MATHEMATICAL MODELLING AND ANALYSIS OF THREE DIMENSIONAL DARCY–BRINKMAN (D-B) MODEL IN AN INCLINED RECTANGULAR POROUS BOX

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ABSTRACT

In this paper, numerical studies on three-dimensional natural convection in an inclined differentially heated porous box employing Darcy-Brinkman flow model are presented. The relative effects of inertia and viscous forces on natural convection in porous media are examined. When Brinkman viscous terms are included in the buoyancy term of the momentum equation, no-slip conditions for velocity at the walls are satisfied. The flow and temperature fields become three-dimensional due to Brinkman viscous term. The governing equations for the present studies are obtained by setting $Da 
eq 0$ and $F_c/Pr = 0$ in the general governing equations for D-B flow description. The system is characterized by the non-dimensional parameters: Rayleigh number ($Ra$), vertical aspect ratio ($AR_Y$), horizontal aspect ratio ($AR_Z$), Darcy number ($Da$) and the angle of inclination ($\phi$). Numerical solutions have been obtained employing the SAR scheme for $200 \leq Ra \leq 2000$, $0.2 \leq AR_Y \leq 5$, $0.2 \leq AR_Z \leq 5$, $10^{-5} \leq Da \leq 10^{-2}$ and $-60^\circ \leq \phi \leq 60^\circ$.

Keywords: Brinkman, buoyancy term, no-slip conditions, viscous term etc

1.0 INTRODUCTION

Studies on natural convection heat transfer in porous media employing non-Darcy extensions to describe the fluid flow which include Forchheimer non-linear inertial terms, convective terms and Brinkman viscous terms, have been reported in the literature. Ghanet, al. [1] included the viscous terms due to Brinkman as an extension to the Darcy equation to describe the fluid flow. They concluded that the average Nusselt number shows a maximum when the aspect ratio is around unity. Tong and Subramaniam [2] developed boundary layer solutions to Brinkman extended Darcy flow model based on the modified Oseen technique and the flow field is found to be governed by a new parameter. Tong and Subramaniam found
that a pure Darcy analysis is applicable when the parameter defined in their study is less than $10^{-4}$. Sen [3] considered the influence of Brinkman viscous terms and convective terms in the limit of vanishingly small aspect ratio. Lauriat and Prasad [4] considered the Brinkman extended Darcy flow model along with convective terms for $AR > 1$. The numerical study [4] concluded that the influence of the Brinkman terms is significant for high Ra and Darcy number and the influence of convective terms on the Nusselt number is insignificant. Kwok and Chen [5] observed from their experimental investigation that the flow becomes unstable at a critical Rayleigh number of 66.2. They performed a linear stability analysis to study the effects of including the Brinkman terms and variable viscosity separately. The study concluded that the effect of variable viscosity has a profound influence on the stability of the thermal convection, whereas the effect of no-slip condition at the wall has minimal influence on the stability. Chang and Hsiao [6] investigated the influence of anisotropic permeability and thermal conductivity on natural convection in a vertical cylinder filled with anisotropic porous material. Holst [7] numerically solved transient, three-dimensional natural convection in a porous box. They concluded that, compared to the two-dimensional values, the three-dimensional heat flow under certain instances is higher at high Rayleigh numbers. At low Rayleigh numbers, the situation may be reversed depending on the initial conditions. Horne [8] investigated the tendency of the flow to be two or three-dimensional. Horne concluded that the flow pattern is more complicated than thought previously. In a numerical study, Singh et al., [9] examined the influence of the Brinkman-extended Darcy model on the fluid flow and heat transfer in a confined fluid overlying a porous layer. Vasseur et al., [10] studied analytically and numerically the thermally driven flow in a thin, inclined, rectangular cavity filled with a fluid saturated porous layer. Sharma R P & Sharma R V has worked on modelling and simulation of three-dimensional natural convection in a porous box and concluded that three-dimensional average Nusselt values are lower than two-dimensional values. [11]

2.0 MATHEMATICAL MODELLING
2.1 Governing Equation

![Fig. 1 Physical model and co-ordinate system]
The physical model is shown in fig.1 is a parallelepiped box of length L, width B and height H filled with fluid saturated porous medium.

