ABSTRACT

The purpose of the present paper is to carry out the seepage analysis for a model to derive the equation of the phreatic line experimentally and thereafter proposing the best solution to trace the line. Sandy silt is used as a base material for this case. Phreatic Line for the model is plotted by carrying out 3 iterations each for two cases, that is, earthen dam without filter and with filter. Their corresponding phreatic line curves are traced on the butter paper on one of the longer side of the model. Plastic channel sections are used to contain the model sealing it air tight with the use of m-seal and rubber stoppers. Further, the traced curves are compared with the Analytical solution of Casagrande. Thereafter, a detailed analysis that is using regression modeling, a more accurate solution to obtain an equation of phreatic line in case of sandy silt earthen dam is proposed. The equation can be used for both the cases to determine the phreatic line more accurately than suggested by Casagrande.

Keywords: Earthfill Dam, Casagrande Method, Seepage, Drainage, Filter, Regression Modeling

1. INTRODUCTION

Seepage is the continuous movement of the water from the upstream face of dam toward its downstream face. The surface in the embankment along which the pore pressure is equal to atmospheric pressure is called the phreatic surface which ought to be kept at or below downstream toe. About 30-35% of earth dams had failed due to the seepage failure like sloughing and piping. The seepage of reservoir water through the body of an earth fill dam or at interfaces of dam with ground/steep ground surface or abutment or both, creates problems like Piping through dam & its foundation, Conduit leakage & also causes excessive water loss, thereby reducing usable storage of reservoir. Therefore, the provision of a drainage system becomes vital that would not only allow easy passage for the seepage flow but also would prevent the phreatic line from emerging at the downstream sloping face. For this purpose a horizontal filter is installed at the bottom of the earthen dam to lower the phreatic line. This paper presents an evaluation of phreatic line location within an earth fill dam with and without filter by the experimental method using channel section as well as
analytical method suggested by A. Casagrande\(^2\) (1940). For this we have constructed three models for each case with same water content to determine the best phreatic line close to the one obtained by the analytical method and the phreatic line was traced using butter paper on the long side of the apparatus. Three models for each case, that is, the case with and without filter were made and the phreatic line obtained is plotted on Ms excel and compared with analytically obtained equation. Model was made in the Geotechnical laboratory of Civil Department, Delhi Technological University. Further, using the results of this experiment, we have suggested the best modified equation for the phreatic in an earthen dam made of sandy silt material. This equation derived is much more accurate than the one given by A. Casagrande.

2. PROBLEM STATEMENT

The real time problem for this study is defined as in Figure 1. The example selected is a simplified representation of a typical homogeneous earth-fill dam geometry with an impervious foundation.

The geometry of the problem: height: 15.75 m, width of crest: 10.5 m, base length: 48.75 m, upstream slope: 1: 1.42, downstream slope: 1: 1 and Upstream water level: 13.5m. The dry and bulk unit weight of earth-fill dam soil are 15.70 KN/m\(^3\) and 19.20 KN/m\(^3\) respectively. Also the permeability of base soil is 5.33 x 10\(^{-7}\) m/sec.

The distorted modelling is used in this case for the experiment purpose and the details of the same are discussed latter in the paper.

![Fig 1. Dimensions of Real time Earthfill Dam Problem](image)

3. CHARACTERISATION OF BASE SOIL

The experiments for determining the physical properties of base soil were conducted prior to the modeling of the earthen dam and the properties of the base soil used have been specified in Table 1.
4. MODEL & EXPERIMENTAL APPARATUS

A small scale model is built after doing carrying out dimensional analysis of real system. This model represents an earth-fill homogenous dam on an impervious foundation. Distorted modeling is done in this case by taking the horizontal scaling ratio as 1/75 for the base dimensions and a vertical scale ratio of 1/45 for the height of dam. Sandy silt soil is used as the embankment material & tamping is done layer by layer of 2.5 cm thickness to obtain a uniform dry density in whole experiments. The phreatic line in the earth fill embankments have been determined by the use of the potassium permanganate solution.

