MAGNETIC FIELD EFFECT ON MIXED CONVECTION FLOW IN A NANOFLOUID UNDER CONVECTIVE BOUNDARY CONDITION

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ABSTRACT

An analysis is carried out to investigate the influence of the prominent magnetic effect on mixed convection heat and mass transfer in the boundary layer region of a semi-infinite vertical flat plate in a nanofluid under the convective boundary conditions. The transformed boundary layer, ordinary differential equations are solved numerically using Runge-Kutta Fourth order method. A wide range of parameter values is chosen to bring out the effect of Magnetic field parameter on the mixed convection process with the convective boundary condition. The effect of mixed convection, Magnetic field and Biot parameters on the flow, heat and mass transfer coefficients is analyzed. The numerical results obtained for the velocity, temperature and volume fraction profiles are presented graphically and discussed.

Keywords: Mixed Convection, Nanofluid, Magnetic Effect, Convective Boundary Condition, Numerical Solution.

1. INTRODUCTION

A Nanofluid is a fluid containing nanometer sized particles, called nanoparticles. These fluids are engineered colloidal suspensions of nanoparticles in a base fluid. The nanoparticles used in nanofluids are typically made of metals, oxides, carbides, or carbon nanotubes. Common base fluids include water, ethylene glycol and oil. Nanofluids have novel properties that make them potentially useful in many applications in heat transfer, including microelectronics, fuel cells pharmaceutical processes, and hybrid-powered engines, engine cooling/vehicle thermal management, domestic refrigerator, chiller, heat exchanger, in grinding, machining and in boiler flue gas temperature
reduction. They exhibit enhanced thermal conductivity and the convective heat transfer coefficient compared to the base fluid. Knowledge of the rheological behavior of nanofluids is found to be very critical deciding their suitability for convective heat transfer applications.

Merkin [1] investigated the mixed convection boundary layer flow on a semi-infinite vertical flat plate when the buoyancy forces aid, and oppose the development of the boundary layer. Ding et al. [2] Nanofluid are also important for the production of nanostructured materials, for the engineering of complex fluids, as well as for cleaning oil from surfaces due to their excellent wetting and spreading behavior. After the pioneering work by Sakiadis [3], a large amount of literature is available on boundary layer flow of Newtonian and non-Newtonian fluids over linear and nonlinear stretching surface. A detailed review of mixed convective heat and mass transfer can be found in the book by Bejan [4]. Recently, Subhashini et al. [5] discussed the simultaneous effects of thermal and concentration diffusion on a mixed convection boundary layer flow over a permeable surface under convective surface boundary condition. The general topic of heat transfer in nanofluids has been surveyed in a review article by Das and Choi [6] and a book by Das et al. [7]. Buongiorno [8] noted that the nanoparticle absolute velocity can be viewed as the sum of the base fluid velocity and a relative velocity. He considered in turn seven slip mechanism: inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage and gravity setting. He concluded that in the absence of turbulent effects, it is the Brownian diffusion and the thermophoresis that will be important. Buongiorno proceeded to write down conservation equations based on these two effects.

The problem of natural convection in a regular fluid past a vertical plate is a classical problem first studied theoretically by E. Pohlhausen in contribution to an experimental study by Schmidt and Beckmann [9]. Unfortunately the boundary layer scaling used by early researchers and textbook authors did not properly incorporate the papers by Kuiken [10, 11]. An extension to the case of heat and mass transfer was made by Khair and Bejan [12]. Makinde, and Aziz [13] considered to study the effect of a convective boundary condition on boundary layer flow, heat and mass transfer and nanoparticle fraction over a stretching surface in a nanofluid. Kuznetsov and Nield [14] studied, the natural convection boundary layer flow, heat and mass transfer of nanofluid past a vertical plate. The transformed non-linear ordinary differential equations governing the flow are solved numerically by the Runge-Kutta Fourth order method.

These authors discussed about the convective heat transport in nanofluids. They studied natural convective flow of nanofluids over a vertical plate and their similarity analysis is identical with four parameters governing the transport process, namely a Lewis number $Le$, a Buoyancy-Number $Gr$, a Brownian motion number $Nb$ and a thermophoresis number $Nt$, A.V. Kuznetsov, D. A. Nield [2010].

Ali et al. [2011] included the idea of induced magnetic field to the problem of Ashraf [2011] and analyzed MHD stagnation-point flow and heat transfer towards stretching sheet with induced magnetic field. The application of Magnetic Field to convection process will play as a control factor in the convection by damping both the flow and temperature oscillation material manufacturing fields.

