NATURAL CONVECTION IN A TWO-SIDED LID-DRIVEN INCLINED POROUS ENCLOSURE WITH SINUSOIDAL THERMAL BOUNDARY CONDITION

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ABSTRACT

Natural convection in a two-sided lid-driven inclined porous enclosure with sinusoidal thermal boundary condition on one wall using staggered grid finite-difference method is studied in this paper. The governing equations are solved numerically for streamlines, isotherms, local Nusselt number and the average Nusselt number for various values of the thermal radiation and heat generation parameters for three different inclination angles. The results indicate that the flow pattern and temperature field are significantly dependent on the physical parameters.

Keywords: Natural convection; two-sided lid-driven inclined cavity; finite-difference method; thermal radiation; heat generation.

1. INTRODUCTION

Fluid flow and heat transfer in closed cavities which are mechanically driven by tangentially moving walls represents a basic problem in fluid mechanics. Flow of fluid in a cavity due to moving lid is a classical problem which has wide applications in engineering such as geothermal energy, lubrication, chemical processes, drying technologies, crude oil production, storage of nuclear waste, compacted beds for the chemical industry and thermal insulation etc. In the past several decades, a number of experimental and numerical studies have been performed to analyze the flow field and heat transfer characteristics of lid-driven cavity flow such as heat exchangers, solar power collectors, packed bed catalytic reactors and so on (Nield and Bejan [1]). Furthermore, heat transfer in a lid-driven cavity flow is widely used in applied mathematics as indicated by Bruneau and Saad [2]. The lid-driven flows with a constant heat flux are frequently faced in the application of cooling of electronic devices (Hsu and Wang [3]). Khanaf er and Chamkha [4] studied the unsteady mixed convection flow in a lid-driven encloser filled with Darcian fluid-saturated uniform porous medium is the presences of internal heat generation. Al-Amiri [5]
investigated the momentum and heat transfer for square lid-driven cavity filled in a porous medium heated from a driving wall. 

Roy and Basak [6] studied the influence of uniform and non-uniform heating of the bottom wall and one vertical wall on flow and heat transfer characteristics due to natural convection within a square enclosure. Later, Basak et al. [7] investigated a natural-convection flow in a square cavity filled with a porous medium considering both uniform and non-uniform heating of cavity wall from below using Darcy-Forchheimer model. Oztop et al. [8] studied numerical simulation in a non-isothermally heated square enclosure. 

Also, the problem of natural convection in an inclined enclosure has considerable attention due to its relevance to a wide variety of application area in engineering and science. The study of mixed convection in inclined lid-driven enclosure filled with viscous fluid was studied by Sharif [9]. He observed that the average Nusselt number increases with increase in the inclination angle. Recently, Ogut [10] investigated a laminar, mixed convection flow in an inclined lid-driven rectangular enclosure heated from one side wall of the cavity with a constant speed and cooled from the stationary adjacent side while the other sides are kept stationary and adiabatic. Sivakumar et al. [11] analyzed numerically the mixed convection heat transfer and fluid flow in a lid-driven cavity for different lengths of the heater and different locations of it. It is found that a better heat transfer rate is obtained on reducing the heater length of the hot wall. Oztop and Varol [12] investigated the flow field, temperature distribution and heat transfer in a lid-driven cavity filled with porous medium in the presence of non-uniformly heated bottom wall. 

Kuhlmann et al. [13] conducted numerical and experimental studies on the steady flow in a rectangular two-sided lid-driven cavity. Alleborn et al. [14] analyzed numerically the mixed convection in a shallow inclined two-sided lid-driven cavity with a moving heated lid. They simulated the problem and found that both heat and mass transports are affected from the change of cavity inclination angle. Moreover, the bifurcation topologies of different creeping flows have been studied by Bruns and Hartnack [15] for the two-sided lid-driven cavity problem by changing the driving or geometry parameters. Understanding the mixed convection heat transfer process in inclined cavities is thus very important for designing purposes in the event when the inclined orientation is required. 