![Figure 2](image2.png)

**Fig 2. Dimensions of the Prototype without filter**

The geometry of the embankment for without any filter is show is shown in Fig 2 and the one with a horizontal filter is shown is Fig 3. The dimensions of the prototype are as follows: height : 0.35 m, width of crest : 0.14 m, base length: 0.65 m, upstream slope : 1 : 0.86, downstream slope : 1 : 0.6 and Upstream water level: 0.30 m.

![Figure 3](image3.png)

**Fig 3. Dimensions of the Prototype with filter**
5. EXPERIMENTAL SOLUTION

The model is built in a plastic sheet (5 mm thick) channel of 0.76 m length, 0.3084 m width, 0.3084 m height, which was used to facilitate the observations of the behavior of seepage water and the process of earth fill embankment failure.

Fig 4. General View of Earth-fill dam

5.1 Determination of Volume of Prototype

V: Volume of Embankment
M1: Mass of Soil Expected of Cube
M2: Mass of Soil Expected for Embankment

\[
V = [(0.5 \times 0.35 \times 0.3) + (0.14 \times 0.35) + (0.5 \times 0.21 \times 0.35)] \times 0.3048 = 0.0421386 \text{ m}^3
\]

\[
M_1 = 0.65 \times 0.35 \times 0.3048 \times 1920 = 133 \text{ kg}
\]

\[
M_2 = 0.0421386 \times 1920 = 80.9 \text{ kg}
\]

5.2 Determining No. of Blows on each layer

For Standard Proctor Compaction Test

\[
W = \text{Weight of compactor} = 2.6 \text{ kg}
\]

\[
h = \text{Height of fall} = 0.310 \text{ m}
\]

\[
V = \text{Volume of cylinder} = 0.001 \text{ m}^3
\]

\[
n = \text{Number of blow} = 25
\]

\[
m = \text{Number of layers} = 3
\]

Taking for the earth-fill dam,

\[
n' = \text{Number of blows in each layer}
\]

\[
m' = \text{Number of layers} = 10
\]

“Compacting effort of standard proctor in lab = compacting effort of standard proctor on embankment” = energy/ volume is constant = \( W \times h \times n \times m / V \)

Compacting effort of standard proctor in lab = \( 2.6 \times 0.31 \times 25 \times 3 / 0.001 = 60450 \text{ J/m}^3 \) …Eq (1)

Compacting effort of standard proctor on embankment using eq (1)

\[
= (2.6 \times 0.31 \times n' \times 10) / 0.0421386 = 191.2 \times n'
\]

Therefore, \( n' = \frac{60450}{191.27} = 316 \)
Assuming Efficiency (\( \eta \)) of tamping as 50 %

Now, the total number of blows = 316 x 2 = 632 blows

6. TRACING OF PHREATIC CURVES

6.1. Case 1: Without filter

In order to determine the position of the phreatic line, three trials are done for 14% (Optimum Moisture Content) without filter. Further, the most accurate position of the phreatic line in the earthfill dam is found out. The following Fig 5 shows the phreatic line obtained on the butter paper for the trial 1. Figure 8 indicates the ideal phreatic line obtained analytically and the phreatic lines obtained for all trials, which are replicated on the excel sheet using the coordinates obtained on the butter paper.

![Fig 5: Phreatic line tracing for Dam without filter](image)

6.2. Case 2: With Horizontal filter

Similarly, in order to determine the position of the phreatic line, three trials are done for 14% (Optimum Moisture Content) using horizontal filter of length 0.21m and thickness 0.022m. Further, the most accurate position of the phreatic line in the earthfill dam is found out. The following Fig 6 shows the phreatic line obtained on the butter paper for the trial 1 and Fig 9 indicates the ideal phreatic line obtained analytically along with the lines obtained for all trials, which are replicated on the excel sheet using the coordinates obtained on the butter paper.

