A NEW APPROACH TO OPTIMIZE TRAFFIC FLOW USING MAXIMUM ENTROPY MODELING

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ABSTRACT:

In this work, maximum entropy model is used for the traffic assignment in Bangalore city for the busiest roads; where multi size vehicles like busses, cars, auto rickshaws and motor bikes use the road. The actual data is collected and the traffic flow is predicted for two different routes which connect two points. The data predicted using the maximum entropy model is compared with the user equilibrium method and found to have reasonable accuracy. The maximum entropy model is defined along with the constraints and mathematical model is solved using MATLAB.

1.0 INTRODUCTION

The planning activity in transportation always relies on simplified representations of roadway networks for traffic analysis in majority of the cases. For example, metropolitan cities usually have networks with large number of coded links, yet they ignore most of the local streets and it is always concentrated to simplify intersection signal timing and other details. In general, level of details used in the modeling of transportation depends on desired accuracy sought in model outputs as well as available computational resources to transform the input into output.
One of the common activities in the transportation planning is to predict the impacts of changes to a large network such as metropolitan cities, sketch planning is often considered as a cost-effective tool. A sketch network may be defined as a skeleton topology covering only major arterials in the region or it may focus on the details of a neighborhood or corridor of larger system to study the impact of the changes to be made to a link in the network model. The benefits of using such an approach would be, it does not require specialized expertise, does not require computationally demanding resources, provides planners with less complex platform to facilitate quick-response and relatively informed decision.

There are number of studies that have already addressed important theoretical and practical issues for network abstraction [1-8], the sub-network analysis i.e. studying a portion of the network for the impact of a change in the neighborhood, is still quite limited [9,10].

Haghani and Daskin [5,6] and Hearn [11] have described a procedure to estimate the trip table of any sub-network by combining path flows from the full network when the network is congested and experiencing user-equilibrium (UE) traffic conditions. This procedure results in a trip matrix for the sub-network which can induce a link flow pattern in the sub-network exactly as the same as that in the full system. One of the drawbacks in this approach is, it requires complete information on paths taken in the full network. Hence it results in sub-network trip matrix which is dependent on the given full-network path flow pattern. In general, in a user equilibrium context, unique path flow patterns do not exist, and hence this approach does not result in a unique sub-network trip matrix. Thus, if such a matrix, which is less accurate for better predictions, was used to evaluate traffic shifts under network modifications, the resulting estimates for the flow will be likely to deviate from results of a full-network. There are two known approaches available to eliminate this non-uniqueness issue. Both of these approaches use entropy maximization principle [12,13] for predicting the flows. In the first approach, most likely and unique path flow pattern in the full network is estimated by means of an entropy-maximizing user equilibrium traffic assignment algorithm [14-18] and then to aggregate corresponding path flows to form a sub-network trip matrix. The second approach, which is relatively simple, requires only a link flow pattern from traffic assignment in the full
network; and the link flows are used as inputs to a maximum entropy method for estimating a most likely and unique flow pattern for the sub-network.

The second approach is much better than the first approach in usage since it requires only link flow information and conducts its maximum entropy optimization on the sub-network level, which is much less computationally demanding. The other reason for the superiority of the second approach is related to availability of input data. In cases where large scale traffic assignment across the full network is not available, one must rely on other data sources. Thus, the second approach is more practical in that it requires just sub-network flow values, either estimated or measured.

