturning Failure Mode

The performance function for overturning mode of failure can be expressed as

\[ G(X) = M - S = W (L - X_w) - (P_aX_p) \]

Where, \( X_w \) is the distance of line of action of \( W \) from the heel; \( X_p \) is the distance of its line of action from the base of the wall; \( L \) denotes the base width of the retaining wall.

\[ g(X) = 71.397X_1 + 21.09X_4 - 121.5X_1\tan^2(45 - \frac{X_2}{2}) \]
Bearing Failure Mode
Factor of safety against bearing capacity failure can be defined as

\[ SF_b = \frac{q_u}{q_{max}} \]

Where \( q_u \) is Ultimate bearing capacity of a shallow foundation below the base slab of the retaining wall.

The performance function for bearing mode of failure can be expressed in terms of the variables

\[ g(X) = 0.6626X_0(\tan^2(\frac{X_0}{2})e^{3.14\tan X_2} - 1)\cot X_2 - 29.16X_1\tan^2(\frac{X_0}{2}) + 2.008X_1 - 1.5873X_4^4 + 5.0X_1 \tan X_2 (\tan^2(\frac{X_0}{2})e^{3.14\tan X_2} + 1)(1 - \frac{2.87}{X_2})^2 \]  

(7)

Equations (5), (6) and (7) present performance function for the sliding, overturning and bearing failure modes respectively. Based on the FORM, SORM and MCS reliability index or probability of failure and factor of safety is calculated for all the three cases. Table 2 presents result of the reliability analysis for the geotechnical modes of failure for the counterfort retaining wall for all the three failure modes.

<table>
<thead>
<tr>
<th>Mode of failure</th>
<th>Deterministic Factor of Safety (FOS)</th>
<th>Result of Reliability analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FORM</td>
<td>SORM</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>Pf</td>
</tr>
<tr>
<td>Sliding</td>
<td>1.80</td>
<td>0.0025</td>
</tr>
<tr>
<td>Overturning</td>
<td>1.905</td>
<td>8.451</td>
</tr>
<tr>
<td>Bearing</td>
<td>3</td>
<td>3.001</td>
</tr>
</tbody>
</table>

Structural Modes of Failure
Each panel of the stem and heel slab between two adjacent counterforts are designed as a two way slabs fixed on three sides, and free on the fourth side (free edge). The toe slab is designed as a cantilever slab to resist the factored moments and shear forces. For all the cases, a load factor of 1.5 is used which ensures the necessary factor of safety by deterministic method. In addition to the variables considered for the geotechnical modes of failure, some more variables are considered for the structural modes of failure and their statistical distributions are shown in Table 3. The amount of steel, depth of the beam and clear cover is different for stem slab, toe slab, heel slab and counterfort slab, but their COV/standard deviation was kept the same values as shown in Table 3. The random variables such as amount of steel, depth of slab and clear cover for toe slab, heel slab and counterfort are shown in Table 3. The generation of performance function with respect to the different random variables and failures modes is present in this section as follows.

(i) stem moment failure, (ii)stem shear failure, (iii) heel moment failure, (iv) heel shear failure (v) toe moment failure (vi) toe shear failure (vii) counterfort moment (viii) counterforts shear failure.
Figure 3 Reinforcement details on stem, toe slab and heel slab