Governing equations for natural convection in the porous box comprising of conservation of mass, momentum and energy are as follows:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}
\]

\[
\frac{\mu}{K} u + \frac{K'}{K} \rho \frac{\partial v}{\partial y} = -\left( \frac{\partial p}{\partial x} + \rho g \sin \phi \right) + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \tag{2}
\]

\[
\frac{\mu}{K} v + \frac{K'}{K} \rho \frac{\partial v}{\partial y} = -\left( \frac{\partial p}{\partial y} + \rho g \cos \phi \right) + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \tag{3}
\]

\[
\frac{\mu}{K} w + \frac{K'}{K} \rho \frac{\partial w}{\partial y} = -\left( \frac{\partial p}{\partial z} \right) + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \tag{4}
\]

\[
\frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \tag{5}
\]

Governing equations are rendered dimensionless introducing the following non-dimensional variables:

\[
X = \frac{x}{L} \quad Y = \frac{y}{L} \quad Z = \frac{z}{L} \quad U = \frac{u}{\alpha / L} \quad V = \frac{v}{\alpha / L} \quad W = \frac{w}{\alpha / L}
\]

\[
\theta = \frac{T - T_e}{T_s - T_e} \quad P = \frac{\rho}{\mu \alpha / K} \quad \overline{\rho} = \frac{\rho}{\rho_m}
\]

The dimensionless conservation of mass, momentum and energy are derived and non-dimensional parameters Ra, the Rayleigh number, Fc, the Forchheimer number, Pr, the Prandtl number and Da, the Darcy number are defined by -

\[
\overline{\rho} = 1 - \beta \Delta T (\theta-0.5); Ra = \frac{KgL\beta \Delta T}{\nu \alpha} ; Fc = \frac{K'}{L} ; Pr = \frac{\nu}{\alpha} ; Da = \frac{K}{L^2} \tag{7}
\]

**Hydrodynamic Boundary Conditions**

(i) With Brinkman terms

\[
U = V = W = 0 \quad \text{ at } \quad X = 0,1 \quad \text{ for } 0 \leq Y \leq AR_Y \quad \text{ and } \quad 0 \leq Z \leq AR_Z
\]

\[
U = V = W = 0 \quad \text{ at } \quad Y = 0, AR_Y \quad \text{ for } 0 \leq X \leq 1 \quad \text{ and } \quad 0 \leq Z \leq AR_Z
\]

\[
U = V = W = 0 \quad \text{ at } \quad Z = 0, AR_Z \quad \text{ for } 0 \leq X \leq 1 \quad \text{ and } \quad 0 \leq Y \leq AR_Z
\]
Thermal Boundary Conditions

\[ \theta = 0 \quad \text{at} \quad X = 0 \quad \text{for} \quad 0 \leq Y \leq AR_Y \quad \text{and} \quad 0 \leq Z \leq AR_Z \]

\[ \theta = 1 \quad \text{at} \quad X = 1 \quad \text{for} \quad 0 \leq Y \leq AR_Y \quad \text{and} \quad 0 \leq Z \leq AR_Z \quad (9) \]

\[ \frac{\partial \theta}{\partial Y} = 0 \quad \text{at} \quad Y = 0, \quad AR_Y \quad \text{for} \quad 0 \leq X \leq 1 \quad \text{and} \quad 0 \leq Z \leq AR_Z \]

\[ \frac{\partial \theta}{\partial Z} = 0 \quad \text{at} \quad Z = 0, \quad AR_Z \quad \text{for} \quad 0 \leq X \leq 1 \quad \text{and} \quad 0 \leq Y \leq AR_Y \]

Where \( AR_Y \), the vertical aspect ratio and \( AR_Z \), the horizontal aspect ratio are defined as

\[ AR_Y = \frac{H}{L} \quad (10) \]

\[ AR_Z = \frac{B}{L} \quad (11) \]

Boundary conditions on temperature (\( \theta \)) are the same as given by Eq (9). The average Nusselt number based on the characteristic length, \( L \) of the box is defined as,

\[ Nu = \frac{\bar{h}L}{k} \quad (12) \]

The average Nusselt number at \( X = 0 \) and \( X = 1 \) is obtained by using a suitable numerical method.

