NEW MODEL OF CFRP-CONFINED CIRCULAR CONCRETE COLUMNS: ANN APPROACH

Dr. Salim T. Yousif
Assistant Prof, Civil Engineering Dept., College of Engineering/University of Mosul, Iraq

ABSTRACT

The application of fiber-reinforced polymer (FRP) composites in civil engineering works has increased in recent years, especially in the area of strengthening concrete columns. The objective of this research is to develop new mathematical models for predicting the confined compressive strength of carbon FRP (CFRP) circular concrete columns using artificial neural networks (ANNs), which is done using 208 incremental data results collected from the literature. Two mathematical models were developed: one depended on six input parameters, whereas the other depended only on three important parameters, namely, unconfined compressive strength of concrete, total thickness of the CFRP, and tensile strength of CFRP along the hoop direction. Comparison of the new two models using experimental data showed a good agreement and accuracy of the developed ANN models in predicting the CFRP-confined compressive strength of circular concrete columns. The new models were also used to perform a parametric study to evaluate the effect of the input parameters on the CFRP-confined compressive strength of circular concrete columns.

Key Words: Artificial neural networks, Concrete, Compressive strength, Fiber-reinforced polymer confinement, Mathematical modeling

1. INTRODUCTION

Externally bonded carbon fiber-reinforced polymer (CFRP) composite sheets and laminates have been used widely in civil engineering construction to strengthen reinforced concrete (RC) components [1] because of their high strength, light weight, ease in use, durability against weather conditions, chemical resistance properties, relative low cost, and ease in repair.
These CFRP composites are used for strengthening of columns [2, 3]. Confinement of the columns using CFRP jackets is done by wrapping the fibers along the hoop direction of the concrete columns.

Concrete expands laterally when subjected to axial compression. The FRP jacket provides a confining pressure to the concrete to resist the expansion caused by the axial compression. Ultimate failure occurs when the FRP jacket ruptures because of the tensile stress along the hoop direction [3]. Because of the FRP confinement, both the compressive strength and ultimate strain of the concrete can be improved [1].

Artificial neural networks (ANNs) have experienced increased interest over the last years and have been successfully applied across a range of engineering problems, including the strengthening of columns [4, 5], increasing the capacity of RC beams strengthened with FRP reinforcements [6, 7], prediction of the compressive strength of concrete [8, 9], linear and nonlinear model updating of RC T-beams [10], predicting the bond strength of FRP-to-concrete joints [11], and many other engineering applications.

Naderpour et al. [2] employed the ANN to generate a model for predicting the compressive strength of FRP-confined concrete independently from the network. The model consisted of an empirical chart and seven mathematical equations.

In the present study, new mathematical models are developed based on ANNs using a database built from existing tests on CFRP-confined circular concrete specimens. This new model is then compared with the experimental data. Finally, the trained network model is used to perform a parametric study to evaluate the effect of various parameters on the CFRP-confined compressive strength of concrete.

2. AVAILABLE EMPIRICAL FRP-CONFINED MODELS

The confining pressure provided by the FRP jacket, as derived from empirical models, is a function of the column’s diameter, stiffness of the FRP jacket, and compressive strength of the unconfined concrete. A lateral confining stress $f_1$ is produced in the concrete when the confining jacket and the member is loaded such that the concrete starts to dilate and expands laterally. The stress is related to the thickness and strength of the FRP by [3]:

$$f_1 = \frac{2f_{FRP}t}{d}$$  \hspace{1cm} (1)

where $f_{FRP}$ is the tensile strength of the FRP along the hoop direction, $t$ is the total thickness of the FRP, and $d$ is the diameter of the confined concrete.

Several existing strength models for FRP-confined concrete take the following form [3]:

$$\frac{f_{cc}}{f_c} = 1 + k_1 \frac{f_1}{f_c}$$  \hspace{1cm} (2)

where $f_{cc}$ and $f_c'$ are the compressive strength of the confined and unconfined concrete, respectively. $f_1$ is the lateral confining pressure, and $k_1$ is the confinement effectiveness coefficient.

A number of strength models have been proposed specifically for the FRP-confined concrete, which employ Eq. 2 with modified expressions for $k_1$. The details of the models can be seen in [12]
3. ARTIFICIAL NEURAL NETWORKS

A neural network is a computer model whose architecture essentially mimics the knowledge acquisition and organizational skills of the human brain [13]. The function of artificial neurons is similar to that of real neurons [14]; they are able to communicate via signals sent among them by a large number of biased and weighted connections. Each neuron has its own transfer function, which describes how to convert a weighted sum of input to output.

