STABILITY ANALYSIS OF A MODIFIED PSK HOMODYNE OPTICAL RECEIVER

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ABSTRACT

In this paper, we present the stability analysis of a modified PSK homodyne optical receiver in the presence of a loop delay. This paper also reports the impact of damping factor and loop gain on the phase margin of the receiver and establishes a relationship between damping factor and phase control parameter to achieve practical phase margin on the order of 45° to 60°. The performance of this receiver is compared to that of conventional homodyne optical receiver and a substantially improved performance is predicted.

Keywords- Modified balanced optical phase-locked loops (MBOPPLL), Binary phase-shift keying (BPSK), Phase modulator (PM), Homodyne receiver, Phase margin (PMR).

1. INTRODUCTION

Optical phase shift-keying homodyne communication systems have been studied over the past few decades and are still in its infancy from the practical point of view. BPSK detection has the best sensitivity of any binary signaling scheme. This high value of receiver sensitivity allows, for example, an increase in repeater spacing in long-haul point to point applications. Cascaded optical amplifiers in long-haul coherent system inevitably degrades the performance of it, because the accumulated amplified spontaneous emission (ASE) noise increases post-detection noise to the PLL receiver [1]. This intensifies the stringency of laser line-width requirements for coherent optical system [2,3]. Additionally, when the source laser line-width exceeds 500 KHz, loop delay is introduced in the system.
The presence of loop propagation delay along with the finite line-width of the laser adds instability to the loop. Loop propagation delay also induces a phase delay at the output of the local oscillator (VCO) and put a restriction on the maximum realizable value of loop natural frequency. This can be minimized by advancing the phase of local oscillator through an external control which is done by electronically controlling the phase of the VCO through phase modulator [5].

Against the above background, it is highly important to examine the relative stability of homodyne optical receiver in the presence of a loop propagation delay. In this respect, the so called phase margin is often used as a measure of the degree of loop stability. In this paper, we present relative stability analysis of a PSK homodyne optical receiver in terms of its phase margin when phase control is used. The analysis takes into account all the parameters affecting the receiver stability.

2. ANALYSIS

The modified balanced optical phase-locked loop (BOPLL) is shown in Fig.1. This homodyne optical receiver contains all the components of a standard BOPLL in conjunction with an additional phase modulator (PM).

Let the electric fields of the received optical signal and local laser source shining on the photo-detector be expressed as

\[ E_r(t) = \sqrt{2P_r} \sin(\omega_r t + \alpha(t)) \]  \hspace{1cm} (1)

\[ E_0(t) = \sqrt{2P_0} \sin(\omega_0 t + \theta_{\text{vco}}(t) + \theta_{\text{vco}}(t) + \beta(t)) \]  \hspace{1cm} (2)

where \( \omega_r \) is received angular frequency; \( \omega_0 \) is angular frequency of the free running local laser source; \( P_r, P_0 \) are received and local laser powers respectively; \( \alpha(t), \beta(t) \) are phase noise of the transmitter and local laser sources respectively; \( \theta_{\text{vco}}(t) \) is phase modulation of the local laser VCO due to the input to the VCO terminal; \( \theta_{\text{vco}}(t) \) is phase modulation of the local laser VCO due to the phase modulation of the phase modulator.

Fig. 1: Block diagram of the modified balanced optical phase locked loop (MBOPPLL)

Assuming a balanced optical phase detector, it is easily shown that the photo-detector output is given by
\[ V_p(t) = A \sin(\phi(t - \tau)) \]  
\[ \phi(t) = (\omega_r - \omega_0) t - \theta_{plan}(t) - \theta_{vco}(t) + (\alpha - \beta) \]  
with \( R = 2rR_0 \sqrt{P_rP_0} \) and \( A = K_{ppl}F(s)(1 + s\tau_p)\sin(\phi(t - \tau)) \) where \( \tau_p = r(\omega_r - \omega_0) + \frac{\beta - \alpha}{\omega_0} \) is the loop propagation delay.

This signal is then processed by a standard first order active filter with transfer function \( F(s) = \frac{1 + s\tau_2}{s\tau_1} \) and finally sent to the VCO laser input. The optical phase modulator possesses linear phase modulation characteristics and is employed to modulate the phase of the voltage controlled laser oscillator.

It can be shown that the governing phase equation of the balanced OPLL is

\[ \frac{d\phi(t)}{dt} = \Omega_o - AK_{ppl}F(s)(1 + s\tau_p)\sin(\phi(t - \tau)) + \frac{d\phi(t)}{dt}(\alpha(t) - \beta(t)) \]  
\[ \Omega_0 = (\omega_r - \omega_0) \]  
where \( \Omega_0 \) is Open loop frequency error; \( k_{ppl} \) is the sensitivity of the VCO (Hz/volt); \( k_p \) is the phase modulator sensitivity (rad/volt) and \( \tau_p = \frac{k_p}{k_{ppl}} \).

