PERFORMANCE ANALYSIS: PID AND LQR CONTROLLER FOR REMUS AUTONOMOUS UNDERWATER VEHICLE (AUV) MODEL

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ABSTRACT

Autonomous underwater vehicles (AUV) are used in many applications like mining, aquaculture and military applications. AUV is highly nonlinear system; modeling and controller performance is challenging issue for the researchers. In this paper a linear quadratic regulator (LQR) and proportional integral derivative (PID) controllers are designed for the depth control system of AUV. These controllers are designed for AUV model, based on REMUS 100 AUV. Design of controllers is simulated on MATLAB / SIMULINK. The results of both are compared in time domain response. Accordingly the performance is compared and observed.

Keywords: LQR control, PID control, AUV, REMUS, MATLAB/SIMULINK.

1. INTRODUCTION

Autonomous underwater vehicle is complex nonlinear system due to hydrodynamic uncertainties involved in it. AUV maneuvering and control is crucial task. Present article represents modeling and controller design aspects. The dynamic model of AUV is based on REMUS100 [1]. REMUS is a small low cost autonomous underwater vehicle. PID and LQR controllers are designed and simulated for REMUS. In this paper the design of LQR is compared with traditional PID controller. Positions of poles and over all characteristics are monitored. In respect MATLAB / SIMULINK is a best tool used for it. PID controller is successfully implemented in various process control systems. It is very simple in structure and has proved very good performance for various systems. But the systems in which uncertainties are more and disturbances are not defined PID is not a suitable controller. Here PID controller is designed to study the behavior of AUV and is compared with LQR controller. The organization of paper is as under.
In 2\textsuperscript{nd} section modeling of AUV is described and is studied through literature. In the subsequent section basic concepts of PID and LQR are explained and are applied for depth control of AUV. In the 4\textsuperscript{th} section simulation results of AUV system with each controller are presented and at lastly in the fifth section the conclusion is obtained. Appropriate controller is which based on the system performance.

2. MODELING OF AUV

2.1 AUV Dynamics
The modeling of AUV involves statics and dynamics. Statics is under equilibrium conditions of vehicle. The motion of vehicle is under dynamics of the vehicle [2]. Dynamics of AUV is under two parts kinematics and kinetics. The equations of motion of AUV are represented in two coordinate frames and can be written as

\[ \dot{\eta} = J(\eta)v \quad (1) \]

\[ M v + C(v)v + D(v)v + g(\eta) = \tau \quad (2) \]

Vector \( v \) can be represented as

\[ v = [v_1^T, v_2^T]^T, \]

\[ v_1 = [u, v, w]^T, \quad v_2 = [p, q, r]^T \]

\( v_1 \) = linear velocity vector
\( v_2 \) = angular velocity vector
\( \eta = [\eta_1^T, \eta_2^T]^T, \]

\[ \eta_1 = [x, y, z]^T, \quad \eta_2 = [\phi, \theta, \psi]^T \]

\( \eta_1 \) = position vector
\( \eta_2 \) = orientation vector

All the variables are described in following Fig.1

![Fig.1.AUV showing all variables](image)

\( J(\eta) \) in equation (1) is transformation matrix based on Euler angles function. Equation (2) gives nonlinear equations of motion in 6DOF. Where

Inertial Matrix

\[ M_{RB} \in R^{6\times6} \]
Coriolis Matrix \[ C(v) \in R^{6\times 6} \]

Damping matrix \[ D(v) \in R^{6\times 6} \]

Control input matrix \[ g(\eta) \in R^{6\times 1} \]

giving forces and moments acting on AUV. The motion of AUV is controlled by stern and rudder angle \( \delta_s, \delta_r \) respectively.

2.2 Subsystems of AUV model

For simplicity the complete model is divided into three subsystems. These [2] non interacting systems are:

- Surge speed control
- Steering control (yaw and yaw rate)
- Depth control (depth, pitch, pitch rate)

Here we are considering linearized depth control system of AUV. The state space depth control model of AUV is as in equation (3)

\[
\begin{pmatrix}
    m - x_v & -(m x_g + z_g) & 0 & 0 \\
    -(m x_g + M \dot{w}) & I_{yy} - M \dot{q} & 0 & 0 \\
    0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
    \dot{w} \\
    \dot{q} \\
    \dot{z} \\
    \dot{\theta}
\end{pmatrix} = 
\begin{pmatrix}
    Z_w \\
    M_w + Z_g \\
    -m x_g U + M \dot{q} \\
    0
\end{pmatrix}
\begin{pmatrix}
    w \\
    q \\
    z \\
    \theta
\end{pmatrix}
\]

The state vector is \( x \) and control input vector is \( u \) as under:

\[
x = \begin{pmatrix} w & q & z & \theta \end{pmatrix}^T
\]

\[
u = \begin{pmatrix} \delta_s \end{pmatrix}^T
\]

State space is typically represented as

\[
\dot{x} = Ax + Bu
\]

