RESEARCH ON PILOT BASED CHANNEL ESTIMATION FOR LTE DOWNLINK USING LS AND LMMSE TECHNIQUE

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ABSTRACT

The main purpose this paper is to study the performance of two linear channel estimators for LTE Downlink systems, the Least Square Error (LSE) and the Linear Minimum Mean Square Error (LMMSE) using QPSK and 16 QAM modulation technique and also studied the effect of channel length on the performance of channel estimation. As LTE Downlink system is a MIMO-OFDMA based system, a cyclic prefix (CP) is inserted at the beginning of each transmitted OFDM symbol in order to mitigate both inter- carrier interference (ICI) and inter-symbol interference (ISI). The inserted CP is usually equal to or longer than the channel length. However, the cyclic prefix can be shorter because of some unforeseen channel behaviour. In the case where the cyclic prefix is equal to or longer than the channel length, LMMSE performs better than LSE but at the cost of computational complexity. In the other case, LMMSE performs also better than LS only for low SNR values. However, LS shows better performance for LTE Downlink systems for high SNR values. MATLAB simulations are used to evaluate the performance of the proposed estimator in terms of Bit Error Rate (BER) for 2x2 LTE Downlink systems.

Index Terms: LTE, Pilot based Channel Estimation, Least square, Linear Minimum Mean Square Error, Inter Symbol Interference, Inter carrier Interference and Orthogonal Frequency Division Multiple Access.
I. INTRODUCTION

Over the last few decades, due to the increasing demand for high speed data and widespread network access in mobile communications, there has been tremendous ongoing research in the field of cellular communications which has resulted in achieving significant developments.

Among them, the multiple-input multiple-output (MIMO) represents the most interest research results. The researches based on MIMO technologies have leading to improve high system capacity without additional bandwidth [1]. Multipath propagation causing selective frequency channels may causes serious problems for mobile. Therefore, Multicarrier modulation (MC), especially Orthogonal Frequency Division Multiplexing (OFDM) [2] which is used to combat the effect of frequency selective fading. OFDM consists of converting a frequency-selective fading channel into parallel flat-fading sub-channels. The propagation over the radio-frequency channel is characterized by a spread of the signal in time due to Doppler effect, so at this time frequency dispersion loss its orthogonality. So, there will be a Inter Symbol Interference and Inter Carrier Interference introduced. So to mitigate that effect cyclic prefix is used in beginning of every OFDM symbol. Cyclic prefix is sometimes shorter and longer than the channel length [3].

LTE is a fourth generation combination of MIMO-OFDM system. In this paper we focus on the LTE downlink System. LTE Downlink system adopts Orthogonal Frequency Division Multiple Access (OFDMA) as a access technique in Downlink system. LTE Downlink provides a data rate of 100 Mbps for 2*2 MIMO system.

Channel estimation is critical in LTE Downlink MIMO-OFDM system. Many works have studied foe this MIMO-OFDM system. At many research works they assume that length of the cyclic prefix should be greater than the channel length. But sometimes because of the channel behaviour cyclic prefix can be shorter than the channel length. At that condition channel estimation is difficult because of ISI and ICI introduced [4].

In this paper, we are using the two estimator technique LS and LMMSE for Block type pilot based channel estimation of LTE Downlink.

In section II, we describe the LTE physical layer. In section III we describe a LTE Downlink Model. In section IV we describe the pilot based channel estimation using LS and LMMSE technique. In section V simulation results are in terms of Bit error rate (BER).