Open-loop transfer function of MBOPPLL system is given by

\[ G(s) = \frac{Ak_{ppl}}{s} \left( \frac{1 + s\tau_2}{s\tau_1} \right) e^{-\tau} \]  
In frequency-domain notation, we may write

\[ G(j\omega) = \frac{Ak_{ppl}}{j\omega} \left( 1 + j\omega\tau_p \right) \left( \frac{1 + j\omega\tau_2}{j\omega\tau_1} \right) e^{-j\tau} \]

The phase margin PMR is determined at the unity-gain frequency of \( G(j\omega) \), i.e., at the frequency \( \omega = \omega_n \) for which \( |G(j\omega)| = 1 \). The system will be unstable if the phase \( \phi(\omega) \) of \( H(\omega) \) exceeds \( -\pi \) rad at \( \omega = \omega_n \). By definition [2], PMR is the margin that is left with respect to this stability limit, i.e.,

\[ PMR = \phi(\omega_n) - (-\pi) = \pi + \phi(\omega_n) \]

Clearly,

\[ |G(j\omega)|^2 = \left( \frac{Ak_{ppl}}{\omega} \right)^2 \left( 1 + \omega_0^2 \tau_p^2 \right) \left( \frac{4\xi^2 \omega_n^2}{k_{ppl}^2} + \frac{1}{\omega^2 \tau_1^2} \right) \]

\[ = \frac{1}{\omega^2} \left[ 1 + \frac{\omega_0^2}{\omega_n^2} p^2 \right] \left[ 4\xi^2 \omega_n^2 + \frac{\omega_n^4}{\omega^2} \right] \]  
where \( \omega_n = \sqrt{\frac{Ak_{ppl}}{\tau_1}} \) and \( \xi = \frac{\omega_n \tau_2}{2} \) are respectively the loop natural frequency and loop damping factor and \( p = \omega_n \tau_p \).

Now, \( |G(j\omega)| = 1 \), if and only if
Equation (9) has only one positive root, which is given by

\[
\left(\frac{\omega}{\omega_n}\right)^2 \left(1 - 4\xi^2 p^2\right) - \left(\frac{\omega}{\omega_n}\right)^2 \left(4\xi^2 + p^2\right) - 1 = 0 \quad \text{..........................(9)}
\]

Therefore, \(\omega_0 = \omega_n F(\xi, P)\) where \(F(\xi, p) = \left[\frac{4\xi^2 + p^2 + \sqrt{(4\xi^2 + p^2)^2 + 4\left(1 - 4\xi^2 p^2\right)}}{2\left(1 - 4\xi^2 p^2\right)}\right]^{1/2}\)

Finally, the phase margin can be expressed in terms of \(\phi\) and \(\omega_0\) as

\[
PMR = \pi + \phi(\omega_0) = \arctan[pF(\xi, p)] + \arctan[2\xi F(\xi, p)] - F(\xi, p)\omega_0\tau \quad \text{....(11)}
\]

Equation (11) is similar to that given by Bergmans[2]. Similarly, when \(F(s)=1\) (i.e. with out loop filter), the phase margin can be expressed as

\[
PMR = \left(\pi/2\right) + \arctan\left[p/(1 - p^2)\right] - \left(Ak_{\text{pll}}\tau\right)/\sqrt{1 - p^2} \quad \text{..................(12)}
\]

The following typical numerical values[7,8,9] are used in our analysis: \(P_r = -53\) dBm, \(P_0 = +0.33\) dBm, \(R = 1\) A/W, \(r = 2.74\) KΩ, \(k_{\text{pll}} = 300\) MHz/volt, \(k_p = 12\) rad/volt, \(\tau_1 = 0.1\) µsec, \(\tau_2 = 16\) nsec.

3. RESULTS and DISCUSSION

Fig. (2), computed from (12), shows a representation of phase margin versus the phase control parameter for 20 nsec loop propagation delay. It can be well understood from the figure that loop stability boundary has been increased from 47º to 75º due to the proposed additional phase control. This is in corroboration with Biswas and Lihiri [11]. Again, the phase control parameter should be of the order of 0.05 to 0.2 to achieve practical phase margin on the order of 45º to 60º deg in this analysis.
In fig.(3), phase margin is plotted against the damping factor for various values of phase control parameter. Irrespective of phase control parameter, for phase margin of practical interest, the curves all have maximum for damping factors in the order of unity. The level off only slowly as damping factor increases beyond this optimum, yet decline rapidly as damping factor increases. This suggests that a proper engineering choice for damping factor would be a value in the order of unity. This result tallies with the results given in Bergmans[6] and Norimatsu and Iwashita[8].