![Fig 6: Phreatic line tracing for Dam with filter](image)
7. ANALYTICAL SOLUTION

7.1. Assumptions
1. Homogenous, isotropic cross section
2. Relatively Impervious base
3. No special toe drainage
4. Darcy’s law is valid i.e. \( q = ky \frac{dy}{dx} \)
5. Steady state seepage,
6. Capillary action is not considered

7.2. Definition of terms
(As shown in Fig 7 & Fig 8)

- \( B_0 \) - Point of Intersection of the basic parabola with the water surface.
- \( h \) - Height of upstream water level
- \( a \) - Slope distance from toe of dam to point of discharge
- \( \alpha \) - Angle of discharge face from the toe =45°
- \( x, y \) - Coordinates of any point on the basic parabola, measured from toe of dam.
- \( a_0 \) - Distance along base line between toe of dam and vertex of basic parabola
- \( y_0 \) - Ordinate of basic parabola at toe of dam
- \( k \) - Coefficient of permeability of soil =5.33 x10-7 m/sec
- \( C \) - Point at which the phreatic line intersect the discharge slope
- \( C_0 \) - Point at which the phreatic line intersect the discharge slope
- \( \Delta a \) - Distance between \( C \) and \( C_0 \)

7.3. CALCULATION

7.3.1 Determining the Phreatic line in a dam without filter:
According to the analytical solution suggested by A.Casagrande \(^2\), we calculate the empirical equation for the phreatic line:

![Fig 7. Determination of phreatic line in an earthfill dam without filter](file)

www.iaeme.com/ijciet.asp 6  editor@iaeme.com
m = 0.30 \times 0.86 = 0.258 \text{ m} \\
BOB = 0.3 \times 0.258 = 0.0774 \text{ m} \\
h = 0.30 \text{ m} \\
And d = 0.65 - 0.7 \times 0.258 = 0.4694 \text{ m} \\

From equation:

\[ p = 0.5 \left( \sqrt{(d^2 + h^2)} - d \right) \] ...... Eq(2) \\
\[ p = 0.5 \left( \sqrt{(0.4694^2 + 0.30^2)} - 0.4694 \right) \]
\[ p = 0.04383 \]

Using the Casagrande equation:

\[ x = \frac{(y^2 - 4p^2)}{4p} \] ...... Eq(3) \\
\[ y^2 = 0.175x + 0.00768 \] ...... Eq(4)

The correction to the parabola is obtained by the following graph:

Taking equation of a line:

\[ y = x \tan \alpha \] ...... Eq (5)

The coordinates of point 'C' can be determined by solving Eq (3) & Eq (5)

Substituting the value of \( z \) from Eq (3),

\[ x = \frac{(x \tan \alpha)^2 - 4p^2}{4p} \]
\[ x^2 \tan^2 \alpha - 4px - 4p^2 = 0 \]

Hence,

\[ x^2 \left( \frac{1}{0.6} \right)^2 - 4(0.04383)x - 4(0.04383)^2 = 0 \]
\[ 2.77x^2 - 0.1692x - 0.00933 = 0 \] ......Eq(6)

The solution of the above Eq(4) gives \( x = 9.6 \times 10^{-2} \text{ m} \).

So, we get

\[ \Delta C_o = \sqrt{(0.096^2 + (0.096(1/0.6))^2)} = 18.66 \times 10^{-2} \text{ m.} = o + \Delta o \]

For 59.032 downstream slope angle, the correction is obtained from the Casagrande Chart:\[2\]

\[ \Delta o/ (o + \Delta o) = 0.32 \]
\[ \Delta o = 0.32 \times 18.66 \times 10^{-2} \text{ m.} = 5.97 \times 10^{-2} \text{ m} \]
\[ o = (o + \Delta o) - (\Delta o) \]
\[ o = (18.66 - 5.97) \times 10^{-2} \text{ m} = 12.69 \times 10^{-2} \text{ m} \]

So, \( o = \Delta C = 12.69 \times 10^{-2} \text{ m} \)

### 7.3.2 Determining the Phreatic line in a dam with filter:

According to the analytical solution suggested by A. Casagrande \[2\], we calculate the empirical equation for the phreatic line:

Length of filter (l) = 21 cm & Thickness of filter = 2.2cm

\[ m = 0.30 \times 0.86 = 0.258 \text{ m} \]
Fig 8. Phreatic lines for trials and ideal case at OMC for earthen dam without filter case.