II. OVERVIEW OF LTE DOWNLINK SYSTEM

Figure 1 illustrates the structure of the LTE radio frame. The duration of one LTE radio frame is 10 ms. It is composed of 20 slots of 0.5 ms described in figure 1. Each time slot consists of either 7 or 6 OFDM symbols depending on the length of the CP (normal or extended). In LTE Downlink physical layer, 12 consecutive subcarriers are grouped into one Physical Resource Block (PRB). A PRB has the duration of 1 time slot. Within one PRB, there are 84 resource elements (12 subcarriers *7OFDM symbols) for normal CP and 72 resource elements (12 subcarriers * 6 OFDM symbols) for extended CP.
A Physical Resource Block (PRB) is defined as 12 consecutive subcarriers for one slot. The subcarriers are spaced by 15 kHz from each other. Therefore, each PRB occupies a bandwidth of 180 kHz (12 x 15 kHz). A PRB consists of 84 resource elements (12 subcarriers during 7 OFDM symbols) or 72 resource elements (12 subcarriers during 6 OFDM symbols).

The LTE specifications [5] define parameters for system bandwidths from 1.25 MHz to 20 MHz as shown in Table 1.
### Table 1: LTE Downlink Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1.25</th>
<th>2.5</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmission Bandwidth (MHz)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sub-frame duration (ms)</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Sub-carrier spacing (KHz)</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Physical resource block bandwidth (KHz)</td>
<td>180</td>
<td>180</td>
<td>180</td>
<td>180</td>
<td>180</td>
<td>180</td>
</tr>
<tr>
<td>Number of available PRBs</td>
<td>6</td>
<td>12</td>
<td>25</td>
<td>50</td>
<td>75</td>
<td>100</td>
</tr>
<tr>
<td>Sampling Frequency (MHz)</td>
<td>1.92</td>
<td>3.84</td>
<td>7.68</td>
<td>15.36</td>
<td>23.04</td>
<td>30.72</td>
</tr>
<tr>
<td>FFT Size</td>
<td>128</td>
<td>256</td>
<td>512</td>
<td>1024</td>
<td>1536</td>
<td>2048</td>
</tr>
<tr>
<td>Number of Occupied sub-carriers</td>
<td>76</td>
<td>151</td>
<td>301</td>
<td>601</td>
<td>901</td>
<td>1201</td>
</tr>
</tbody>
</table>

### III. LTE DOWNLINK MODEL

LTE Downlink system is a MIMO-OFDM based system. The system model is given in Figure 2. We consider a MIMO system with N transmits antennas and N receives antennas [6].

![LTE Downlink model](image-url)
LTE Downlink systems employ OFDMA scheme for multiple access. OFDMA is a multiple access scheme based OFDM modulation technique. We consider a OFDM system with \( NBW \) carriers occupying the bandwidth. OFDMA allocates a number of time/frequency resources to different users. Its objective is to split the data stream to be transmitted onto narrowband orthogonal subcarriers by the means of the inverse discrete Fourier Transform (IDFT) operation, which allows for an increased symbol period. A cyclic prefix with the length of \( LCP \) is appended at the beginning of each OFDM symbol. The CP consists of a repetition of the last part of an OFDM symbol. In order to avoid degradations due to inter-symbol interference (ISI) and inter carrier interference (ICI), the inserted CP is generally equal or longer than the maximum excess delay of the channel [7].

Figure 4 illustrates a baseband OFDM system model. The \( N \) complex constellation symbols are modulated on the orthogonal sub-carriers by mean of the Inverse Discrete Fourier. The \( N \) subcarriers are spaced by \( \Delta f = 15 \) KHz. To combat the effect of frequency-selective fading, a Cyclic prefix (CP) with the length of \( Lcp \) is inserted at the beginning of each OFDM symbol.

![Baseband OFDM System](image)

Figure 4: baseband OFDM System

Each OFDM symbol is transmitted over frequency-selective fading MIMO channels assumed independents of each other. Each channel is modeled as a Finite Impulse Response (FIR) filter with \( L \) taps.

Here \( X \) is a input data of IDFT block at transmitter, \( Y \) is a output data of IDFT block at receiver. \( F \) is a DFT matrix. \( W \) is a sampled additive noise. \( g \) is a sampled channel impulse response.