When the damping factor is large, it is better to use the loop gain \( k_h \) and the damping factor \( \xi \) to describe the performance of a PLL, as suggested by Norimatsu and Iwashita[8]. Thus, we adopt the parameter \( k_h \) [Hz], which is the loop gain at an infinite frequency and it is defined as 
\[
k_h = \frac{A k_{pl} \tau_2}{\tau_1} = 2 \xi \omega_o \quad \text{………………..(13)}
\]

Dependence of phase margin on loop gain is shown in Fig.(4) and Fig.(5), showing the effect of phase modulator control and the damping factor for the second order MBOPPLL respectively. The plots show that phase margin falls with the increase of loop gain. It is found that at 50º phase margin and \( \xi = 1, \tau = 20 \) nsec, loop gain increases from 21 MHz to 31 MHz in presence of phase modulator with normalized phase modulator sensitivity \( p = 0.1 \). It is vivid from the figure that loop gain increases with the increase of damping factor in presence of phase modulator with \( p = 0.2 \). Thus we can increase the PLL bandwidth by increasing the damping factor. As system bandwidth increases, large volume of information can be received by the improved system. The normalized required line-width can be expressed as
\[
\delta v = \alpha (k_h \tau) \beta \quad \text{………………..(14)}
\]
where \( (\alpha, \beta) = (4.19 \times 10^{-5}, 9.60 \times 10^{-1}) \). It is clear that we can broaden the line-width requirements by increasing the loop gain of the receiver.

![Fig.4 The required phase margin versus loop gain for the Phase control parameters of 0, 0.1 and 0.2. Damping factor \( \xi = 1 \) and loop propagation delay \( \tau = 20 \) nsec.](image)

![Fig.5 The required phase margin versus loop gain for the Damping factors 0.707 and 1.0. Loop delay \( \tau = 20 \) nsec and \( p = 0.2 \).](image)
For 20nsec and 40nsec loop propagation delay, Fig.(6) portrays the combinations of damping factor (ξ) and normalized phase control parameter (p) that yield maximum phase margin on the order of 45º to 60º. The curve plotted in Fig.(6) can be approximated as

\[ \xi = a + bp - cp^2 \]  

……………  (15)

where a,b and c are loop propagation delay dependent constant co-efficient.

(a,b,c) = (0.8690,1.0107,3.336) for 20nsec loop delay,(a,b,c) = (0.8458,0.8430,2.8461) for 40nsec loop delay.

Fig.(7),computed from (11),depicts the maximum loop delay that is permissible for a given phase margin. The plots show that phase margin falls with the increase of loop delay. For a practical damping factor in the order of unity and with additional phase control p =0.2, the allowable loop delay should apparently be of the order of 13nsec to 23 nsec to achieve practical phase margin on the order of 45º to 60º. In presence of optical phase modulator, the stability of the receiver is enhanced and system can tolerate larger loop delay.

<table>
<thead>
<tr>
<th>Loop Delay = 20 nsec</th>
<th>Loop Delay = 25 nsec</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>Maximum PMR(deg)</td>
</tr>
<tr>
<td>0.08</td>
<td>42.3449</td>
</tr>
<tr>
<td>0.09</td>
<td>43.3242</td>
</tr>
<tr>
<td>0.1</td>
<td>44.2879</td>
</tr>
<tr>
<td>0.11</td>
<td>45.2350</td>
</tr>
<tr>
<td>0.12</td>
<td>46.1646</td>
</tr>
<tr>
<td>0.13</td>
<td>47.0755</td>
</tr>
<tr>
<td>0.14</td>
<td>47.9669</td>
</tr>
<tr>
<td>0.15</td>
<td>48.8376</td>
</tr>
</tbody>
</table>
The effect of signal power and local oscillator laser signal power are presented in Fig.(8) and Fig.(9) respectively. It is observed that phase margin increases with signal power and local oscillator laser power almost linearly. Consider phase margin as 50º, power requirement is relaxed for source laser from -52dBm to -56dBm and local oscillator laser from +1dBm to -2dBm with $k_p = 10 \text{ rad/volt}(\text{Table-2}).$ Thus, improvement in power sensitivity of source and local oscillator laser is 4dBm and 3dBm respectively at same value of phase margin with additional phase control.

Table 2: Laser power budget [ $\delta v = 10 \text{ MHz}, \tau = 20 \text{ nsec}$]

<table>
<thead>
<tr>
<th>Phase Margin (deg)</th>
<th>Received power requirement(dBm) at $P_0 = +0.33 \text{ dBm}$</th>
<th>VCO laser power requirement(dBm) at $P_r = -53 \text{ dBm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>With out Phase control $k_p = 10 \text{ rad/v}$</td>
<td>MBOPPLL $k_p = 10 \text{ rad/v}$</td>
</tr>
<tr>
<td>50 deg</td>
<td>-52.0</td>
<td>-56.0</td>
</tr>
<tr>
<td>45 deg</td>
<td>-55.5</td>
<td>-58.5</td>
</tr>
</tbody>
</table>
4. CONCLUSIONS

In this paper we have considered a homodyne PSK optical receiver with an additional arrangement of phase modulator in the presence of nonnegligible loop propagation delay. A general expression of phase margin, the measure of loop relative stability, has been derived in terms of all the parameters affecting the receiver stability. In presence of optical phase modulator, the stability of the receiver is enhanced and system can tolerate larger loop delay. In optical communication systems, the white frequency noise of optical sources is much larger than random-walk or flicker noise within the PLL bandwidth. Therefore, large damping factors are preferable. Also, loop gain increases with the increase of damping factor in presence of phase modulator. This permit the required line-width to be correspondingly increased. Another finding of the receiver is the improvement in power budget of laser sources for cost-effective PSK receiver is promising one for future optical communication networks.

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6. REFERENCES


