THE SINGLE INDEX MODEL AND THE CONSTRUCTION OF OPTIMAL PORTFOLIO WITH CNXPHARMA SCRIP

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ABSTRACT

Risk and return plays an important role in making any investment decisions. Decision include Investment should be done or not and which securities should be included in portfolio. Determining efficient portfolios within an asset class (e.g., stocks) can be achieved with the Single index (beta) model proposed by Sharpe. Sharpe's single-index model was applied by using the monthly closing prices of 10 companies listed in NSE and CNX PHARMA price index for the period from September 2010 to September 2014. From the empirical analysis it can be concluded that out of 10 companies only one company is selected for investment purpose on the basis of Cut-off point which is -0.11182.

Keywords: Risk & return, Efficient Portfolio, Sharpe Index Model, Cutoff point.

INTRODUCTION

Portfolio is a bundle of or a combination of individual assets or securities. The portfolio theory provides normative approach to investors to make decisions to invest their wealth in assets or securities under risk (See Mullins 1982 and Butters et. al.). It is based on the assumption that investors are risk-averse. This implies that investors hold well diversified portfolios instead of investing their entire wealth in a single or few assets. Investors who are risk-averse reject investment portfolios that are fair games or worse. Risk-averse investors are willing to consider only risk-free or
speculative prospects with positive risk premiums. Loosely speaking, risk-averse investors “penalize” the expected return of the risky portfolio by certain percentage to account for risk involved. Greater the risk, greater the penalty. A rational investor is a person that desires to maximize their return with less risk on his investment in a portfolio. For this purpose investor has to construct a portfolio of assets which is an efficient portfolio (minimum risk for a given expected return) which comprises of different classes of assets. Determining efficient portfolios within an asset class (e.g., stocks) can be achieved with the single index (beta) model proposed by Sharpe. In the early 1960s, the investment community talked about risk, but there was no specific measure for the term. To measure risk or to avoid risk investor had to quantify their risk variable by building a basic portfolio model. The basic portfolio model was developed by Harry Markowitz (1952, 1959), who derived the expected rate of return.

Markowitz showed that the variance of the rate of return was a meaningful measure of portfolio risk under reasonable set of assumptions, and he derived the formula for computing the variance of a portfolio as variance is a measure of risk. The Markowitz model is based on several assumptions regarding investor behavior. (1) Investor considers each investment alternative as being represented by a probability distribution of expected returns over some holding period. (2) Investors maximize one-period expected utility, and their utility curves demonstrate diminishing marginal utility of wealth. (3) Investors estimate the risk of the portfolio on the basis of the variability of expected return. (4) Investors base decisions solely on expected return and risk, so their utility curves are a function of expected return and the expected variance of returns only. (5) For a given level of risk, investor prefers higher returns to lower returns. Under these assumptions, a single asset or portfolio of assets is considered to be efficient if no other assets offers higher expected return with the same (or lower) risk or lower risk with the same (or higher) expected return. Harry Markowitz model has certain drawbacks. These are:

The model requires huge number of estimates to fill the covariance matrix.

The model does not provide any guidelines to the forecasting of the security risk premiums that are essential to construct the efficient frontier of risky assets.

Identification of this, several studies and research had been done to develop simple index model for portfolio. William Sharpe is among many who tried to simplify the Markowitz model and developed Sharpe index model which reduces substantially its data and computational requirements. The simplified model assumed that the fluctuations in the value of stock relative to other stocks do not depend on the characteristics of those two securities alone. The two securities are more suitable or appropriate to describe business condition. Relationship between securities occurs only through their individual relationships with some indexes of business activity. The reduction in the number of covariance estimates needed eases considerably he job of security analysis and portfolio-analysis computation. Thus the covariance data requirement reduces from \( \frac{N^2 - N}{2} \) under the Markowitz technique to only \( N \) measures of each security as it relates to the index.