We define the frequency channel given by,

\[
H = DFT(g) = F \cdot g
\]  

(1)

Noise in the frequency domain given by,

\[
W = DFT(w) = F \cdot w
\]  

(2)
The received OFDM symbol, after removing the CP and performing the DFT, at one receive antenna can be written as:

\[ Y = DFT \left( IDFT(x) \ast g \right) + W \]
\[ = X F g + w \]
\[ = XH + W \]  \hspace{1cm} (3)

\( Y \) = Received symbol  \\
\( X \) = Diagonal matrix containing references symbols  \\
\( H \) = Channel frequency response  \\
\( W \) = An additive white Gaussian noise with zero mean

Noise is assumed to be independent of transmitted symbols.

**IV. CHANNEL ESTIMATION**

Channel estimation has important impact on the receiver performance. In LTE systems, channel estimation is performed based on the reference signals. The reference symbols are placed in the first OFDM symbol and the fifth OFDM symbol of every time slot when short CP is used while they are placed in the first and the fourth OFDM of every time slot when long CP is used. Figure 5 shows reference signal pattern for two antennas [8].

The received pilot signals can be written as:

\[ Y = X_p H_p + W_p \]  \hspace{1cm} (4)

(\( . \) \( p \) denotes the position where the references signals transmitted.

![Figure 5: LTE Downlink reference signal pattern for two antennas](image-url)
The task now is to estimate the channel responses given the pilot signals X and the received signals Y. In this paper, two linear channel estimators are studied: the Least Square (LS) and the Linear Minimum Mean Square Error (LMMSE).

A. Least Square LS

The least square (LS) channel estimation is a simple estimation technique with very low complexity. It does not require any prior knowledge of the channel statistics. It is widely used because of its simplicity. However, it suffers from a high mean square error. The LS estimation channel frequency response is obtained by [9],

\[ h_{LS} = x^{-1}y \]  

Where, X = Transmitted Symbols 
Y = Received Symbols

The LS estimate of such system is obtained by minimizing the square distance between the received signal and the original signal.

B. Linear Minimum Mean Square Error LMMSE

The LMMSE channel estimation employs the channel statistics to minimize the MSE estimate of the channel responses given by,

\[ H_P^{LMMSE} = R_{HHP} \left( R_{HPHP} + \sigma^2 \right)^{-1} H_P^{LS} \]

\[ R_{HHP} = \text{Cross correlation matrix between all subcarriers and the subcarriers with reference signals} \]
\[ R_{HPHP} = \text{Autocorrelation matrix of the subcarriers with reference signals} \]

The high complexity of LMMSE estimator is due to the inversion matrix lemma. Every time data changes, inversion is needed. The complexity of this estimator can be reduced by averaging the transmitted data. Therefore we replace the term,

\[ (XX^H)^{-1} = E \left[ (XX^H)^{-1} \right] \]

Therefore the simplified LMMSE estimator becomes,
\[ \hat{H}_p^{\text{LMMSE}} = R_{\hat{H}_p} \left( R_{\hat{H}_p} + \frac{\beta}{S\text{NR}} I_p \right)^{-1} \hat{H}_p^{\text{LS}} \]  

(7)

\( \beta \) = scaling factor depending upon the constellation

Value of 1 for QPSK and 17/9 for 16 QAM

V. SIMULATION RESULT

In this section, we compare the performance of the LS and the LMMSE estimation techniques for 2*2 LTE-5 MHz Downlink systems under the effect of the channel length. The transmitted signals are quadrature phase-shift keying (QPSK) and 16 QAM modulated. The number of subcarriers in each OFDM symbol is \( N = 300 \), and the length of CP is \( \text{LCP} = 36 \). 100 LTE radio frames are sent through a frequency-selective channel. The frequency-selective fading channel responses are randomly generated with a Rayleigh probability distribution.

