THE DESIGN OF VERTICAL PRESSURE VESSELS SUBJECTED TO APPLIED FORCES AND VIBRATIONAL CONDITIONS

Mane S.S\textsuperscript{1}, Prof. Wankhede P.A\textsuperscript{2}

\textsuperscript{1,2}Department of Mechanical Engineering, VJTI, Matunga, Mumbai 400019, Maharashtra, India,
E-mail: meshardul@gmail.com; pawankhede@vjti.org.in

ABSTRACT

Pressure-vessel codes do not give design methods except for the relatively simple case of cylindrical shells with standard-type heads and openings under uniform pressure. The designer must apply engineering principles when he deals with more complicated structures and loading systems. The paper deals with vessels that are subjected to various applied forces acting in combination with internal or external pressure as well as concerned with the vibration of vertical pressure vessels known as columns or towers. The type of vessels considered is cylindrical shells with the longitudinal axis vertical. The procedure for estimating the period of first mode of vibration for columns which are the same diameter and thickness for their entire length is outlined. A method for designing vessels considered as dynamic structures is described as well as a detailed procedure for estimating the period of vibration of multi thickness (stepped shell) vessels and/or vessels built to two or more diameters with conical transitions where the difference in diameter is small. The design procedure outlined will be useful to the practical vessel designer confronted with the task of investigating vibration possibilities in vertical pressure vessels.

Keywords: Cylindrical shell, Pressure vessel, Dynamic structure

1. INTRODUCTION

The pressure-vessel codes give a list of the principal loading conditions that the designer should consider in designing a vessel. These conditions may be divided into pressure loadings and applied forces. Pressures are applied either internally or externally over the surface of the vessel. Applied forces act either at local points or throughout the mass of the vessel. In order to design a self-supporting vessel of this type, the following problems must be carefully analyzed:
1. When it is necessary to deviate from the common practice of designing a vertical vessel as a static structure it is considered as a dynamic structure.

2. What is the most practical method of designing to meet dynamic conditions?

3. Does the method used produce consistent results and does it provide additional strength to resist the force due to the mass acceleration resulting from the motion of the vessel?

4. Is the period of vibration of the dynamically designed vessel such that prevailing winds are not severe to cause excessive movement?

5. Are the external attachments such as piping, ladders, and platforms distributed all around the vessel are contributing to the disturbing vessel system?

These problems will be discussed during the outline of a design procedure presented in this paper. This paper discusses some problems of design of cylindrical pressure vessels that have their axes vertical and are subjected to applied forces in addition to internal or external pressure. The vertical forces considered are the weight of the vessel and its contents and the weight of any attachments to the vessel. The horizontal forces include wind pressures, seismic forces, and piping thrusts.

It should be pointed out that vessel vibrations induced by earthquakes are infrequent in occurrence and this paper is more concerned with vibrations induced by wind or other forces which may occur every day or many times during the day may depending upon the location. This paper discusses some problems of design of cylindrical pressure vessels that have their axes vertical and are subjected to applied forces in addition to internal or external pressure. The vertical forces considered are the weight of the vessel and its contents and the weight of any attachments to the vessel. The horizontal forces include wind pressures, seismic forces, and piping thrusts.

2. LOADS

The vertical loads consist primarily of forces due to gravity, that is, to weight. The vertical component of piping thrusts also must be considered. Liquid contents normally are carried by the bottom head and the vessel supports. But in fractionating columns, the weight of the liquid on internal trays is transferred into the shell. Part of the weight of stored solids is transferred into the shell by friction. The weights of attachments that are eccentric to the axis of the vessel produce bending moments which must be considered in the design.