The purpose of this paper is to construct the optimum portfolio by using single index model on Indian Stock market. Even there are 22 stock exchanges in India, National Stock Exchange (NSE), Mumbai is in the top position based on the market turnover as well as the technology to trade on the stocks. In NSE short selling is not allowed on the spot market.
LITERATURE REVIEW

Markowitz (1952 and 1959) performed the pioneer work on portfolio analysis. The major assumption of the Markowitz's approach to portfolio analysis is that investors are basically risk-averse. This means that investors must be given higher returns in order to accept higher risk. Markowitz then developed a model of portfolio analysis. Markowitz (1952) and Tobin (1958) showed that it was possible to identify the composition of an optimal portfolio of risky securities, given forecasts of future returns and an appropriate covariance matrix of share returns. Sharpe (1963) attempted to simplify the process of data input, data tabulation, and reaching a solution. He also developed a simplified variant of the Markowitz model that reduces data and computational requirements. William Sharpe (1964) has given model known as Sharpe Single Index Model which laid down some steps that are required for construction of optimal portfolios. Elton and Gruber (1981), and Elton, Grube and Padberg [1976, 1977A, 1978A, 1978B, 1979] have established simple criteria for optimal portfolio selection using a variety of models, such as single index, multi-index, and constant correlation models. These models are used to provide solution to portfolio problems by disallowing short sales of risky securities in portfolios and this can be done by using simple ranking procedures. Elton, Grube, Padberg (1977B) have also extended their analysis using a constant correlation model, as well as a single index model incorporate upper limits on investment in individual securities. Haugen (1993) stated that Index models can handle large population of stocks. They serve as simplified alternatives to the full-covariance approach to portfolio optimization. Although the Single Index Model offers a simple formula for portfolio risk, it also makes an assumption about the process generating security returns. According to Terol et al. (2006) Markowitz model is a conventional model proposed to solve the portfolio selection problems by assuming that the situation of stock markets in the future can be characterized by the past asset data. In addition, Briec & Kerstens (2009) stated that Markowitz model contributes in geometric mean optimization advocated for long term investments. On the other hand, the Simple Index Model is no longer good approximations to multi period. As seen by Frankfurter et al. (1976) according to this study, under conditions of certainty, the Markowitz and Simple Index Model approaches will arrive at the same decision set in the experiment. These results demonstrate that under conditions of uncertainty, Simple Index Model approach is advantageous over the Markowitz approach. It was found that variation in performance is explained in terms of the two essential differences in the models. First, fewer and different estimators are used in the Simple Index Model to summarize past history. Second, the linear assumption of the Simple Index Model does not necessarily hold. They finally found that in experiments, the Simple Index Model process performs worse than Markowitz process, and gives superior results when only short data histories are available. Omet (1995) argued that the two models are similar. Simple Index Model can be used, which is more practical than the Markowitz model in generating ASE efficient frontier. Dutt (1998) used Sharpe single index model in order to optimize a portfolio of 31 companies from BSE (Bombay Stock Exchange). Nanda, Mahanty, and Tiwari (2012) selected stocks from the clusters to build a portfolio, minimizing portfolio risk and compare the returns with that of the benchmark index i.e. Sensex. Saravanan and Natarajan (2012) used Sharpe single index model in order to construct an optimal portfolio of 4 companies from NSE (National Stock Exchange of India) and used NSE NIFTY as market index. Meenakshi and Sarita (2012) stated that Sharpe's single index model is of great importance and the framework of Sharpe's single index model for optimal portfolio construction is very simple and useful.
RESEARCH DESIGN

GENERAL OBJECTIVE

The main objective of the research is to construct an optimum portfolio in CNX Pharma index stocks.

SPECIFIC OBJECTIVES

1. To calculate the Mean and standard deviation of all securities which constitutes the CNX pharma index.
2. To find out the excess return from the securities.
3. To calculate the cutoff rate which serves as a benchmark to select stocks to be included in a portfolio.
4. To select the stocks which are more than the cutoff value.