Maximum entropy theory was first used by Willumsen [19] and Van Zuylen and Willumsen [20] for the most-likely-trip-matrix estimation problem, based on traffic counts. These models resort to a simple iterative balancing method for solutions by assuming the underlying traffic flows to follow a known proportional routing pattern. In similar research methodology, Nguyen [21] formulated a maximum entropy problem considering both traffic count data and trip production and attraction data. It is always useful to have more information for an accurate prediction, but potential inconsistencies across constraints can result in no feasible solutions. Fisk [22] imposed the user-equilibrium routing principle to a similar matrix estimation problem, resulting in a maximum entropy model subject to a variational inequality constraint. His contribution is mostly theoretical, rather than practical, however; its nonconvex feasible region makes the problem hard to solve. Nguyen [23,24] and others [24-27] proposed an alternative approach, which incorporates equilibrium traffic flows in to the formulation. With this approach, having known flows and link cost functions, link travel costs and path travel costs can be readily calculated. The model does not make any assumptions about trip distribution patterns; however, alternative optimal solutions may exist. To ensure convergence to a unique matrix solution, some extra information is generally needed. There are other trip matrix estimation methods which are based on the combined usage of traffic count and target matrix information, such as the Bayesian inference approach [28,29], least squares approach [30-33], maximum likelihood approach [29, 34-36], constrained regression approach [37], least absolute norm approach [38-40], and integrated squared error approach [41]. Despite various implicit parameter assumptions
and optimization principles, all seek a matrix that represents some form of trade-offs between a target trip matrix and observed traffic counts. These trade-offs appear in the model constraints or objective functions. Given the fact that link flows (either estimates or measured counts) are the only data source available in the context under study, we propose an ME model for subnetwork trip matrix estimation. This approach is based on the early work of Willumsen [19] and Van Zuylen and Willumsen [20].

Recently, MAXENT model is becoming very popular in the area of traffic flow optimization in the developing countries. It is essential to optimize the traffic flow in the road networks to meet the sustainable development of the transport system especially having vehicles of multi sizes. There has been much interest in optimizing traffic flows of the road networks when the sizes of the vehicle range from small to large. Accordingly, many various mathematical models of traffic flows are forwarded to solve the optimization problem [42]. However, most algorithms rely on detailed modeling information that often requires knowing explicit system performance such as the nodes, the length of links, type of facility, location in the area, number of lanes, speed, maintenance condition and travel time, etc. But real-world systems are often too complex to admit such a detailed analytical description. In the current study the Maximum Entropy Model, related to traffic flow and modified by Shannon entropy, would be employed to optimize traffic flow in a road network when different sizes of the vehicles are there on the roads.

**2.0 MAXIMUM ENTROPY MODEL**

The distribution of the flows among the different classes possesses a great deal of disorder or chaos [42]. The statistical measure of this disorder is given by modified entropy. Let let \( r \) and \( s \) are two nodes connected by link \( a \). Then one can construct the maximum entropy problem:

\[
\max \left\{ -\sum_{ij} q_{ij} \ln q_{ij} - q_{ij} \right\} \tag{1}
\]

or

\[
\min \left\{ \sum_{ij} q_{ij} \ln q_{ij} - q_{ij} \right\} \tag{2}
\]
In other words,
\[
\min \left\{ \sum_j K_j \sum_i q_{ij} \ln q_{ij} - q_{ij} \right\}
\]

where \( K \) is a constant and depends on the unit of measurement of entropy. Without loss of generality, we can take \( K = 1 \). Eq.1 represents the Shannon’s Entropy Equation in Information Theory. In the above equation, \( j \) is the index of the size of the vehicle and \( i \) is the index of the vehicle under a particular size. Hence, the ME problem can be stated as:

\[
\min \left\{ \sum_j \sum_i q_{ij} \ln q_{ij} - q_{ij} \right\}
\]

Assume that there are multiple sizes of vehicles moving from region I to region j. Hence the entropy of the route between node I and j are dependent on the size of the vehicles too, as the size of the vehicles decides he maximum number of vehicles of that size on the route. Hence the ME problem for the multi size vehicles can be stated as

\[
\min \left\{ \sum_k \sum_j \sum_i q_{ij}^k \ln q_{ij}^k - q_{ij}^k \right\}, \quad (5)
\]

In terms of the probabilities, the above equation can be stated as

\[
\min \left\{ \sum_k \sum_j \sum_i P_{ij}^k \ln P_{ij}^k - P_{ij}^k \right\}
\]

\[\text{Subject to}\]
\[
\sum_{j=1}^{N} P_{ij}^k = \frac{U_i^k}{T}; \quad \forall i, \forall k\]  \quad (7)

\[
\sum_{i=1}^{M} P_{ij}^k = \frac{U_j^k}{T}; \quad \forall j, \forall k \quad \text{and} \]

\[P_{ij}^k \geq 0; \quad \forall i, \forall j, \forall k\]