When technology processes take place at high temperatures thermal radiation heat transfer become very important. Recent developments in hypersonic flights, missile reentry rocket combustion chambers and gas cooled nuclear reactors have focused attention of researchers on thermal radiation and emphasize the need for inclusion of heat transfer in these processes. Very recently, Mahapatra et al. [16] studied natural convection in a lid-driven square cavity filled with fluid-saturated porous medium in the presence of thermal radiation considering Darcy-Forchheimer model. Not much of attention has been given on the study of laminar natural convection flow in an inclined two-sided lid-driven enclosure with thermal boundary conditions as far as authors’ knowledge. Thus the present study deals with the unsteady laminar natural convection flow in an inclined enclosure heated non-uniformly in the presence of thermal radiation and heat generation from the left vertical wall, heated uniformly from bottom wall and cooled from the top wall by keeping right wall in adiabatic state. The numerical results for streamlines, isotherms and the heat transfer rate at the heated walls in terms of local Nusselt number and average Nusselt number are presented graphically and in tabular form.  

II. GOVERNING EQUATIONS AND BOUNDARY CONDITIONS  

The two-sided lid-driven inclined enclosure under investigation is filled with a fluid-saturated porous medium and with impermeable walls. The schematic configuration of the problem is illustrated in Fig. 1, with H denoting the length of the sides of the square enclosure. The fluid is assumed as incompressible and the porous medium is considered to be homogeneous and isotropic. Furthermore, the porous medium is assumed to be in local thermal equilibrium with the fluid. The top and bottom walls of the cavity are of different temperatures, the right side wall is adiabatic and the left side wall temperature varies sinusoidally. The Brinkman-Darcy model is adopted for the fluid flow in the porous medium. 

The dimensionless governing equations take the following form:
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1) \]

\[ \frac{\partial u}{\partial t} = -\left( \frac{Pr \sigma}{Da} \frac{\partial p}{\partial x} \right) + Pr \varepsilon \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \sigma \left( \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} \right) - \frac{Pr \varepsilon}{Da} \frac{u}{Da} \Omega \sin \varphi, \quad (2) \]

\[ \frac{\partial v}{\partial t} = -\left( \frac{Pr \sigma}{Da} \frac{\partial p}{\partial y} \right) + Pr \varepsilon \sigma \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\partial v^2}{\partial x} + \frac{\partial uv}{\partial y} - \frac{Pr \varepsilon}{Da} \frac{v}{Da} \Omega \cos \varphi, \quad (3) \]

\[ \frac{\partial \theta}{\partial t} = \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) + \frac{4}{3N_R} \frac{\partial^2 \theta}{\partial y^2} + He \theta. \quad (4) \]

Now we define the following non-dimensional variables:

\[ x = \frac{X}{H}, \quad y = \frac{Y}{H}, \quad u = \frac{UH}{\alpha_m}, \quad v = \frac{VH}{\alpha_m}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \quad p = \frac{KP}{\mu \alpha_m}, \quad Pr = \frac{\nu}{\alpha_m}, \quad Da = \frac{K}{H^2}, \quad N_R = \frac{k k_*/4 \sigma T_c^3}{\kappa}, \quad He = \frac{QH^2}{\kappa}. \]

Where \( Pr, \ N_R \) and \( He \) are Prandtl, thermal radiation parameter and heat generation parameter, respectively. Here \( \varphi \) is the inclined angle of the cavity. The heat capacity ratio \( \sigma \) and Rayleigh- Darcy number \( Ra^* \) (Wang et al. [19]) are defined as
\[
\sigma = \frac{(\rho c_p)_f}{(\rho c_p)_m}, \quad Ra^* = g\kappa\beta(T_h-T_c) \frac{H^2}{\nu \alpha_m}.
\]