Table 3 Statistical properties of random variables for structural failure modes

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Random Variable</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Distribution</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Concrete strength, $f_{ck}$ (X7) N/mm$^2$</td>
<td>25</td>
<td>2.5</td>
<td>Normal</td>
<td>Ranganathan (1999)</td>
</tr>
<tr>
<td>2.</td>
<td>Steel strength, $f_y$ (X8) N/mm$^2$</td>
<td>415</td>
<td>20.75</td>
<td>Normal</td>
<td>Shivkumar Babu and Basha (2008)</td>
</tr>
<tr>
<td>3.</td>
<td>Percentage steel, $A_s$ (X9) %</td>
<td>0.018</td>
<td>0.009</td>
<td>Normal</td>
<td>Shivkumar Babu and Basha (2008)</td>
</tr>
<tr>
<td>4.</td>
<td>Depth of the slab, D (X10) m</td>
<td>0.6</td>
<td>0.025</td>
<td>Normal</td>
<td>Ranganathan (1999)</td>
</tr>
<tr>
<td>5.</td>
<td>Clear cover to reinforcement, $S_v$ (X11)</td>
<td>70</td>
<td>0.025</td>
<td>Normal</td>
<td>Ranganathan (1999)</td>
</tr>
<tr>
<td>6.</td>
<td>Thickness of counterforts, $D_c$ (X12) mm</td>
<td>0.5</td>
<td>0.025</td>
<td>Normal</td>
<td>Ranganathan (1999)</td>
</tr>
<tr>
<td>7.</td>
<td>Thickness of stem at top, S (X13) m</td>
<td>0.3</td>
<td>0.025</td>
<td>Normal</td>
<td>Ranganathan (1999)</td>
</tr>
</tbody>
</table>

Stem Moment Failure

The factor of safety against stem moment is the ratio of the resisting moment (RM) to the overturning moment (OM), where the resisting moment can be computed as:

$$RM = 0.87f_yA_s(D - S_v - 0.42X_u)$$

Where $X_u$ is the neutral axis depth and can be computed as:

$$X_u = \frac{0.87f_yA_s}{0.36f_{ck}b}$$

The maximum negative moment ($M_{u-ve}$) occurs in the stem at the counterfort location and the moment can be computed as
Figure 4 Loading considerations for simplified analysis of stem

\[ M_{u-ve} = W_u \frac{L_{cc}^2}{12} \text{ and } W_u = P_a \gamma_s h/2 \]

Where \( P_a \) is active earth pressure coefficient and \( L_{cc} \) is the effective span between counterforts.

The performance function for moment failure mode at stem can be written as

\[
g(X) = 0.87X_8X_9(X_{10} - X_{11} - \frac{0.001015X_8X_9}{X_7})10^{-6} - 8.8765X_1 \tan^2(45 - \frac{X_2}{2}) \]  

(8)

**Stem Shear Failure**

The factor of safety against shear mode of failure is the ratio of the shear capacity of the section (\( RF \)) to the shear at the section due to the loads (\( SF \)).

The RF can be computed as

\[ RF = \tau_c b d \]  

(9)

Where the magnitude of \( \tau_c \) (shear capacity of a section) depends on the grade of concrete and percentage tension steel \( P_{st} = 100 A_{st} / b d \)

The SF for stem can be computed as

\[ SF = W_c(L_c/2 - D + S_v) \]  

(10)

The performance function for shear failure mode at stem can be written as

\[
g(X) = 0.5 \frac{X_7}{X_8} (1 - \sqrt{1 - \frac{1.119}{X_7}}) - 68 (1.5 - X_{10} + X_{11}) \]  

(11)

**Heel slab Moment Failure**

The resisting moment (\( RM \)) and the resisting shear force (\( RF \)) can be computed. Therefore the performance function for negative and positive moments

\[
g(X) = 0.87X_8X_9(X_{10} - X_{11} - \frac{0.001015X_8X_9}{X_7})10^{-6} + 0.905X_1 + 0.5188X_4 - 20.804X_1 \tan^2(45 - \frac{X_2}{2}) \]  

\[ g(X) = 0.87X_8X_9(X_{10} - X_{11} - \frac{0.001015X_8X_9}{X_7})10^{-6} + 0.679X_1 + 0.384X_4 \]  

(12)
Reliability Analysis of Counterfort Retaining Wall

- \(15.604X_i \tan^2 (45 - \frac{x}{2}) \quad (13)\)

**Figure 5** Loading considerations for simplified analysis of heel slab

The performance function of heel slab for cantilever action (moment failure)

\[ g(X) = 0.87X_0X_9(X_{10} - X_{11} - \frac{0.001105X_8X_9}{x_7})10^6 + 0.966X_1 + 1.858X_4 \]

\[-3.791X_i \tan^2 (45 - \frac{x}{2}) \quad (14)\]