Replacing the probabilities with the trip matrix,

\[
\min \left\{ \sum_k \sum_j \sum_i T_{ij}^k \ln \frac{T_{ij}^k}{T} \right\}
\]

\[\text{Subject to}\]
\[
\sum_{j=1}^{N} T_{ij}^k = O_i^k \quad \forall \ i, \forall \ k
\]  
(10)

\[
\sum_{i=1}^{M} T_{ij}^k = D_j^k \quad \forall \ j, \forall \ k
\]  
(11)

\[
T_{ij}^k \geq 0 \quad \forall \ i, \forall \ j, \forall \ k
\]  
(12)

where

\[\begin{align*}
O_i & : \text{Traffic flow produced from region } i \\
D_j & : \text{Traffic flow destined to region } j \\
U_i & : \text{Amount of traffic originated from source } i \\
V_j & : \text{Amount of traffic attracted by destination } j \\
M & : \text{number of origins generating traffic}; \\
N & : \text{number of destinations attracting traffic}; \\
T & : \text{total amount of traffic} \\
T_{ij} & : \text{Traffic flowing from origin } i \text{ to destination } j; \ \text{and}
\end{align*}\]

\[P_{ij} : \text{Probability of traffic from origin } i \text{ to destination } j = T_{ij}/T\]

\(S_k : \text{Particular size of the vehicle, } k = 1,2,...Q \) and \(Q\) is maximum number of varieties of sizes of the vehicles under consideration.

And one more constrain can be the relation between different sizes of vehicles based on the area each one occupies. For example, a normal government bus can be substituted with 4 small cars, or 10 motor bikes which occupy the same area as the bus.

\[
T_{ij}^k = n_{k,p} T_{ij}^{k+p} \quad \forall \ p, \forall \ k
\]  
(13)

Here index \(p\) refers to the index of other size of vehicles with respect to a particular size of vehicle in interest. For example, when the vehicles of different sizes are arranged in order according to their size, the one size of vehicle can be expressed in terms of the other vehicles. All smaller size vehicles can be expressed in terms of number of equivalent busses that occupy the same area on the road. One bus may be substituted for four small cars or 10 motor bikes or 6 auto rickshaws. Since the models are based on entropy, 10 motor bikes create more entropy than one single bus on the road. Hence, by
neglecting this error, computations can be made to predict the optimum number of vehicles on the road. Entropy is minimum, when all the vehicles on the road are expressed in terms of largest size of vehicle on the road and maximum when all the vehicles are expressed in terms of smallest size of vehicles. In this work, predictions are made based on considering different sizes of vehicles into equivalent number of busses, cars and auto rickshaws.

The optimality conditions of this sub-network trip matrix estimation problem with ideal input data set can be analyzed by using the Lagrangian model formulation, incorporating the link flow conservation constraint:

\[
L(f, \lambda_a) = \left\{ \sum_k \sum_j \sum_i q_{ij}^k \ln q_{ij}^k - q_{ij}^k \right\} + \sum_a \lambda_a \left( v_a - \sum_k \sum_j \sum_i P_{ij}^k \delta_{a,ij}^k \right) 
\]

Where \( \lambda_a \) is the Lagrangian multiplier on link \( a \)'s flow conservation constraint.

Since the first order condition of the Lagrangian with respect to Probability of traffic from origin \( i \) to destination \( j \), \( P_{ij} \) is

\[
\frac{\partial L(f, \lambda)}{\partial P_{ij}} = \ln q_{ij} - \sum_a \lambda_a \delta_{a,x}^{ij} 
\]

Note that - \( \ln q_{ij} \) denotes the minimum path entropy impedance among all paths connecting pair \((i, j)\). One can also define \( -\lambda_a \) as the entropy impedance of link \( a=(i, j) \) and define \( -\sum_a \lambda_a \delta_{a,x}^{ij} \) as the entropy impedance of path \( x \) between pair \((i, j)\), which is the sum of the entropy impedances of all links of along this path.