RESEARCH METHODOLOGY

The Single Index Model

The risk return model suggested by Sharpe is:

\[ R_i = \alpha_i + \beta_i I + e_i \ldots \]

Where:
\( R_i \) = expected return on security \( i \)
\( \alpha_i \) = intercept of a straight line or alpha coefficient
\( \beta_i \) = slope of straight line or beta coefficient
\( I \) = expected return on index (market)
\( e_i \) = error term with the mean of zero and a standard deviation which is a constant

RETURN

The daily return on each of the selected stocks is calculated with the following formula.

\[ R_{it} = \frac{P_t}{P_{t-1}} - 1 \]

Where \( P_t, P_{t-1} \) are the share price at time \( t \) and \( t-1 \) for security \( i \).

STANDARD DEVIATION

The standard deviation is the most common measure of variability, measuring the spread of the data set and the relationship of the mean to the rest of the data. If the data points are close to the mean, indicating that the responses are fairly uniform, then the standard deviation will be small.
Conversely, if many data points are far from the mean, indicating that there is a wide variance in the responses, then the standard deviation will be large. If all the data values are equal, then the standard deviation will be zero. The standard deviation is calculated using the following formula:

\[
\sigma = \sqrt{\frac{\sum_{i=1}^{n} (P_i - R_i)^2}{n-1}}
\]

Where,
- \(P_i\) - value of the security at ‘t’;
- \(R_i\) - mean value of the security i

**BETA**

It is a measure of the volatility, or systematic risk, of a security or a portfolio in comparison to the market as a whole. In other words, beta gives a sense of a stock's market risk compared to the greater market. Beta is also used to compare a stock's market risk to that of other stocks. Beta coefficient is a measure of sensitivity of a share price to movement in the market price. It measures systematic risk which is the risk inherent in the whole financial system. Beta coefficient is an important input in capital asset pricing model to calculate required rate of return on a stock. It is the slope of the security market line.

\[
\beta = \frac{\text{Correlation Coefficient Between Market and Stock}}{\text{Standard Deviation of Stock Returns}} \times \frac{\text{Standard Deviation of Stock Returns}}{\text{Standard Deviation of Market Returns}}
\]

**EXCESS RETURN**

The excess return is the difference between the expected return on the stock and the riskless rate of interest such as the rate offered on the government security or Treasury bill. The excess return to beta ratio measures the additional return on a security (excess of the riskless assets return) per unit of systematic risk or non-diversifiable risk. This ratio provides a relationship between potential risk and reward.

\[
\text{Excess return} = \frac{R_i - R_f}{\beta_i}
\]

where
- \(R_i\) - return of the stock I ; \(R_f\) - Risk free return ; \(\beta_i\) - Systematic risk of stock i

**OPTIMUM PORTFOLIO WHEN SHORT SALES ARE NOT ALLOWED**

Ranking of the stocks is done on the basis of their excess return to beta. Portfolio managers would like to include stocks with higher ratios. The selection of the stocks depends on a unique cut-off rate such that all stocks with higher ratios of \((R_i - R_f) / \beta_i\) are included and the stocks with lower ratios are left out.

The cutoff point is denoted by $C^*$.

$$
C^* = \frac{\sum_{l=1}^{m} \frac{(R_l - R_f)\beta_l}{\sigma_{R_l}}}{1 + \sum_{l=1}^{m} \frac{\sigma_{R_l}}{\sigma_{\epsilon_l}}}
$$

Where,
- $\sigma_{R_l}$ - Variance of the market index
- $\sigma_{\epsilon_l}$ - Variance of a stock’s movement that is not associated with the movement in the market index; this is the stock’s unsystematic risk.
- $R_l$ - return of the stock $i$
- $R_f$ - Risk free return
- $\beta_l$ - Systematic risk of stock $i$

**DATA**

Here in this study daily data have not been used for analysis purpose reason for this are being mentioned below:

**Non normality** The daily stock return for an individual security exhibits substantial departures from normality that are not observed with monthly data. The evidence generally suggests that distributions of daily returns are fat-tailed relative to a normal distribution [Fama (1976, p. 21)].