The multi-layer perceptron is the most widely used type of ANN [15]. It is both simple and based on solid mathematical grounds. The input quantities are processed through successive layers of “neurons.” An input layer (with the number of neurons equal to the number of variables in the problem) and an output layer always exist. The layers in between are called “hidden” layers. Without a hidden layer, the perceptron can only perform linear tasks. All problems, which can be solved by a perceptron, can be solved with only one hidden layer; however, using two or more hidden layers is sometimes more efficient.

3.1 Back-propagation neural network

The back-propagation (BP) neural network is a multi-layered feed-forward [15, 16]. The BP neural network adjusts internally the weight values to set the non-linear relationships between the input and the output without giving explicitly the function expression. Further, the BP neural network can be generalized for the input that is not included in the training patterns.

The BP algorithm is used to train the BP neural networks. This algorithm looks for the minimum error function in the weight space using the method of gradient descent. The combination of weights that minimizes the error function is considered to be a solution to the learning problem. The input feed forward can be described by the following steps [15, 17]:

Once the input vector \( \mathbf{x} \) is introduced into the input layer, it can calculate the input to the hidden layer \( h^H_j \) as

\[
h^H_j = b_j + \sum_{i=1}^{N_I} w_{ji} x_i
\]

where \( b_j \) is the bias and \( w_{ji} \) is the synaptic weight that connects input neuron \( i \) to hidden neuron \( j \).

Each neuron of the hidden layer takes its input \( h^H_j \), uses it as the argument for a function, and produces an output \( y^H_j \) given by

\[
y^H_j = f(h^H_j)
\]

The input to the neurons of output layer \( h^O_k \) is calculated as

\[
h^O_k = b_k + \sum_{j=1}^{N_H} w_{kj} y^H_j
\]

and the network output \( y_k \) is given by

\[
y_k = f(h^O_k)
\]

where \( f \) represents the activation function. Then
3.2 Neural network design and training

A set of test results were collected from the literature for the axial compressive strength of the circular confined concrete columns [18–41]. The selected database contains 208 test results.

The data collected from the field were divided randomly into two groups. The first group, which contained 188 results, was used in the training of the neural network, and the other data group, which contained 20 results, was used to test the obtained networks. The multi-layer feed-forward BP technique was implemented in the current research to develop and train the neural network, where the sigmoid transform function was adopted.

Different training functions are available in MATLAB [42]. The Levenberg–Marquardt (LM) technique have been proven to be an efficient training function and are therefore used to construct the ANN model. This training function is one of the conjugate gradient algorithms that start the training by searching in the steepest descent direction (negative of the gradient) on the first iteration. The LM algorithm is known to be significantly faster than the more traditional gradient descent-type algorithms for training ANNs.

The input, as well as the output, was scaled in the range of 0.1 to 0.9. The scaling of the training data sets was carried out using the following equation:

\[
y = \frac{0.8(x - x_{\text{max}})}{x_{\text{max}} - x_{\text{min}}} + 0.9.
\] (8)

Any new input data should be scaled before being introduced to the network and the corresponding predicted values should be unscaled before use.

For each model, several architectures of the ANN models were examined by varying the number of hidden layers and the training function parameters to establish a suitable and stable network for the project. Each network must be tested and analyzed, and the most appropriate network must be chosen for a particular project.

The parameters used for the input nodes in the ANN modeling were as follows: diameter \(d\) of the circular concrete specimen (mm), height \(h\) of the circular concrete specimen (mm), compressive strength \(f'_c\) of the unconfined concrete (MPa), total thickness \(t\) of the CFRP (mm), tensile strength \(f_{\text{FRP}}\) of the CFRP along the hoop direction (MPa), and elastic modulus \(E_{\text{FRP}}\) of the CFRP (MPa). The target node was the compressive strength of the confined concrete \(f_{\text{CC}}\).

The range of the input data used is listed in Table 1. The architecture of the developed ANN model is shown in Fig. 1.

**Table 1**: Range of input data used in the ANN models

<table>
<thead>
<tr>
<th>Input parameter</th>
<th>d (mm)</th>
<th>h (mm)</th>
<th>(f'_c) (MPa)</th>
<th>t (mm)</th>
<th>(f_{\text{FRP}}) (MPa)</th>
<th>(E_{\text{FRP}}) (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>70</td>
<td>140</td>
<td>18</td>
<td>0.11</td>
<td>580</td>
<td>38</td>
</tr>
<tr>
<td>Maximum</td>
<td>200</td>
<td>788</td>
<td>169.7</td>
<td>2.06</td>
<td>4400</td>
<td>415</td>
</tr>
</tbody>
</table>
A regression analysis was conducted between the network response and the corresponding targets and a correlation coefficient was found. This option is a measure of how well the variation in the output is explained by the targets. If this number is equal to one, then a perfect correlation exists between the target and output predictions.