The performance function for shear failure of stem

\[ g(X) = 0.5 \frac{x^7}{x_8} \left(1 - \sqrt{1 - \frac{1.119}{x_7}}\right) + 0.995X_1 + 0.439X_4 - 25.078X_i \tan^2 (45 - \frac{x}{2}) \quad (15)\]

**Table 4** Random variables for toe, heel and counterfort slabs.

<table>
<thead>
<tr>
<th>Section</th>
<th>Amount steel</th>
<th>Depth section</th>
<th>Clear cover</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toe slab</td>
<td>0.32</td>
<td>720mm</td>
<td>75mm</td>
</tr>
<tr>
<td>Heel slab (-ve moment)</td>
<td>0.28</td>
<td>500mm</td>
<td>75mm</td>
</tr>
<tr>
<td>Heel slab (+ve moment)</td>
<td>0.17</td>
<td>500mm</td>
<td>75mm</td>
</tr>
<tr>
<td>Counterfort</td>
<td>0.35</td>
<td>2287mm</td>
<td>5mm</td>
</tr>
</tbody>
</table>
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Figure 6 Pressure distribution on toe slab

**Toe Moment and Shear Failure Modes**
Performance function for toe moment can be expressed as

\[
g(X) = 0.87X_8X_9(X_{10} - X_{11} - \frac{0.001105X_8X_9}{X_7})10^{-6} + 0.6479X_1 + 1.8514X_4
\]
\[
- 102.35X_1\tan^2 \left(45 - \frac{X_2}{2}\right) (3000 + X_{12})10^{-3}
\]

Performance function for toe shear can be expressed as

\[
g(X) = 0.5 \frac{X_7}{X_9} (1 - \sqrt{1 - \frac{1.119}{X_7}}) + 0.5158X_1 + 0.8778X_4
\]
\[
- 17.496X_1\tan^2 \left(45 - \frac{X_2}{2}\right) (2000 - X_9 - X_{10})10^{-3}
\]

**Counterfort Moment and Shear Failure Mode**
Performance function for counterfort moment can be expressed as

\[
g(X) = 0.87X_8X_9(X_{10} - X_{11} - \frac{0.001105X_8X_9}{X_7})10^{-6}
\]
\[
- 102.35X_1\tan^2 \left(45 - \frac{X_2}{2}\right) (3000 + X_{12})10^{3}
\]

Performance function for counterfort shear can be expressed as

\[
g(X) = 0.5 \frac{X_7}{X_9} (1 - \sqrt{1 - \frac{1.119}{X_7}}) - 36.125X_1\tan^2(45 - \frac{X_2}{2})(3 + X_{12})10^{-3}
\]
\[
- 32.467X_1\tan^2 \left(45 - \frac{X_2}{2}\right) \frac{(3000+X_{12})(X_{10}-X_{11})}{X_{10}-X_{11}}
\]

Equations (8) to (19) present performance function for the various structural modes of failure respectively. Based on the FORM, SORM and MCS reliability index or probability of failure and factor of safety is calculated for all the nine cases. Table 5 presents result of the reliability analysis for the structural modes of failure for the counterfort retaining wall for all the nine failure modes.
A Civil structure consist of number of structural elements such as beam, column, slab, footing etc. and its strength depends on the strength of these elements. The probabilistic behavior of the structure depends upon the performance of individual component which is random in nature. If the performances of structures are compared with a system under certain condition, reliability of its element must be known to us. The associated reliability of the component is evaluated based on the performance under which it is made. For a system, failure can occurred in various ways and depends on the combination of its component failure. Combining this failure modes structure reliability can be obtained. In a broad way system is classified in three classes: (i) series system, (ii) parallel or redundant system and (iii) mixed system

With reference to the counterfort retaining wall problem for all the failure modes i.e. geotechnical and structural failure modes, reliability index/probability of failure is calculated. Table 4 and Table 5 present reliability index and probability of failure for the geotechnical and structural failure mode respectively. The system reliability is calculated based on all the probability of failures for all the modes. The following section presents evaluation of system reliability for all the three cases.