2.1. Wind Load:

The force per unit area exerted by the wind depends on a number of factors, including wind velocity, height above ground, and drag coefficient. This last includes height-to-diameter ratio and shape factor. The discussion does not consider the effect of velocity on the drag coefficient. The drag coefficient, or friction factor, for a given shape varies with the Reynolds number. For a circular cylinder, the coefficient is practically constant for values of Reynolds number between $R = 20,000$ and $R = 200,000$. Above $R = 500,000$, the coefficient drops to less than half its value in the lower range. The values of wind pressure used by designers are usually taken from structural codes or from a purchaser's specification. All such values appear to be based on the use of a drag coefficient for Reynolds number in the region between $R = 20,000$ and $R = 200,000$. The value of Reynolds number for a circular cylinder is equal to $9100 DV$, where $D$ is the
diameter in feet and $V$ is the wind velocity in miles per hour. With a value of $DV$ greater than 55, the drag on the vessel would be less than half that specified in the codes. Thus the codes might well consider the advisability of reducing the wind pressures to be wed with circular vessels.

The usual simplified approach to the problem is based on the assumption that the structure is a rigid body which undergoes the accelerations of the supporting ground. The horizontal force which acts on the structure is equal to its mass times the ground acceleration, and has the same ratio to the weight as the ground acceleration has to that of gravity. Structural codes give values of this ratio that are based on engineering experience and judgment.

2.2 Seismic loading:

Seismic loading means application of an earthquake-generated excitation on a structure (or geo-structure). It happens at contact surfaces of a structure either with the ground, or with adjacent structures, or with gravity waves from tsunami. Earthquake or seismic performance defines a structure’s ability to sustain its due functions, such as its safety and serviceability, at and after a particular earthquake exposure. A structure is, normally, considered safe if it does not endanger the lives and well-being of those in or around it by partially or completely collapsing. A structure may be considered serviceable if it is able to fulfill its operational functions for which it was designed. Basic concepts of the earthquake engineering, implemented in the major pressure vessel codes, assume that a building should survive a rare, very severe earthquake by sustaining significant damage but without globally collapsing. On the other hand, it should remain operational for more frequent, but less severe seismic events. Engineers need to know the quantified level of the actual or anticipated seismic performance associated with the direct damage to an individual vessel subject to a specified ground shaking. Such an assessment may be performed either experimentally or analytically.

2.3. Supports and external attachments:

Vessels not larger than 1 m diameter or 2 m high may be supported on legs. Vessels subjected to severe vibrations arising through process flow, reciprocating machinery etc. shall not be supported on legs. Vertical vessels shall normally be provided with a support skirt, which shall be in accordance with the following requirements:-

1. Subject to there being no overriding statutory requirements, vessel skirts of 2400 mm diameter or greater shall be provided with two unobstructed access openings of not less than 600 mm diameter. When the skirt is less than 2400 mm diameter, one unobstructed access opening of 600 mm diameter shall be provided. In some circumstances, a lockable, removable grill may be required on each opening to prevent unauthorized access.

2. Provision shall be made for proper ventilation of the skirt.

3. Pipe work shall not be routed through an access opening in a skirt.

4. The skirt shall be attached so that its mean diameter coincides approximately with the vessel mean diameter. Attachment welds shall be continuous and shall not cover a shell to head weld.
5. On high temperature reactors: an insulated air space shall be provided at the skirt to shell junction to minimize thermal stress, the attachment of the skirt to the shell shall be designed to provide adequate resistance to fatigue from the thermal stresses due to start-up and shutdown.

3. DESIGN OF PRESSURE VESSEL

3.1. Design with Internal Pressure:

The axial stresses set up in the shell may be classified under three types: (a) The longitudinal stress produced by the internal pressure; (b) the uniform compressive stress produced by the sum of the weights assumed to act along the axis of the vessel; (c) the bending stress produced by the horizontal loads and by the resultant weight when eccentric to the axis of the vessel. Literature indicates that a somewhat higher computed stress is required to produce failure under combined bending and compression than under compression alone. Thus we may safely combine compressive stresses due to bending with those due to uniform compression, and design the vessel shell as though these stresses were all due to uniform compression. The tension side of the shell has its highest stress when the vessel is under pressure. On the compression side, the highest stress occurs when the internal pressure is not acting. The stresses set up in the shell for these two conditions are,