All the hydrodynamic coefficients are of REMUS and physical dimensions are as per its Myring hull shape. The forward velocity of vehicle is \( U=1.54m/s\) (knots). The state space parameters are as under:

\[
A = \begin{bmatrix}
-2.3792 & 1.5680 & 0 & 0.0431 \\
4.2367 & -1.1880 & 0 & -0.7027 \\
1.0000 & 0 & 0 & -1.5400 \\
0 & 1.0000 & 0 & 0
\end{bmatrix}
\]

[4]
B = \begin{pmatrix} -1.3744 \\ -3.8352 \\ 0 \\ 0 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix} \quad D = [0]

3. CONTROL METHODS

Here Proportional Integral Derivative (PID) and Linear Quadratic Controller (LQR) control strategies are applied to AUV system. In this section PID and LQR Controllers are explained [5]. For applying controller the system is to be controllable and observable for all states in state space model.

3.1 PID Controller

Proportional Integral Derivative controller is a feedback controller. Different methods are used for the tuning $K_p$, $K_i$ & $K_d$, i.e. the proportional, integral & derivative gains of PID controller, but here we have tuned these gains using trial and error method. The transfer function of PID Controller is:

$$T_{PID} = K_p + K_i/s + K_d * s$$

The reference input and measured output are compared and generated error is used to control the system. $K_p$, $K_i$, and $K_d$ are tuned [9] for best performance of the controller.

3.2 LQR Controller

Linear Quadratic Regulator (LQR) is well known control technique which provides practical feedback gain [8].

For LQR derivation from the state space model of system

$$\dot{x} = Ax + Bu$$

all n states are available for controller. The design of LQR means to design state feedback gain factor $K$. The objective is considered as $J$ [10] where $J$ is minimized in such a way that response will be stable. $J$ can be written as:

$$J = \int_0^\infty (x^T Qu + u^T Ru)dt$$

$Q$ and $R$ are deciding weights. $Q$ and $R$ matrices are symmetric diagonal matrices.

$$Q, R \geq 0$$

$Q$ is related to $J$. The objective of LQR is to provide optimal feedback $K$ as under:

$$u = -Kx$$
K can be given by matrix algebraic Riccati equation (MARE) as under:

$$u = -R^{-T} B^T P x$$

also P is needed for optimal feedback gain K. Closed loop poles are moved for ideal performance of the system.[7]

4. SIMULATION AND RESULTS

The open loop response of depth control system is shown in Fig.2. The pole zero positions are seen in pole-zero map of AUV depth system is as in Fig.3.

![Fig.2 Open Loop Time Response](image)

![Fig.3 Pole Zero map of Open loop system](image)

<table>
<thead>
<tr>
<th>Eigen value</th>
<th>Damping</th>
<th>Freq.(rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00e+000</td>
<td>-1.00e+000</td>
<td>0.00e+000</td>
</tr>
<tr>
<td>3.96e-001 + 4.30e-001i</td>
<td>-6.78e-001</td>
<td>5.84e-001</td>
</tr>
<tr>
<td>3.96e-001 - 4.30e-001i</td>
<td>-6.78e-001</td>
<td>5.84e-001</td>
</tr>
<tr>
<td>-4.36e+000</td>
<td>1.00e+000</td>
<td>4.36e+000</td>
</tr>
</tbody>
</table>
Eigen values and damping values for open loop system of AUV are mentioned above. From the time response of open loop system demonstrated in Fig.2 and pole positions in Fig.3 it is seen that the behavior of system is unstable. For easy and proper evaluation of the system both PID and LQR controller’s responses are taken for closed loop system for unit step input in time domain.

Designing and applying PID Controller to the depth control system of AUV following closed loop response is obtained [6]. Simulation model is in SIMULINK as in Fig.6 shown below.

Simulation of SIMULINK model is based on the values of gains as follows.

\[ K_p = 0.090, \ K_I = 0.00094, \ K_D = 0.09 \]

In LQR controller design [8] the best LQR parameters are as under:

\[ Q = \begin{bmatrix} 1 & 0 & 0 & 0; 0 & 1 & 0 & 0; 0 & 0 & 1 & 0; 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ R = 1 \]
The pole zero positions in LQR controller and closed loop step response for depth control system of AUV is shown below in Fig.7 and Fig.8 respectively.

From the closed loop responses for PID and LQR controllers it is seen that both give zero steady state error. Response time is faster in PID and in LQR it is comparatively slow. Overshoot in PID is remarkable than LQR. From the results it is seen that LQR acts better in terms of overshoot. Response time wise PID is better but proper tuning is required to minimize overshoots.

5. CONCLUSION AND FUTURE SCOPE

Both the PID and LQR control approaches can be applied to the depth control system of AUV based on the mathematical model of AUV. From the simulation results of both the controllers it is seen that proper tuning of PID controller is required for better performance and to reduce overshoots. LQR is comparatively
steady in its behavior. In addition to above control strategies other control methods like fuzzy, sliding mode, Fuzzy PID, Derivative sliding mode also can be applied for AUV system in future.

6. REFERENCES


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