Non-dimensional boundary conditions are

\[ u = v = 0 \quad \text{and} \quad \theta = \sin(\pi y) \quad \text{at} \quad x = 0, \]
\[ u = v = 0 \quad \text{and} \quad \frac{\partial \theta}{\partial x} = 0 \quad \text{at} \quad x = 1, \]
\[ u = A, \quad v = 0 \quad \text{and} \quad \theta = 1 \quad \text{at} \quad y = 0, \]
\[ u = A, \quad v = 0 \quad \text{and} \quad \theta = 0 \quad \text{at} \quad y = 1. \]

where \( \Omega = \left(\frac{H U_0}{\alpha_m}\right) \) is a parameter.

The heat transfer coefficient in terms of the local Nusselt number (\( Nu \)) is defined by
\[ Nu = -(\partial \theta / \partial n), \]
where \( n \) denotes the normal direction on a plane. The local Nusselt number at the bottom wall (\( Nu_b \)) and left vertical wall (\( Nu_l \)) are defined as
\[ Nu_b = -(\partial \theta / \partial y)|_{y=0} \quad \text{and} \quad Nu_l = -(\partial \theta / \partial x)|_{x=0}. \]

The average Nusselt number at the hot walls is given by,
\[ Nu_{H|y=0} = \int_0^1 Nu_b \, dx \quad \text{and} \quad Nu_{H|x=0} = \int_0^1 Nu_l \, dy. \]

### III. Solution Procedure and Numerical Stability Criteria

Control-volume based finite-difference discretization of the above equation has carried out in the present work in staggered grid, popularity known as MAC cell. In this type of grid alignment, the velocities and the pressure are evaluated at different locations of the control volume, the pressure and temperature are evaluated at same locations of control volume as shown in Figure 1(b). The difference equations have been derived in distinct types of cells for the four equations, viz., (i) continuity cell, (ii) u-momentum cell, and (iii) v-momentum cell [24], (iv) temperature cell. These distinct cells have been shown in the Figures 2(a) and 2(b). We now describe the iteration process to obtain the solutions of the basic equations with appropriate boundary conditions. In the derivation of pressure Poisson equation, the divergence term at n-th time level \( (D^*_y) \) is retained and evaluated in the pressure-Poisson iteration. It is done because the discretized form of divergence of velocity field, i.e, \( (D^*_y) \) is not guaranteed to be zero. The solution procedure starts with the initializing the velocity field. This is done either from the result of
previous cycle or from the prescribed initial and boundary conditions. Using this velocity field pressure-Poisson equation is solved using Bi-CG-Stab method. Knowing pressure field u-momentum, v-momentum and equation for temperature are updated to get u, v, θ at (n + 1)th time level. Using the values of u and v at (n + 1)th time level, the value of the divergence of velocity field is for its limit. If its absolute value is less than 0.5 × 10^5 and steady state reaches then iteration process stops, otherwise again pressure-Poisson equation is solved for pressure. Linear stability of fluid flow is

\[
\delta t_1 \leq \text{Min} \left[ \frac{\delta x}{u} \cdot \frac{\delta y}{v} \right],
\]

which is related to the convection of fluid, i.e., fluid should not move more than one cell width per time step (Courant, Friedrichs and Lewy condition). Also, from the Hirt’s stability analysis, we have

\[
\delta t_2 \leq \text{Min} \left[ \frac{1}{2 \text{Pr} \left( \frac{\delta x^2 + \delta y^2}{\delta x^2} \right)} \right].
\]

This condition roughly stated that momentum cannot diffuse more than one cell width per time step. The time step is determined from \( \delta t = FCT \times \text{Min} (\delta t_1, \delta t_2) \), where the factor FCT varies from 0.2 to 0.4. The upwinding parameter \( \beta \) is governed by the inequality condition. \( 1 \leq \beta \leq \text{Max} \left[ \frac{u \delta t}{\delta x}, \frac{v \delta t}{\delta y} \right] \). As a rule of thumb, \( \beta \) is taken approximately 1.2 times larger than what is found from the above inequality condition.