A. Tension
\[ \text{St} = \frac{P_D}{4\pi} - \frac{W}{\pi D^2} - \frac{4We}{\pi D^4} - \frac{4M}{\pi D^6} \quad \text{........... (1)} \]

B. Compression
\[ \text{Sc} = -\frac{W}{\pi D^2} - \frac{4We}{\pi D^4} - \frac{4M}{\pi D^6} \quad \text{........... (2)} \]

The factor \( W \) includes all the vertical loads and the factor \( M \) includes all the moments due to horizontal loads for the loading condition under consideration. A value of shell thickness must be selected so that these stresses are not greater than the allowable stress values, taking into account the applicable joint efficiencies.

3.2. Design with External Pressure:

The code charts for determining the required thickness of shells under external pressure have been developed for the condition of a uniform pressure on the cylindrical surface and the heads of the vessel. The longitudinal compressive stresses set up in the shell by weight and lateral forces have an effect similar to that produced by subjecting the heads of the vessel to a higher external pressure than that which acts on the shell. The charts can be used to give an approximate solution for such a load condition by a suitable change in the vertical scale on which the factor \( B \) is read. The nature of the approximations will be discussed later. In practice, it is simpler to leave the scale unchanged and make the adjustment in the value of pressure to be used in reading the chart.

In reference (7) Sturm gives a method of dealing with end loads on the heads. This method can be used as the basis for the design of a vessel under external pressure and subjected to applied loads.

Using Sturm's Equation, the ratio of the collapsing pressure \( W_e \) and \( W_a \) is equal to
\[ r = - \frac{W_e}{W_e} = \frac{F}{F \frac{2R^2}{L^2}} \]

or

\[ r = \frac{n^2 - 1}{n^2 - 1 + m} \] ............. (3)

Where, \( m = \frac{\pi^2 R^4}{2L^2} = \frac{\pi^2 D^4}{2L^2} = \frac{1.28 D^2}{L^2} \)

and \( F \) is given as approximately equal to \( n^2 - 1 \), where \( n \) is the number of lobes into which the shell may buckle. By Sturm's Equation [46], the ratio of collapsing pressures \( W_e \) and \( W_e' \) is equal to,

\[ r = \frac{W_e}{W_e'} = \frac{F}{F \frac{2R^2}{L^2} + \frac{\pi^2 R^4}{2L^2} + \frac{1.28 D^2}{L^2}} \]

or

\[ r = \frac{n^2 - 1}{n^2 - 1 + m + m} \] ............. (4)

In this equation \( \alpha = 2P/(W_e R) \), where \( P \) is the axial compression per lineal inch due to the externally applied loads, and \( W_e R/2 \) may be looked on as the axial compression per lineal inch in the shell, if the collapsing value of the external pressure were acting on the ends of the vessel.

Since the ASME Code charts are made up on the basis of the collapsing pressure \( W_e' \), the ratio of \( W_e' \) to \( W_e'' \) is needed to make use of the charts. This is

\[ r'' = \frac{W_e}{W_e'} = \frac{n^2 - 1 + m + m}{n^2 - 1 + m} \] ............. (5)

Windenburg and Trilling (9) have developed a chart which gives \( n \) as a function of \( t/D \) and \( L/D \) for pressure on the sides and ends of the vessel. This chart is reproduced in Fig. 1. A comparison of Sturm's Figs. 4 and 8 indicates that the values of \( n \) for different values of \( t/D \) and \( L/D \) change very little between the condition of pressure on the sides only and that of pressure on both sides and ends. Thus Fig. 1 should give satisfactory values of \( n \) for external loading conditions for which \( \alpha \) is not much greater than one. For large values of \( \alpha \), the shell will buckle in fewer lobes.
than the number given in Fig. 1, or it may even fail by plastic flow without the formation of any lobes. For values of $\alpha$ greater than one, the vessel should also be checked as a cantilever beam, including the axial stress due to external pressure in the computations.