**VARIANCE ESTIMATION**

The first issue is the time-series properties of daily data. As a consequence of non-synchronous trading, daily excess returns can exhibit serial dependence. Attempts to incorporate such dependence into variance estimates have appeared in the event study literature [see, Ruback (1982)]. The second issue is stationarity of daily variances. There is evidence that the variance of stock returns increases for the days immediately around events such as earnings announcements [see, Beaver (1968), Patell and Wolfson (1979)].

Construction of optimal portfolio by using Sharpe’s Single Index Model is the aim of this paper. For this purpose monthly closing price of stocks listed on National Stock Exchange (NSE) and monthly closing index value of CNX PHARMA INDEX have been used for construction. This study takes 10 Pharmaceutical companies listed on NSE. Selection is done on the basis of market capitalization. This study has used secondary data and for risk free securities 91 days T-Bill has been used as a proxy for risk free rate and sourced from Reserve Bank of India.

**OVERVIEW OF CNX PHARMA INDEX**

CNX Pharma Index captures the performance of the pharmaceutical sector. The Index comprises of 10 companies listed on National Stock Exchange of India (NSE).
CNX Pharma Index is computed using free float market capitalization method, wherein the level of the index reflects the total free float market value of all the stocks in the index relative to particular base market capitalization value. CNX Pharma Index can be used for a variety of purposes such as benchmarking fund portfolios, launching of index funds, ETF’s and structured products. Index Variants: CNX Pharma Total Returns Index.

**PORTFOLIO CHARACTERISTICS**

<table>
<thead>
<tr>
<th>Methodology</th>
<th>Free Float Market Capitalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Constituents</td>
<td>10</td>
</tr>
<tr>
<td>Launch Date</td>
<td>July 01, 2005</td>
</tr>
<tr>
<td>Base Date</td>
<td>January 01, 2001</td>
</tr>
<tr>
<td>Base Value</td>
<td>1000</td>
</tr>
<tr>
<td>Calculation Frequency</td>
<td>Real-time Daily</td>
</tr>
<tr>
<td>Index Rebalancing</td>
<td>Semi-Annually</td>
</tr>
</tbody>
</table>

**STATISTICS**

<table>
<thead>
<tr>
<th></th>
<th>QTD</th>
<th>YTD</th>
<th>1 Year</th>
<th>5 Years</th>
<th>Since Inception</th>
</tr>
</thead>
<tbody>
<tr>
<td>Returns</td>
<td>11.86</td>
<td>11.80</td>
<td>26.34</td>
<td>26.64</td>
<td>17.21</td>
</tr>
<tr>
<td>SD</td>
<td>16.46</td>
<td>14.87</td>
<td>20.06</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**RESULT AND DISCUSSION**

Firstly the securities are ranked according to their excess return to beta ratio from highest to lowest. Among 10 companies 9 companies offer less return then risk free rate. And then $C_i$ is calculated in order to find out the optimum $C_i$. The highest $C_i$ value is considered as the optimum $C_i$. And this is known as the cut-off point $C^*$. From table 1 it can be seen that out of 10 companies one company is having excess return and their values are more than cut-off value.

**TABLE 1: OPTIMAL PORTFOLIOS**

<table>
<thead>
<tr>
<th>Companies</th>
<th>Rank</th>
<th>Mean Return</th>
<th>$R_i - R_f$</th>
<th>Beta</th>
<th>Unsystematic Risk</th>
<th>Excess return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piramal Enterprises Ltd.</td>
<td>1</td>
<td>0.2442</td>
<td>0.038</td>
<td>-0.098</td>
<td>0.077</td>
<td>0.38720</td>
</tr>
<tr>
<td>Lupin Ltd</td>
<td>2</td>
<td>0.3343</td>
<td>-0.029</td>
<td>0.651</td>
<td>0.0626</td>
<td>-0.04455</td>
</tr>
<tr>
<td>Dr. Reddy’s Laboratories Ltd</td>
<td>3</td>
<td>0.0205</td>
<td>-0.042</td>
<td>0.73</td>
<td>0.0716</td>
<td>-0.05753</td>
</tr>
<tr>
<td>Cadila Healthcare Ltd.</td>
<td>4</td>
<td>0.0183</td>
<td>-0.044</td>
<td>0.3454</td>
<td>0.0789</td>
<td>-0.12739</td>
</tr>
</tbody>
</table>

Risk free rate - 6.2%

From the above table it can be seen that on the basis of excess return ranks are given to different companies. There are 10 securities in the table 1. They are already ranked. Ranking is done on the basis of excess return. Selecting a security in an optimal portfolio it is necessary to compare excess return.