Fig. 2 shows the plot of the experimental compressive strength against the corresponding ANN predictions for the test data. A linear correlation can be observed, and the correlation coefficients are 0.977 and 0.964 for the training and the test data, respectively. Therefore, we can conclude that the model successively predicts accurately the compressive strength of the confined concrete.
4. IMPORTANCE OF INPUT PARAMETERS

Because the weight of the BP neural network cannot be easily understood in a numerical matrix form, it could be transformed into code values in percentage form by dividing the weights by the sum for all the input parameters, which yields the relative importance of each input parameter to the output parameter. The method of partitioning weights, proposed by Garson [17] and adopted by Goh [13], was used in this study to determine the relative importance of the various input parameters (Fig. 3). The major important parameter that influences the compressive strength of the confined concrete ($f_{cc}$) is the tensile strength ($f_{FRP}$) of the CFRP along the hoop direction with an importance of 30.67%, followed by the total thickness (t) of CFRP with an importance of 20.78% and the compressive strength ($f'_c$) of the unconfined concrete with an importance of 19.13%. The diameter (d) of the circular concrete specimen does not affect the compressive strength of the confined concrete ($f_{cc}$) because its importance is only 4.162%. Most mathematical models consider the column diameter as one of the main factors.

![Importance of input parameters of the first ANN model](image)

**Fig 3.** Importance of input parameters of the first ANN model
5. MODEL DEVELOPMENTS FOR COMPRESSIVE STRENGTH OF CONFINED CONCRETE

Another application of ANNs is in building a mathematical model. The present study contains six input and one output parameters. A model equation can be established using the weights as the model parameters [43]. The mathematical equation can be written as

\[
f_{cc} = \frac{1}{1 + e^{-\left[\theta_4 + \left(\frac{w_{q4}}{1 + e^{-x_1}}\right) + \left(\frac{w_{q4}}{1 + e^{-x_2}}\right) + \left(\frac{w_{q4}}{1 + e^{-x_3}}\right)\right]}}
\]  

(9)

where:

\[
x_1 = \theta_1 + w_{11} \times d + w_{21} \times h + w_{31} \times f'c + w_{41} \times t + w_{51} \times f_{FRP} + w_{61} \times E_{FRP}
\]

(10)

\[
x_2 = \theta_2 + w_{12} \times d + w_{22} \times h + w_{32} \times f_c + w_{42} \times t + w_{52} \times f_{FRP} + w_{62} \times E_{FRP}
\]

(11)

\[
x_3 = \theta_3 + w_{13} \times d + w_{23} \times h + w_{33} \times f_c + w_{43} \times t + w_{53} \times f_{FRP} + w_{63} \times E_{FRP}
\]

(12)

The values of weights \(w_{ij}\) and threshold \(\theta_j\) are shown in Table 2.

<table>
<thead>
<tr>
<th>Table 2: Weights and threshold levels of the ANN model</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Weights from node (i) in the input layer to node (j) in the hidden layer</td>
</tr>
<tr>
<td>nodes</td>
</tr>
<tr>
<td>(J=1)</td>
</tr>
<tr>
<td>(J=2)</td>
</tr>
<tr>
<td>(J=3)</td>
</tr>
<tr>
<td>b. Weights from node (i) in the hidden layer to node (j) in the output layer</td>
</tr>
<tr>
<td>node</td>
</tr>
<tr>
<td>(J=4)</td>
</tr>
</tbody>
</table>

Equation (9) is long and complex because it contains six independent variables. On the other hand, it can predict accurately the compressive strength \((f_{cc})\) of the confined concrete (Fig.2) with a correlation coefficient equal to 0.964.

The equation length depends on the number of nodes in the input and hidden layers. To simplify the equation, the most importance input parameters, which are the compressive strength \((f_c)\), of the unconfined concrete the total thickness \((t)\) of CFRP, and the tensile strength \((f_{FRP})\) of the CFRP along the hoop direction, were used in training the second ANN model with two nodes in the hidden layer. The result was the development of an ANN model with a regression of 0.9259 (Fig. 4). The small number of connection weights of the neural network enables the ANN model to be translated into a relatively simple formula in which the compressive strength of the confined concrete \((f_{cc})\) can be expressed as follows:
\[ f_{cc} = \frac{1}{1 + e^{-281.45 + \frac{137.75}{t_{FRP}} + \frac{288.86}{t_{FRP}^2}}} \]  

(13)

where:

\[ x_1 = -2.50 + 0.40f_{c} - 0.64t + 1.90f_{FRP} \]  

(14)

\[ x_2 = 2.90 - 0.36f_{c} + 0.34t - 1.42f_{FRP}. \]  

(15)

Before using Eqs. 10–12, 14, and 15, all input variables must be scaled between 0.1 and 0.9 using Eq. 8 for the data ranges shown in Table 1. The predicted values obtained from Eqs. 9 and 13 are scaled between 0.1 and 0.9. To obtain the actual values, these had to be unscaled using Eq. 8.