The relation between $W_e'$ and $W_e''$ given by $\Gamma'$ is a ratio. Hence the value of $\Gamma'$ can be used equally well for the relation between allowable external working pressures. Thus the code charts can be used to determine the required thickness with external loads and moments by using an equivalent design external pressure $W_e$ equal to

$$W_e = \frac{n^2 - 1 + m + ma}{n^2 - 1 + m} \quad \text{......... (6)}$$

Where, $W_e'$ and $W_e''$ are equal to the values of $W_e'$ and $W_e''$ divided by the factor of 4 against collapse. The use of Equation [a] in connection with the code charts implies that the pressure on the sides of the vessel is increased in the same ratio that the applied vertical forces increase the axial compression in the shell. Since the applied loads do not increase the circumferential compression in the shell, the use of Equation [a] gives answers that are somewhat on the side of safety.

4. DISCUSSION OF WIND EFFECTS

Tall, cylindrical structures such as pressure vessels and stacks are subject to being put in oscillatory motion by wind currents. The motion is at right angles or normal to the direction of the wind. The relationship between wind velocity and frequency of eddy shedding is given by the equation.

$$V = \frac{fiD}{k} = \frac{D}{Tk} \quad \text{............. (7)}$$
Since we are primarily concerned with the resonant condition which occurs when the frequency of eddy shedding equals or is in the neighborhood of the natural frequency of vibration for the vessel the symbols for the vessel frequency and period of vibration are shown in equation (7). From this equation, we can estimate the critical wind velocity for most vessels.

The value for $k$ was first determined in 1878 by V. Strouhal as 0.185 and is known as the Strouhal number [9]. It is assumed by some authorities to be within 0.18 to 0.27 and dependent upon the velocity of flow. The reproduced graph shown in Fig. 2 gives the variation of the Strouhal number with the Reynolds number as obtained experimentally by Relf and Simmons [9]. Research engineers, employed by the same company as the author, reported the following values for $k$ obtained from full size vessels after erection,

$k = 0.133$, for a 7.67 O.D. insulated vessel at a wind velocity of 39.6 ft/sec (27 mph)
$k = 0.189$, for a 3.0 O.D. vessel at a wind velocity of 32.25 ft/sec (22 mph)

The difference between the values reported from field data and the graph is probably due to the size of cylinders tested and the method of support. When the velocity of the wind is such that the frequency $f$ in the equation corresponds to the natural frequency of the vessel, resonance occurs and the vessel will oscillate at excessive amplitude. Since aerodynamic stability theory and calculation methods are beyond the scope of this paper.

![Graph showing the variation of Strouhal number with Reynolds number.](image)

Figure 2 gives the effect of wind loads on vertical pressure vessels

5. CONCLUSIONS

The design method to be used for vertical pressure vessels depends on whether the longitudinal stress in the shell is tension or compression, and on whether the vessel is...
subjected to internal or external pressure. Self-supporting vertical pressure vessels should always be investigated regarding their possible behavior under vibrating conditions. The evaluation of wind velocity effects should include considerations pertaining to the distribution of external vessel attachments as well as the surrounding equipment and terrain. It should be borne in mind that liquid loading in vessels having trays will help dampen vibration, but should not be relied upon as a cure all.

If vibration trouble does occur, careful analysis of any proposed remedy must be made in order to avoid trouble from some other source. External loads applied to vertical pressure vessels produce axial loading and bending moments on the vessel. These result in axial tensions and compressions in the shell, which must be combined with the effects of the pressure loading to give the total longitudinal stress acting in the shell.

6. REFERENCES

7. “A Study of the Collapsing Pressure of Thin-Walled Cylinders,” by R. G. Sturm, University of Illinois, Engineering Experiment Station, Bulletin No. 329