3.0 NUMERICAL SOLUTION

The following sequence of instruction clearly describe the procedure to solve a ME problem numerically.

![Figure 1 The traffic flow of typical sub-network model](image-url)
**Step 1:** List down the links or traffic pairs, as (1,2), (1,3), (1,4), (2,3) and (4,3).

**Step 2:** Define the maximum entropy problem as

$$
\min \left\{ \sum_{ij} q_{ij} \ln q_{ij} - q_{ij} \right\}
$$

**Step 3:** List down the constaints

$$
\begin{align*}
T_{1-2} + T_{1-2-3} &= 2 \\
T_{2-3} + T_{1-2-3} &= 2 \\
T_{1-3} &= 3 \\
T_{1-4} + T_{1-4-3} &= 1 \\
T_{4-3} + T_{1-4-3} &= 1 \\
T_{1-2}, T_{1-3}, T_{1-4}, T_{1-2-3}, T_{1-4-3}, T_{2-3}, T_{4-3} &\geq 0
\end{align*}
$$

**Step 4:** Compute the flow variables as

$$
\begin{align*}
q_{12} &= T_{1-2} \\
q_{23} &= T_{2-3} \\
q_{13} &= T_{1-3} + T_{1-2-3} \\
q_{14} &= T_{1-4} \\
q_{43} &= T_{4-3}
\end{align*}
$$

**Step 5:** Solve the problem numerically as follows:

Given $q_{12} = q_{23}, q_{14} = q_{41}$ and $q_{12} + q_{13} + q_{14} = 6$, the objective function can be reduced to

$$
2(q_{12} \ln q_{12} - q_{12}) + 2(q_{14} \ln q_{14} - q_{14}) + (6 - q_{12} - x_{14}) \ln(6 - q_{12} - q_{14}) - (6 - q_{12} - q_{14})
$$

This single objective minimization function can be readily solved by checking its partial gradient subject to $0 \leq q_{12} \leq 2$ and $0 \leq q_{14} \leq 1$, which results in

$$
\begin{align*}
q_{12} &= 1.789 \\
q_{14} &= 1 \\
q_{23} &= 1.789 \\
q_{43} &= 1 \\
q_{13} &= 3.211
\end{align*}
$$

The Frank-Wolfe [43] algorithm may be used to solve the ME problem described in the above sections. The procedure used for the solving the ME problem is as follows:

**Step 1:** Find an initial feasible trip matrix. One possible initial trip matrix can be obtained by setting $q_{ij} = v_x$. 

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Step 2: Find the auxiliary trip matrix $\tilde{q}_{ij}$ by solving the following linearized problem:

$$\min \left\{ \sum \sum \sum k_{ij} \ln q_{ij} \right\}$$

Subject to

$$\sum \sum \sum P_{ij}^k s_{a,m} = v_a$$

and $P_{ij}^k \geq 0$

Where trip rate $\tilde{q}_{ij}$ is defined as

$$\tilde{q}_{ij} = \sum_{m} P_{ij}^m ; \quad m = \text{path between pair } (i,j).$$

Step 3: Find and optimal value of $\alpha$ for $0 \leq \alpha \leq 1$ by solving the line search problem:

$$\min \left\{ \sum \sum \sum \left[ d_{ij}^k + \alpha (\tilde{q}_{ij}^k - q_{ij}^k) \right] \ln \left[ q_{ij}^k + \alpha (\tilde{q}_{ij}^k - q_{ij}^k) \right] - \left[ d_{ij}^k + \alpha (\tilde{q}_{ij}^k - q_{ij}^k) \right] \right\}$$

subject to $0 \leq \alpha \leq 1$

Step 4: Set $q_{ij}^{k+1} = q_{ij}^k + \alpha (\tilde{q}_{ij}^k - q_{ij}^k)$

Step 5: If a convergence criteria is met, then stop, else go to Step 2.