Fig 9. Determination of phreatic line in an earthfill dam with filter

\[ \text{BOB} = 0.3 \times m = 0.0774 \text{m} \]

\[ h = 0.3 - 0.022 \times m = 0.278 \text{m} \]

And \( d = 0.65 - 0.7 \times m - l = 0.2594 \text{ m} \)

From equation:

\[ p = 0.5 \left( \sqrt{(d^2 + h^2)} - d \right) \]

\[ p = 0.5 \left( \sqrt{(0.2594^2 + 0.278^2)} - 0.2594 \right) \]

\[ p = 0.0604 \]

Using the Casagrande equation:

\[ x = \frac{(y^2 - 4p^2)}{4p} \]

\[ y^2 = 0.242x + 0.0146 \]
Fig10. Phreatic lines for trials and ideal case at OMC for earthen dam with filter case

Fig11. Comparison of phreatic lines for best trial and ideal case at OMC for both cases
8. EVALUATION OF EQUATION OF PHREATIC LINE FOR EXPERIMENTAL DATA

MS-EXCEL Statistical Package has been used to design regression models. Subsequently, their statistical feature i.e. R square for the model (coefficient of determination) were checked as shown in Tables 2 and 3 to satisfy the statistical robustness.

8.1. Test Regression Models Statistically

R square for the model represents the coefficient of determination that measures the proportional reduction of total variation in "y" of phreatic line using “x” as independent variable where x, y- Coordinates of any point on the basic parabola ,measured from toe of dam or toe of horizontal filter. In other words, it represents the total variability in y explained by x.

Checking the models in the two tables- Table 2 and 3 indicates that R square is above 0.9 for all of them. This implies that data varies little around the fitted models.

From Table 2, Trial 3+No Filter case which has regression equation- y = 7.3567 ln(x) - 0.7871 ,has R square as 0.94 that is higher than that for the analytical solution suggested by A. Casagrande [2].

Similarly, for the Trial 2 +Filter case which has regression equation- y = 28.465 ln(x) - 78.724 ,has R square as 0.9858 that is higher as compared to the Casagrande [2] analytical solution as seen from Table 3.

Table 2. Models for y coordinate for No filter case

<table>
<thead>
<tr>
<th>No.</th>
<th>Case</th>
<th>Model</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TC+ No Filter</td>
<td>y = 17.5x + 0.768</td>
<td>0.9391</td>
</tr>
<tr>
<td>2</td>
<td>T1+No Filter</td>
<td>y = 7.466ln(x) - 0.4175</td>
<td>0.9303</td>
</tr>
<tr>
<td>3</td>
<td>T2+No Filter</td>
<td>y = 6.7849ln(x) - 0.8776</td>
<td>0.9334</td>
</tr>
<tr>
<td>4</td>
<td>T3+No Filter</td>
<td>y = 7.3567ln(x) - 0.7871</td>
<td>0.9400</td>
</tr>
</tbody>
</table>

Table 3. Models for y coordinate for filter case

<table>
<thead>
<tr>
<th>No.</th>
<th>Case</th>
<th>Model</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TC+Filter</td>
<td>y = 24.2x + 1.46</td>
<td>0.9814</td>
</tr>
<tr>
<td>2</td>
<td>T1+Filter</td>
<td>y = 27.724ln(x) - 77.321</td>
<td>0.9817</td>
</tr>
<tr>
<td>3</td>
<td>T2+Filter</td>
<td>y = 28.465ln(x) - 78.724</td>
<td>0.9858</td>
</tr>
<tr>
<td>4</td>
<td>T3+Filter</td>
<td>y = 24.524ln(x) - 67.174</td>
<td>0.9841</td>
</tr>
</tbody>
</table>