In contrast to all previous models, the second ANN model depends on the compressive strength \((f'_{c})\) of the unconfined concrete, the total thickness \((t)\) of CFRP, and the tensile strength \((f_{FRP})\) of the CFRP in the hoop direction, whereas the geometry of the column is not considered in this ANN model.

**Fig. 4.** Experimental and corresponding ANN compressive strength of the test data of second Model

Gaussian distributions are perhaps the most important model for studying the quantitative phenomena in the natural and behavioral sciences, such as the problems encountered in structural analysis and design. To determine the suitability of the developed CFRP-confined model, all 208 experimental and predicted confined compressive strength values were taken, and the results of the confined compressive strength ratio
(predicted/experimental) were statistically analyzed with a 0.5% level of significance using the SPSS software V.16.

The average and variance of all values were found to be 92% and 0.02, respectively. Fig. 5 shows the frequency histogram of the confined compressive strength ratio \( \frac{f_{cc,\text{pred}}}{f_{cc,\text{Exp}}} \) population curve. The \( \frac{f_{cc,\text{pred}}}{f_{cc,\text{Exp}}} \) values are distributed around their mean values. The magnitude of the frequency becomes smaller when the value moves away from the mean central value. The probability of obtaining a ratio between 90% and 95% is 85%.

![Histogram and normal distribution curve for the second mathematical model](image)

**Fig. 5.** Histogram and normal distribution curve for the second mathematical model

6. **PARAMETRIC STUDY**

One of the advantages of the ANN models is that parametric studies can be easily conducted by simply varying one input parameter while all other input parameters are set to constant values. Parametric studies can verify the performance of the model in simulating the physical behavior of the CFRP-confined concrete due to the variation in certain parameter values.

The second ANN model and Eq. 13 were used to complete this parametric study.

Figs. 6 and 7 show the relationship between the compressive strength \( f_{cc} \) of the unconfined concrete and that of the CFRP-confined concrete \( f_{cc} \) under different values of tensile strength \( f_{FRP} \) of the CFRP and total thickness \( t \) of the CFRP, respectively. In general and regardless of the other parameters, the compressive strength \( f_{cc} \) increases with the increasing compressive strength \( f'_{c} \) of the unconfined concrete.

Fig. 6 shows the effect of the tensile strength \( f_{FRP} \) of the CFRP on the compressive strength of the CFRP-confined concrete \( f_{cc} \) under different values of compressive strength \( f'_{c} \) of the unconfined concrete with a constant total thickness \( t \) equal to 0.22mm.
The compressive strength of the confined concrete ($f_{cc}$) strongly affects the tensile strength ($f_{FRP}$) of the CFRP, especially under high unconfined compressive strength and high tensile strength of the CFRP. For the unconfined compressive strength of 25 MPa, changing the tensile strength of the CFRP from 2,500 MPa to 4,000 MPa led to an increase in the confined compressive strength from 55.18 MPa to 56.6 MPa (increasing by 2.57%), whereas for the unconfined compressive strength of 65 MPa, the same change in the tensile strength ($f_{FRP}$) of the CFRP led to an increase in the confined compressive strength ($f_{cc}$) from 75.64 MPa to 87.27 MPa (increasing by 15.37%).

![Graph showing the effect of tensile strength on confined compressive strength](image)

**Fig 6.** Effect of the tensile strength of the CFRP on the compressive strength of the FRP-confined concrete

Fig. 7 shows the effect of the total thickness ($t$) of the CFRP on the compressive strength of the CFRP-confined concrete ($f_{cc}$) for different values of compressive strength ($f'_c$) of the unconfined concrete with a constant tensile strength ($f_{FRP}$) equal to 3,000 MPa. For different thicknesses ($t$) of CFRP, the curves are parallel, that is, a low or high unconfined compressive strength yields the same effect on the compressive strength of the CFRP-confined concrete with different thicknesses.

7. **CONCLUSIONS**

Two mathematical models for predicting the confined compressive strength of an CFRP circular concrete column have been developed using the ANN approach. The importance study showed that the diameter and height of the specimen and the elastic modulus of CFRP had little effect on predicting the confined compressive strength of the CFRP circular concrete column; hence, they were excluded from building the second ANN model, leaving only three input parameters. Both parametric and importance studies showed that the tensile strength of CFRP had an effect on predicting the confined compressive strength of the CFRP circular concrete column. Finally, the ANN approach was proven to be good and efficient in developing the mathematical models.
Fig 7. Effect of the total thickness of the CFRP on the compressive strength of the CFRP-confined concrete

REFERENCES


[12] Y.A. Al-Salloum, Experimental and analytical investigation of compressive strength of FRP-confined concrete, Project No. 11 / 426, Research Center, College of Eng., King Saud University, 2007, pp. 26