For series system reliability,
\[ P_{fs} = 1 - \prod_{i=1}^{n} P_f \]
\[ P_{fs} = 1 - (1 - 0.0025) \times (1 - 1.4 \times 10^{-15}) \times (1 - 0.0010) \times (1 - 8.0 \times 10^{-23}) \times (1 - 8.4 \times 10^{-19}) \times (1 - 1.2 \times 10^{-7}) \]
\[ \times (1 - 0.0059) \times (1 - 0.0021) \times (1 - 1.6 \times 10^{-5}) \times (1 - 1.5 \times 10^{-9}) \times (1 - 2.3 \times 10^{-4}) \times (1 - 5.5 \times 10^{-6}) \]
\[ P_{fs} = 7.9064 \times 10^{-19} \]

For series system reliability, \( P_{fps} = 1 - \prod_{i=1}^{n} P_f \)
\[ P_{fps} = 1 - (0.0025 \times 1.4 \times 10^{-15} \times 0.0010 \times 8.0 \times 10^{-23} \times 8.4 \times 10^{-19} \times 1.2 \times 10^{-7} \times 0.0059 \times 0.0021 \]
\[ \times 1.6 \times 10^{-5} \times 1.5 \times 10^{-9} \times 2.3 \times 10^{-4} \times 5.5 \times 10^{-6} \]
\[ P_{fps} = 1.02 \times 10^{-3} \]

Combined system reliability,
\[ P_c = (E1)P(E2) \]
6. DISCUSSION FOR THE SYSTEM RELIABILITY

In the given system reliability, the probabilistic behavior of the structure depends upon the performance of individual component which is random in nature. For a system reliability of counterfort retaining wall, different failure modes can be occurred in various ways such as geotechnical and structural failure modes. Combining these failure modes, structure reliability can be obtained by using series reliability, parallel reliability and combine reliability of system as given.

7. CORRELATION BETWEEN FAILURES

It is very important to know the correlation between different failure modes. This correlation between random variable could be possible through correlation coefficient. For example, sliding failure and overturning failure are the two geotechnical modes of failures. The possible random variables are \(x_1, x_2\) and \(x_4\) that represents unit weight of soil, angle of internal friction and unit weight of concrete respectively for the given modes of failures. Table 6 presents the mean (\(\mu\)), standard deviation (\(\sigma\)) and coefficient of variation (\(\delta\)) for sliding and overturning failures respectively. Table 7 presents correlation coefficients between random variables within each failure modes. The correlations between pairs of parameters belonging to different failure mode were assumed to be zero.

### Table 6 Shear strength parameters and Unit weight of soils (Pillai and Menon, 2009)

| \(\gamma_s\) | 1 | -0.70 | 0.12 |
| \(\gamma_c\) | -0.70 | 1 | 0.015 |
| \(\phi\) | 0.12 | 0.015 | 1 |

Notation:  \(\mu\)- mean,  \(\sigma\)- standard deviation,  \(\delta\)- coefficient of variation

### Table 7 Correlation coefficients between random variables within each failure modes

| Sliding failure mode | 16 | 30 | 25 | 16 | 30 | 25 |
| Overturning failure mode | \(\gamma_s\) | \(\phi\) | \(\gamma_c\) | \(\gamma_s\) | \(\phi\) | \(\gamma_c\) |
| \(\mu\) | 16 | 30 | 25 |
| \(\sigma\) | 11.2 | 3 | 1 | 11.2 | 3 | 1 |
| \(\delta\) | 0.7 | 0.1 | 0.04 | 0.7 | 0.1 | 0.04 |

In order to include correlation between geotechnical parameters which are regarded as random variables, the use of a special procedure, orthogonal transformation, is necessary before the correlation between safety margins associated with different failure modes can be calculated.

8. DISCUSSION FOR THE CORRELATION COEFFICIENT

Table 7 present correlation coefficient between random variables which are involve in the sliding failure mode and overturning failure mode. The correlation coefficient measures a degree to which two variables are related, it only measures the linear relationship between the variables. A value of exactly 1.0 means there is a perfect positive relationship between the two variables. For a positive increase in one variable, there is also a positive increase in the second variable. A value of exactly -1.0 means there is a perfect negative relationship between the two variables. This shows the variables move in opposite directions; for a positive increase in one variable, there is a
decrease in the second variable. If the correlation is 0, this simply means there is no relationship between the two variables. The strength of the relationship varies in degree based on the value of the correlation coefficient.