IV. RESULTS AND DISCUSSIONS

Numerical results for contours of the streamlines and isotherms inside the inclined square cavity and the average Nusselt number distribution at the heated surface of the cavity for various values of radiation parameter, heat generation parameter and inclination angle \( \phi \) have been examined and presented graphically and in tabulated form. The working fluid is chosen as air with Prandtl number \( \text{Pr} = 0.7 \) and the value of \( \epsilon = 0.6 \) are taken in the present study. The inclination angle \( \phi \) of the enclosure are chosen as \( 20^0, 45^0, 60^0, 80^0, 90^0 \).

In order to get a grid independent solution to the present problem, a grid refinement study is performed for \( \text{Ra}^* = 10^3, \text{Da} = 10^{-3}, \phi = 20^0, 45^0, 80^0 \) and a parameter A \((1.0 \leq A \leq 200)\). The results of grid independence test are shown in Table 1. As observed from the results, 80×80 grid is sufficient to achieve good results in the entire study. As shown in Table 2, the average Nusselt numbers are in good agreement with the Darcy-Brinkman solutions reported by Lauriat and Prasad [20]. These comprehensive verification efforts demonstrated the accuracy of the present numerical method. Table 3 presents the computed values of Nu_{H|y=0} and Nu_{H|x=0} for various values of \( \phi, \text{N}_{R} \) and He, keeping the other parameters fixed. From this table, it is seen that Nu_{H|y=0} and Nu_{H|x=0} decreases with increase in \( \text{N}_{R} \) and He for all the three inclination angle, \( \phi = 20^0, 45^0, 80^0 \).

<table>
<thead>
<tr>
<th>Grid points</th>
<th>20×20</th>
<th>40×40</th>
<th>80×80</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi )</td>
<td>Iteration</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>17469</td>
<td>34729</td>
<td>101051</td>
</tr>
<tr>
<td>450</td>
<td>16475</td>
<td>31954</td>
<td>74572</td>
</tr>
<tr>
<td>800</td>
<td>12714</td>
<td>24601</td>
<td>58398</td>
</tr>
</tbody>
</table>

Stream function and isotherm contours for various values of \( \phi, \text{N}_{R} \) and He with non-uniform heating of the left wall and uniform heating of the bottom wall in the two-sided driven cavity flow are displayed in Figs. 2-5. As expected, due to hot left vertical wall, fluids rise up along the side of the hot which then flow down along the cold wall,
forming a roll with anti-clockwise rotation inside the cavity. Counter-clockwise circulations are shown with positive sign of stream functions. Fig. 2(a) depicts the plot of the stream function for $N_R = 1.0$ and $He = 1.0$, keeping the other parameters fixed. It is seen from this figure that the values of stream function in the core decreases with increasing the inclination angle of the cavity i.e, the flow rate decreases and it is also observed that the streamlines are more concentrated near the side walls due to stronger circulation which results in lower heat transfer rate due to convection and increase in the inclination angle. Similar flow pattern is observed in the Figs. 2(b) and 2(c). It is interesting to note that when inclination angle are fixed (for $20^\circ$ and $80^\circ$), the values of stream function in the core increases with increase in the value of $N_R$ i.e, the flow rate increases with increase in the radiation parameter. But when $\phi = 45^\circ$ there is not much change in the value of stream function in the core for the value of $N_R > 3.0$.

Table 2: Comparison of average Nusselt number predictions with the computed data of Lauriat and Prasad [20] when $Pr = 1.0$ and $A = 0.0$.