Table 4. Comparison for Casagrande, Experimental & Proposed Solution for No filter case

<table>
<thead>
<tr>
<th>X</th>
<th>Y casagrande</th>
<th>Y experimental</th>
<th>Y proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>9.40</td>
<td>10.50</td>
<td>11.05</td>
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<tr>
<td>10</td>
<td>13.26</td>
<td>14.30</td>
<td>16.15</td>
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<td>15</td>
<td>16.23</td>
<td>17.45</td>
<td>19.14</td>
</tr>
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<td>18.73</td>
<td>19.80</td>
<td>21.25</td>
</tr>
<tr>
<td>25</td>
<td>20.93</td>
<td>22.10</td>
<td>22.89</td>
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<td>30</td>
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<td>24.23</td>
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<td>24.76</td>
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<td>40</td>
<td>26.47</td>
<td>27.77</td>
<td>26.35</td>
</tr>
<tr>
<td>45</td>
<td>28.08</td>
<td>29.50</td>
<td>27.22</td>
</tr>
</tbody>
</table>
Table 5. Comparison for Casagrande, Experimental & Proposed Solution for filter case

<table>
<thead>
<tr>
<th>x [x 10^{-2} m]</th>
<th>y_{casagrande} [x 10^{-2} m]</th>
<th>y_{experimental} [x 10^{-2} m]</th>
<th>y_{proposed} [x 10^{-2} m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>12.11</td>
<td>13.50</td>
<td>12.90</td>
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<tr>
<td>30</td>
<td>17.01</td>
<td>18.90</td>
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<td>22.90</td>
<td>22.48</td>
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<tr>
<td>45</td>
<td>26.33</td>
<td>29.20</td>
<td>29.63</td>
</tr>
</tbody>
</table>

9. RESULTS AND DISCUSSION

The phreatic lines for the trials are plotted on the excel sheet and compared with the analytical solution given by A Casagrande as shown in Fig 8 and Fig 10 for no filter and filter case. For both cases we find that the phreatic line obtained by the experimental method is more accurate than that suggested by Casagrande.

In this section, by making use of regression models for the trials having a greater R square than the Casagrande solution. We obtain y coordinates for analytical, experimental and proposed solution as shown in Table 4 and Table 5. Further, a modified equation for the cases of with & without filter have been proposed in Section 9.1. which can be used in the case of sandy silt soil.

9.1 Modified equation for without filter case

The proposed equation in this case is presented in the logarithmic form as follows:

\[ y = a \ln(x) + b \]  \[ \text{......Eq (10)} \]

Here, a & b are constants and are dependent on the soil type & soil density.

These values are taken as

\[ a = 7.3567; \quad b = -0.7871 \] in the case of sandy silt soil.

Table 4 displays the accuracy of the points of our proposed phreatic line equation compared to that of the experimentally acquired points.

8.2 Modified equation for the filter case

The proposed equation in the filter case is presented in the logarithmic form as follows:

\[ y = c \ln(x) + d \]  \[ \text{......Eq (11)} \]

Here, c & d are constants and are dependent on the soil type & soil density.

These values are taken as

\[ c = 28.465; \quad d = -78.724 \] in the case of sand silt soil.

Table 5 displays the accuracy of the points of our proposed phreatic line equation compared to that of the experimentally acquired points.

9. CONCLUSION

The proposed modified equations for the phreatic line i.e. Eq (10) and Eq (11) are more accurate than the ones suggested by the A. Casagrande\(^2\). The tables 4 and 5 in the above section display the accuracy of the proposed equations compared to that of the experimentally obtained points. The modified phreatic line equations can be used for an earthen dam made of sandy silt material. The line obtained will be more accurate and close to the actual phreatic line. For both cases, the equations obtained are more accurate and we propose that a similar relation might be valid for other materials also. Further research can be carried out for different materials and more accurate equations can be developed for each case. After developing equations, a correlation between these
lines can be made to propose a common equation that can be used for every material. This proposal can be a future scope of work.

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