### 9. SENSITIVITY ANALYSIS

Sensitivity analysis finds the most sensitive random variables influencing the probability of failure. It is expressed in FORM analysis through the so-called ‘sensitivity factor’ \( \alpha \), which is measured in terms of the directional cosines of the position vector of the “design point in the transformed U-space, as shown in Figure 2 for a case involving two random variables.

#### Table 8 Sensitivity factors for geotechnical modes of failure.

<table>
<thead>
<tr>
<th>Random Variable</th>
<th>Sliding failure</th>
<th>Overturning failure</th>
<th>Bearing failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_s ) (X1)</td>
<td>0.061</td>
<td>0.128</td>
<td>0.076</td>
</tr>
<tr>
<td>( \Phi ) (X2)</td>
<td>-0.824</td>
<td>-0.988</td>
<td>-0.833</td>
</tr>
<tr>
<td>( \mu ) (X3)</td>
<td>-0.567</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \gamma_c ) (X4)</td>
<td>-0.035</td>
<td>-0.0814</td>
<td>0.019</td>
</tr>
<tr>
<td>C (X5)</td>
<td>-</td>
<td>-</td>
<td>-0.398</td>
</tr>
<tr>
<td>( \Phi_1 ) (X6)</td>
<td>-</td>
<td>-</td>
<td>0.385</td>
</tr>
</tbody>
</table>

#### Table 9 Sensitivity factors for structural modes of failure.

<table>
<thead>
<tr>
<th>Random variable</th>
<th>Stem moment</th>
<th>Stem shear</th>
<th>Heel moment</th>
<th>Heel moment</th>
<th>Heel shear</th>
<th>Toe moment</th>
<th>Toe shear</th>
<th>Counter moment</th>
<th>Counter shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_s ) (X1)</td>
<td>0.3906</td>
<td>0.420</td>
<td>0.415</td>
<td>0.4057</td>
<td>0.4278</td>
<td>0.4113</td>
<td>0.4693</td>
<td>0.449</td>
<td>0.482</td>
</tr>
<tr>
<td>( \Phi ) (X2)</td>
<td>-0.714</td>
<td>-0.80</td>
<td>-0.80</td>
<td>-0.786</td>
<td>-0.828</td>
<td>-0.759</td>
<td>-0.801</td>
<td>-0.80</td>
<td>-0.87</td>
</tr>
<tr>
<td>( \gamma_c ) (X4)</td>
<td>-0.017</td>
<td>-0.04</td>
<td>-0.02</td>
<td>-0.018</td>
<td>-0.044</td>
<td>-0.044</td>
<td>-0.068</td>
<td>-</td>
<td>-0.04</td>
</tr>
<tr>
<td>F_{ck} (X7)</td>
<td>-0.382</td>
<td>-0.24</td>
<td>-0.285</td>
<td>-</td>
<td>-0.385</td>
<td>-</td>
<td>-0.37</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>A_{at} (X9)</td>
<td>-0.032</td>
<td>-0.01</td>
<td>-0.02</td>
<td>-0.027</td>
<td>-0.013</td>
<td>-0.035</td>
<td>-0.018</td>
<td>-0.03</td>
<td>-0.01</td>
</tr>
<tr>
<td>D (X10)</td>
<td>-0.029</td>
<td>-0.40</td>
<td>-0.33</td>
<td>-0.364</td>
<td>-0.335</td>
<td>-0.321</td>
<td>-0.361</td>
<td>-0.08</td>
<td>-0.01</td>
</tr>
<tr>
<td>S_c(X11)</td>
<td>0.038</td>
<td>0.40</td>
<td>0.03</td>
<td>0.036</td>
<td>0.035</td>
<td>0.032</td>
<td>0.036</td>
<td>0.08</td>
<td>-</td>
</tr>
<tr>
<td>D_c(X12)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.05</td>
<td>0.05</td>
<td>-</td>
</tr>
<tr>
<td>S(X13)</td>
<td>-0.295</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