<table>
<thead>
<tr>
<th>$Ra^*$</th>
<th>$Da$</th>
<th>$\epsilon$</th>
<th>Present Study</th>
<th>Lauriat &amp; Prasad [20]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^4$</td>
<td>$10^{-4}$</td>
<td>0.4</td>
<td>25.74</td>
<td>25.70</td>
</tr>
<tr>
<td>$10^3$</td>
<td>$10^{-6}$</td>
<td>0.4</td>
<td>13.30</td>
<td>13.22</td>
</tr>
<tr>
<td>$10^2$</td>
<td>$10^{-6}$</td>
<td>0.4</td>
<td>3.08</td>
<td>3.06</td>
</tr>
</tbody>
</table>

Table 3: Computed values of $Nu_{H|y=0}$ and $Nu_{H|x=0}$ when $Pr = 0.7$, $A = 1.0$, $Ra^* = 10^3$, $Da = 10^{-3}$, $\epsilon = 0.6$ for various values of $\phi$, $N_R$ and $He$.

| $\phi$ | $N_R$ | $He$ | $Nu_{H|y=0}$ | $Nu_{H|x=0}$ |
|--------|------|------|--------------|--------------|
| $20^\circ$ | 1.0 | 1.0 | 6.9437 | -4.2071 |
| | 3.0 | 1.0 | 5.1820 | -5.2032 |
| | 5.0 | 1.0 | 4.4149 | -6.0774 |
| | 1.0 | 3.0 | 6.9437 | -4.2071 |
| | 5.0 | 3.0 | 6.7958 | -4.3654 |
| | 1.0 | 5.0 | 6.6331 | -4.5459 |
| $45^\circ$ | 1.0 | 1.0 | 7.3142 | -3.7731 |
| | 3.0 | 1.0 | 5.3929 | -4.8953 |
| | 5.0 | 1.0 | 4.5677 | -5.8316 |
| | 5.0 | 3.0 | 7.3142 | -3.7731 |
| | 1.0 | 5.0 | 7.1752 | -3.8693 |
| | 1.0 | 5.0 | 7.0286 | -3.9744 |
| $80^\circ$ | 1.0 | 1.0 | 6.9926 | -3.1420 |
| | 3.0 | 1.0 | 5.1191 | -4.4739 |
| | 5.0 | 1.0 | 4.2956 | -5.5441 |
| | 5.0 | 3.0 | 6.9927 | -3.1420 |
| | 1.0 | 5.0 | 6.8542 | -3.1826 |
| | 1.0 | 5.0 | 6.7113 | -3.2256 |
Figure 1. Geometry and the coordinate system

- U = U₀, V = 0, T = T_c
- U = V = 0, δT/δx = 0 (Adiabatic wall)
- U = V = 0
- T = (T_h - T_c)sin(πy/H) (Sinusoidally varying temperature)
- H = Porous Medium
- H = moving wall
- U = U₀, V = 0, T = T_h
Figure 2. Streamlines for He=1.0, A=1.0 and (a) N R =1.0, (b) N R =3.0, (c) N R =5.0.
Figure 3. Streamlines for $N_R=1.0$, $A=1.0$ and (a) $He=1.0$, (b) $He=3.0$, (c) $He=5.0$. 
Figure 4. Isotherms for $He=1.0$, $A=1.0$ and (a) $N_R=1.0$, (b) $N_R=3.0$, (c) $N_R=5.0$. 
Figure 5. Isotherms for $N_R=1.0$, $A=1.0$ and (a) $He=1.0$, (b) $He=3.0$, (c) $He=5.0$. 
Figure 6. Local Nusselt number at bottom wall for different values of $\phi$, $N_R$ and $He$ when $Pr=0.7$, $Ra^*=10^3$, $Da=10^3$, $A=1.0$ and $\epsilon=0.6$. 
Figure 7. Local Nusselt number at left vertical wall for different values of $\phi$, $N_R$ and $He$ when $Pr=0.7$, $Ra^* = 10^3$, $Da=10^{-3}$, $A=1.0$ and $\varepsilon=0.6$. 
Figure 8. Plot of average Nusselt number vs. inclination angle of the cavity
Fig. 3 shows that as inclination angle increases the flow patterns are different from those shown in Fig. 2. Three cases arise when inclination angle is fixed i.e. (i) when $\varphi = 20^\circ$, the value of stream function in the core increases with increase the value of He (ii) when $\varphi = 45^\circ$, there is not much flow rate change after He $> 3.0$ (iii) when $\varphi = 80^\circ$, the value of stream function in the core changes with increase in value of He, whereas not effect is seen near the walls.