RESULTS & DISCUSSION

Variation of average Nusselt number with \( AR_Z \) for \( Ra = 1000 \), \( AR_Y = 1.0 \) and \( Da = 10^{-2} \) is shown in Fig. 2. The three-dimensional effects are pronounced for \( AR_Z < 1 \) even for inclined porous enclosure. Variation of average Nusselt number (\( Nu \)) with angle of inclination (\( \phi \)) for \( Ra = 500, 1000, 2000 \) and \( Da \) upto \( 10^{-2} \) are shown in Fig. 3. From this figure, it is evident that in all cases the average Nusselt number (\( Nu \)) is increasing up to an angle (\( \phi = -30^o \)) and then it is decreasing. This is due to the fact that with increasing inclination, the multicellular fluid motion in the porous matrix gets changed gradually to unicellular one. The multicellular circular cells (Fig. 4) and buoyancy forces are equally significant up to the critical angle of inclination (\( \phi_c \)). As these increase slowly up to \( \phi_{critical} \), the average Nusselt number (\( Nu \)) shows marginal increase. After critical angle of inclination (\( \geq \phi_c \)), as the motion turns to unicellular one (Fig. 5) and the average Nusselt number (\( Nu \)) values start declining with increasing inclination angle. Multicellular convection augments the heat transferred through the porous material. The main mechanism of heat exchange is due to the motion of the fluid in direction perpendicular to the isothermal walls whereas in the unicellular mode this occurs only in regions close to the isothermal wall (Fig. 5).

In unicellular circulation situation the temperature difference between fluid and hot surface (i.e. the driving force for heat transfer) is smaller compared to multicellular circulation situation. Further, in multicellular system the fluid mixing is more efficient. As a result, average heat transfer rate during multicellular circulation is more than that during unicellular situation.
**Fig. 2:** Variation of Nu with ARz for Ra=1000, ARy=1.0, Da=10^-2 and φ=0°

**Fig. 3:** Variation of Nu with φ for Da=10^-2, ARy=1.0, and ARz=1.0

**Fig. 4:** Iso-vector-potential ( ) plot for Ra = 1000, ARY = ARZ = 1.0, Da = 10^-2 and φ = -30°.

**Fig. 5:** Iso-vector-potential ( ) lines for Ra = 1000, ARY = ARZ = 1.0, Da = 10^-2 and φ = 0°.
CONCLUSIONS

The flow description is within the framework of Darcy-Brinkman flow model. The non-dimensional parameters required to describe the system are \( Ra, AR_Y, AR_Z, Da \) and \( \phi \). The numerical solutions have been obtained for \( 100 < a < 2000, 10^{-5} < Da < 10^{-2}, 0.2 < AR_Y, AR_Z < 5 \) and \( -60^\circ < \phi < 60^\circ \). Due to inclusion of Brinkman viscous term, which satisfies the no-slip boundary condition, the flow becomes three-dimensional particularly for low horizontal aspect ratio, \( AR_Z < 1 \). The three-dimensional average Nusselt number values are lower than that of two-dimensional average Nusselt number values for \( AR_Z < 1 \). For \( Da < 10^{-5} \), the average Nusselt number values are close that of Darcy’s values. The critical angle of inclination for Darcy – Brinkman flow description is \(-30^\circ \) irrespective of Darcy number and Rayleigh number.

REFERENCES