Table 2: Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTE Bandwidth</td>
<td>5 MHz</td>
</tr>
<tr>
<td>Number of used subcarriers</td>
<td>300</td>
</tr>
<tr>
<td>Cyclic Prefix length</td>
<td>36</td>
</tr>
<tr>
<td>Number of transmitted frames</td>
<td>100</td>
</tr>
<tr>
<td>Number of transmitted antennas</td>
<td>2</td>
</tr>
<tr>
<td>Number of received antennas</td>
<td>2</td>
</tr>
<tr>
<td>Modulation schemes</td>
<td>QPSK, 16 QAM</td>
</tr>
<tr>
<td>Channel Model</td>
<td>Rayleigh</td>
</tr>
</tbody>
</table>
**Case 1: L < LCP**

For L=6 and LCP = 36

**QPSK:**

![Graph showing BER vs. SNR for L=6 and LCP = 36 of QPSK]

**Figure 5:** BER vs. SNR for L=6 and LCP = 36 of QPSK

**16 QAM:**

![Graph showing BER vs. SNR for L=6 and LCP = 36 of 16 QAM]

**Figure 6:** BER vs. SNR for L=6 and LCP = 36 of 16 QAM

In this case, the cyclic prefix is longer than the channel which means that ISI and ICI are completely suppressed. Figure 5 and Figure 6 shows that LMMSE estimation technique is better than the LS estimator. Although, LMMSE gives the best performance but its complexity is higher due to the channel correlation and the matrix inversion lemma.
For L=20 and LCP =36

**QPSK:**

![Figure 7: BER vs. SNR for L=20 and LCP =36 of QPSK](image)

**16 QAM:**

![Figure 8: BER vs. SNR for L=6 and LCP =36 of 16 QAM](image)

From figure 7 and 8 shows that LMMSE is better than LS technique but with the drawback of computational complexity. LMMSE works better at low SNR and LS is better at higher SNR.
**Case 2: L > LCP**  
For L=40 and LCP =36

**QPSK:**

![Figure 9: BER vs. SNR for L=40 and LCP=36 of QPSK](image)

**16 QAM:**

![Figure 10: BER vs. SNR for L=40 and LCP=36 of 16 QAM](image)

In this case, the cyclic prefix is shorter than the channel. ISI and ICI will be introduced. Figure 9 and 10 shows that more the channel is longer than CP, more the performance is lost in terms of BER at the cost of more complexity for LMMSE estimation technique.
For L=50 and LCP=36

**QPSK:**

![Graph showing BER vs. SNR for QPSK](image)

**16 QAM:**

![Graph showing BER vs. SNR for 16 QAM](image)

Figure 11 and 12 demonstrates that LMMSE that; even the cyclic prefix is shorter than the channel length; LMMSE shows also better performances than LS for LTE Downlink systems but only for low SNR values. For high SNR values, LMMSE loses its performance in terms of BER and LS estimator seems to perform better than LMMSE for this range of SNR values.
VI. CONCLUSION

In this paper, we propose to evaluate the performance of LS and LMMSE estimation techniques for LTE Downlink systems under the effect of the channel length. The cyclic prefix inserted at the beginning of each OFDM symbol is usually equal to or longer than the channel length in order to suppress ICI and ISI. However, the CP length can be shorter than the channel length because of some unforeseen behaviour of the channel. Simulation results show that in the case where the CP length is equal to or longer than the channel length, the LMMSE performs better than LS estimator but at the cost of the complexity because it depends on the channel and noise statistics. In the other case, LMMSE provides better performance only for low SNR values and begins to lose its performance for higher SNR values. In other hand, LS shows better performance than LMMSE in this range of SNR values. And by comparing QPSK and 16 QAM modulated technique, we can see that the BER performance is lost in 16 QAM than the QPSK technique as increasing SNR values. So we can conclude that QPSK technique is better than the 16 QAM techniques in terms of BER performance.

REFERENCES

5. 3GPP, “Evolved Universal Terrestrial Radio Access (E-UTRA); Physical channels and modulation,” TS 36.211, 3rd Generation Partnership Project (3GPP), Sept. 2008.