Figs. 4(a) presents isotherms are concentrated near the edges of hot walls and cold walls due to stronger circulation, which results in higher heat transfer rate due to convection. It is seen from this figure that as inclination angle increases, the boundary layers are relatively thick and a very small core region occurs such that the isotherms become almost parallel to the vertical walls indicating that conduction regime is approached due to the fact that the middle cells of the cavity get enhanced compressing shear cells generated due to moving lids. However, the shear cells near the walls become more vigorous. The isotherm patterns realize the effect of shear cell near the walls. But in the middle portion of the enclosure, the feature of the natural convection is maintained. Similar flow patterns are shown in Figs 4(b), 4(c) with less concentrated streamlines near the hot and cold walls. Fig. 5 depicts the various streamlines patterns for different values of He for fixed value of NR = 1.0. The flow patterns are almost similar to those shown in Fig. 4 which shown the isotherm for different values of thermal radiation parameters. Careful observation of Figs. 5(a) and 5(c), show that there is significant change in the streamlines pattern by increasing the values of internal heat generation. The effect of NR and He on the local Nusselt numbers at the bottom and left vertical walls ($N_{ub}$ and $N_{ul}$) for different values of the inclination angle ($20^\circ$, $45^\circ$ and $80^\circ$) by keeping the other parameters fixed are displayed in Figs. 6 and 7. In the case when uniform heating at the bottom wall (Fig. 6) is very high at the left edge of the bottom wall due to the discontinuity present at this edge in the temperature boundary condition then the heat transfer rate reduces towards the right corner of the bottom wall for all values of NR and He. It is also seen that heat transfer rate decreases with increase in NR and He, for any inclination angle. It is also
observed from Fig. 7 that in the case of non-uniform heating condition at the left wall, the heat transfer rate is very high at both left edge and right edge and it is minimum near the left corner of the left wall. The effect of the inclination angle on average Nusselt number is investigated and demonstrates in Fig. 8. As can be seen from the Fig. 8(a), the average Nusselt number on walls reaches its maximum value at the inclination angle of around 55° due to uniform heating at bottom wall. This angle is nothing but a 'critical angle' (Nor Azwadi et al. [21]) where the natural convection is at the maximum point. But there is no 'critical angle' due to non-uniform heating at left vertical wall which is seen in Fig. 8(b). It is also seen that average Nusselt number increases with increase in inclination angle of the cavity. This phenomenon is explained in the discussion for Table 2. Fig. 9 shows that as parameter A increases from 1 to 200, the local Nusselt number at the bottom wall (Nub) increases upto a certain value of x and thereafter opposite trend is observed. This is occurs due to change of the speed of the lid of the cavity.

V. CONCLUSIONS

The present paper deals with the study of thermal radiation and heat generation effects on unsteady two-dimensional laminar natural convection flow in a two-sided lid-driven inclined enclosure non-uniformly heated from the left vertical wall, uniformly heated from bottom wall and cooled from the top wall while right vertical wall is kept adiabatic by using staggered grid finite-difference method. Following conclusions are drawn from this study:

(i) Flow rate decreases with increase in the inclination angle.
(ii) Increasing in the thermal radiation parameter enhances the flow rate by keeping the inclination angle fixed.
(iii) The value of stream function in the core does not change for any value of He when φ = 80°.
(iv) Average Nusselt number decreases with increasing the value of both NR and He.
(v) ‘Critical angle’ is formed due to uniform heating at the bottom wall, whereas no ‘Critical angle’ is found in the case of non-uniform heating at the bottom wall.

REFERENCES