Convergence Criteria: $$\sum \sum \frac{q_{ij}^{k+1} - q_{ij}^k}{q_{ij}^{k+1}} < \varepsilon$$

3.0 NUMERICAL RESULTS

Let two points are chosen in Bangalore and call them as node 1 and node 2. Node 1 is selected as Kanteerava Studio and Node 2 is selected as Jalahalli Circle. Node 1 and node 2 are linked by two routes. Constraints are given by for certain time duration (~ 12 hrs) on a working day.

- The free flow of traffic into node 1 out of node 2 is 5000 in terms of number of equivalent busses.
- Free flow time of link 1 is 5 min and that of link 2 is 7 mins for equivalent busses.
- Capacities on link 1 and link 2 are 2000 and 3500 in terms of equivalent busses.
- Each bus is equivalent to 4 small size Cars.
- Each bus is equivalent to 3 large size Cars
• Each bus is equivalent to 10 Motor Bikes
• Each bus is equivalent to 6 Auto Rickshaws

The problem is converted into number of equivalent busses. But the actual problem can be like this:
• The free flow of traffic into node 1 out of node 2 is ~2000 busses, ~4000 small cars, ~6000 auto rickshaws, and ~10000 motor bikes.

The traffic assignment for these two routes are found by solving the Maximum entropy model using MATLAB and the results are given in the following table. Also the results are compared with the user equilibrium method for validation. The user equilibrium traffic assignment could be found by All-Or-Nothing (AON) and Method of successive averages (MSA) (Zuidgeest, 2003).

Table 1 Traffic Assignment based on Equivalent Busses, Cars and Auto Rickshaws

<table>
<thead>
<tr>
<th>Method</th>
<th>Equivalent Busses - Traffic Assignment</th>
<th>Equivalent Small Cars - Traffic Assignment</th>
<th>Equivalent Auto Rickshaws - Traffic Assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bus-q1</td>
<td>Bus-q2</td>
<td>Cars-q1</td>
</tr>
<tr>
<td>User Equilibrium Method</td>
<td>1881</td>
<td>3119</td>
<td>7524</td>
</tr>
<tr>
<td>Maximum Entropy Method</td>
<td>1893</td>
<td>3107</td>
<td>8140</td>
</tr>
</tbody>
</table>

Table 1 lists the traffic assignment for different sizes of vehicles. Reader must note that these are equivalent vehicles by substituting the actual size of vehicles with equivalent number of different size vehicles for modeling purposes as explained above. Fig. 2 shows the graphical comparison of the maximum entropy model with the user equilibrium model for all varieties of sizes. It shows a good agreement between the predicted values based two approaches.
The generalized cost for each route is calculated using the following relation.

\[
T_i = T_{F_i} \left(1 + \alpha \left(\frac{q_i}{K_i}\right)^\beta\right)
\]

(26)

where

- \(T_i\) is the time required for the traffic flow \(q_i\).
- \(T_{F_i}\) is the time required for the free flow.
- \(K_i\) is the capacity of the routes.
- \(\alpha\) is a constant chosen as 0.15.
- \(\beta\) is another constant chosen as 4.

Maximum entropy model can be used for assigning the traffic flow for different routes for a given source and destination. For example, if there are 5 routes to reach a destination from a given source then the traffic can be diverted into these five different routes. The MAXENT model takes care of the maximum capacity of the routes, generalized cost for the users and time of travel. Hence it gives the best assignment and is proven to be one of the good models for traffic assignment.
4.0 CONCLUSIONS

In this work, maximum entropy model is used for the traffic assignment in Bangalore city for the busiest roads; where multi size vehicles like buses, cars, auto rickshaws and motor bikes use the road. The actual data is collected and the traffic flow is predicted for two different routes which connect two points. The data predicted using the maximum entropy model is compared with the user equilibrium method and found to have reasonable accuracy. The maximum entropy model is defined along with the constraints and mathematical model is solved using MATLAB. Computations are made by considering equivalent vehicles since it is difficult, however not impossible, to model the different sizes of vehicles on the road at the same time. The results show a very good agreement with the user equilibrium model.

5.0 REFERENCES