TABLE 2: DETERMINING THE OPTIMAL PORTFOLIO
(FROM SEPTEMBER 2011 TO SEPTEMBER 2014)

<table>
<thead>
<tr>
<th>Companies</th>
<th>Rank</th>
<th>(Rt - Rf)βl</th>
<th>(\sum_{i=1}^{n} \frac{(Rt - Rf)βl}{\sigma_{ei}^2})</th>
<th>(\sum_{i=1}^{n} \frac{\sigma_{ei}^2}{\sigma_{ei}^2})</th>
<th>Cutoff value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piramal Enterprises Ltd.</td>
<td>1</td>
<td>0.38720</td>
<td>0.62899</td>
<td>0.04</td>
<td>-0.11067</td>
</tr>
<tr>
<td>Lupin Ltd</td>
<td>2</td>
<td>-0.04455</td>
<td>-4.18861</td>
<td>0.04</td>
<td>-0.11317</td>
</tr>
<tr>
<td>Dr.Reddy’s Laboratories Ltd</td>
<td>3</td>
<td>-0.05753</td>
<td>-10.126</td>
<td>0.04</td>
<td>-0.04968</td>
</tr>
<tr>
<td>Cadila Healthcare Ltd</td>
<td>4</td>
<td>-0.12739</td>
<td>-12.5673</td>
<td>0.04</td>
<td>-0.055</td>
</tr>
<tr>
<td>Divi’s Lab</td>
<td>5</td>
<td>-0.14008</td>
<td>-15.4759</td>
<td>0.04</td>
<td>-0.14855</td>
</tr>
<tr>
<td>Glaxo smithkline Pharmaceuticals Ltd</td>
<td>6</td>
<td>-0.14167</td>
<td>-19.6908</td>
<td>0.04</td>
<td>-0.01475</td>
</tr>
<tr>
<td>Glenmark Pharmaceuticals Ltd</td>
<td>7</td>
<td>-0.14547</td>
<td>-21.1111</td>
<td>0.04</td>
<td>-0.14746</td>
</tr>
<tr>
<td>Sun Pharmaceutical Industries Ltd</td>
<td>8</td>
<td>-0.20571</td>
<td>-21.675</td>
<td>0.04</td>
<td>-0.01181</td>
</tr>
<tr>
<td>Ranbaxy Laboratories Ltd</td>
<td>9</td>
<td>-0.75833</td>
<td>-21.8096</td>
<td>0.04</td>
<td>-0.01113</td>
</tr>
<tr>
<td>Cipla Ltd.</td>
<td>10</td>
<td>-1.7179</td>
<td>-36.7928</td>
<td>0.04</td>
<td>-0.14968</td>
</tr>
</tbody>
</table>

The cutoff value C* is -0.11182.

From the above table it can be seen that Piramal Enterprises Limited, Ranbaxy Laboratories Ltd, Sun Pharmaceutical Industries Ltd and Cadila Healthcare Ltd can be included into the portfolio because all these scrips will be more than the cutoff value. But Piramal Enterprises have the only
positive excess return to beta ratio. So one stock can be included in the portfolio from all Pharmaceutical companies which are selected for inclusion.

CONCLUSION

This study aims at analyzing the opportunity that are available for investors as per as returns are concerned and the investment of risk thereof while investing in equity of firms listed in the national stock exchange. Sharpe's single-index model was applied by using the monthly closing prices of 10 companies listed in NSE and CNX PHARMA price index for the period from September 2010 to September 2014. From the empirical analysis it can be concluded that out of 10 companies 1 company is selected for investment purpose.

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