### 10. RESULTS AND DISCUSSION

**Geotechnical Mode of Failure:** Table 2 presents result of the reliability analysis based on FORM, SORM and MCS for the geotechnical modes of failure for the counterfort retaining wall are presented in the form of reliability index or probability of failure and factor of safety. It has been observed that the probability of failure obtained from all the three methods (FORM, SORM and MCS) fall in the same range for sliding, overturning and bearing failure modes respectively. In general, the factor of safety of 1.5 (usually adopted in the design practice of...
many countries) is not sufficient to get a ‘target’ reliability index of 3, that is required to avoid any unforeseen structural and geotechnical modes of failure which is desirable for engineering problems. From Table 2, it has been concluded that the sliding and bearing modes of failure resulted in critical modes of failure. The variation of reliability index with factor of safety for sliding, and bearing modes of failure are shown fig.8. The value of reliability index corresponding to FOS of 1.5 for sliding failure is1.5 whereas for bearing failure it is 2.6. In contrast, target reliability of 3 is required to maintain the stability of structures. Therefore, to achieve target reliability of 3 the FOS must be in the range 2.

![Figure 8 Variation of FS and reliability index for sliding and bearing mode of failure](image)

**Structural Mode of Failure**

Table 5 presents comparison of reliability index for all the nine modes of failure using FORM, SORM and MCS for the structural modes of failure. In majority of the cases, results of FORM, SORM and MCS can be comparable, because the performance functions have a distribution, which is close to the normal distribution. Whereas, MCS is not able to catch probability of failure for some failure modes such as stem moment, shear, toe shear, counterfort shear as the probability of failure is very less and a very large number of simulations are required to capture the probability of failure which cannot be achieved by using ordinary computers. In case of structural modes of failure mid span moment at heel (positive moment) and heel shear fall in critical modes of failure as their reliability index is less than 3 that is required for the target reliability. In order to achieve higher reliability index factor of safety has to be increase from the standard values. Fig.9 shows the variation of reliability index vs. FOS for heel moment and heel shear. It has been observed that the for FOS 1.5 the value of reliability index1.5 and 2.7 is obtained for the heel moment and heel shear respectively. Whereas the FOS of 2.8 is required to achieve the target reliability index of 3.Among the various modes of failure, mid span moment at heel (positive moment) is the most critical one.
Sensitivity Analysis
Table 8 and Table 9 present sensitivity analysis of geotechnical and structural modes of failure corresponding to the different random variables \((X_1, X_2 \ldots X_{13})\) respectively. It has been observed that angle of internal friction of the backfill soil is the most sensitive random variable affecting the reliability index or probability of failure of both geotechnical and structural failure modes.

11. CONCLUSION
In conclusion this paper presents the reliability analysis and sensitivity analysis of counterfort retaining wall based on FORM, SORM and MCS methods. The following conclusions have been drawn from the present study are as:

1. Probability of failure is a more appropriate measure than the deterministic factor of safety FoS for expressing geotechnical and structural stability of the retaining walls, since it considers uncertainties such as the variability in the parameters used for FS calculation.

2. The results of the reliability analysis obtained for the various geotechnical and structural failure modes of the counterfort retaining walls using the FORM, SORM and MCS methods are comparable.

3. Among the geotechnical modes of failure, sliding is the most critical one and a higher factor of safety (than the traditionally adopted FoS of 1.5) is required. Among the structural modes of failure, positive moment failure in heel slab is the critical one and FS of 1.7 is required to get a reliability index of 3.0.

4. Based on the reliability analysis geotechnical failures are more critical than the structural modes of failure.

5. Angle of internal friction of the backfill soil is the most sensitive random variable and has to be evaluated more realistically since a wide variation in reliability index occurs with the variation in the value of angle of internal friction.

6. Correlations between performance functions of individual failure modes (sliding and overturning failure) must be evaluated to determine the system reliability.

7. System reliability of counterfort retaining wall are also computed by taking into consideration different probability of failure of subsystem i.e. all the failure modes.
REFERENCES