Motivated by the above referenced work, and the vast possible industrial applications, it is paramount interest to consider the effect of magnetic parameter on mixed convective flow along a vertical plate in a nanofluid under the convective boundary condition. A similarity solution is presented. This solution is depends on Prandtl number $Pr$, a Lewis number $Le$, a Brownian motion number $Nb$, a nanoparticle buoyancy ratio $Nr$, thermophoresis number $Nt$, Biot number $Bi$, mixed convection number $\lambda$ and a magnetic number $M$. The dependency of the Skin friction coefficient, Nusselt number, nanoparticle Sherwood number and regular Sherwood number on these six parameters is numerically investigated. Consideration of the nanofluid and the convective boundary conditions enhanced the number of non-dimensional parameters considerably. The effect
of mixed convection, Magnetic and Biot parameters on the physical quantities of the flow, heat and mass transfer coefficients are analyzed. To examine the convergence of the numerical code written to solve the present problem, we compare the present result for the clear fluid mixed convection results with previously published works with convective boundary conditions and the comparison shows that the results are in very good agreement.

2. MATHEMATICAL FORMULATION

Choose the coordinate system such that the x-axis is along the vertical plate and y-axis normal to the plate. The physical model and coordinate system are shown in fig. 1. Consider the steady laminar two-dimensional mixed convection heat and mass transfer along a flat vertical surface embedded in a nanofluid having \( T_\infty \) and \( \phi_\infty \) as the temperature and nanoparticle volume fraction respectively in the ambient medium. Also assume that a free stream with uniform velocity \( u_\infty \) goes past the flat plate. The plate is either heated or cooled from left by convection from a fluid of temperature \( T_f \) with \( T_f > T_\infty \) corresponding to a heated surface (assisting flow) and \( T_f > T_\infty \) corresponding to a cooled surface (opposing flow) respectively. On the wall the nanoparticle volume fraction is taken to be constant and is given by \( \phi_w \) respectively.

By employing Oberbeck-Boussinesq approximation, making use of the standard boundary layer approximation and eliminating pressure, the, the governing equations for the nanofluid are given by

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\rho_f u \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \frac{\partial^2 u}{\partial y^2} + g \rho_f \left( 1 - \phi_\infty \right) \left[ \beta_T (T - T_\infty) \right]
\]

Fig: 1
\[-(\rho_p - \rho_{f\infty})g(\phi - \phi_w) - \frac{\sigma B_s^2 u}{\rho} \]  

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[ D_B \frac{\partial \phi}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_{\infty}} \left( \frac{\partial T}{\partial y} \right)^2 \right] \]  

\[ u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = D_B \frac{\partial^2 \phi}{\partial y^2} + \frac{D_T}{T_{\infty}} \frac{\partial^2 T}{\partial y^2} \]  

where \( u \) and \( v \) are the velocity components along the \( x \) and \( y \) axes, respectively, \( T \) is the temperature, \( \phi \) is the nanoparticle volume fraction, \( g \) is the gravitational acceleration, \( \alpha_m = k/(\rho c)_f \) is the thermal diffusivity of the fluid, \( \nu = \mu/\rho_{f\infty} \) is the kinematic viscosity coefficient and \( \tau = (\rho c)_p/(\rho c)_f \). Further, \( \rho_{f\infty} \) is the density of the base fluid and \( \rho, \mu, k, \beta_r \) and \( \beta_v \) are the density, viscosity, thermal conductivity, volumetric thermal expansion coefficient and volumetric solutal expansion coefficient of the nanofluid, while is \( \rho_p \) the density of the nanoparticles, \( (\rho c)_f \) is the heat capacity of the fluid and \( (\rho c)_p \) is the effective heat capacity of the nanoparticle material. The coefficients that appear in Eqs. (3) and (4) are the Brownian diffusion coefficient \( D_B \), the thermophoretic diffusion coefficient \( D_T \). For, details of the derivation of equations (1) - (4), one can refer the papers by Buongiorno [8] and Nield and Kuznetsov ([14, 15]). 

The boundary conditions are

\[ u = 0, v = 0, -k \frac{\partial T}{\partial y} = h_f (T_{\infty} - T), \phi = \phi_w \text{ at } y=0 \]  

\[ u \rightarrow u_{\infty}, \quad T \rightarrow T_{\infty}, \quad \phi \rightarrow \phi_{\infty} \text{ as } y \rightarrow \infty \]  

here, \( h_f \) is the convective heat transfer coefficient and the subscripts \( w \) and \( \infty \) indicate the conditions at the surface and at the outer edge of the boundary layer respectively.

In view of the continuity equation (1), we introduce the stream function \( \psi \) by

\[ u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \]  

Substituting (6) in eqs. (2)-(4) and then using the following non-dimensional transformation

\[ \eta = \sqrt{u_{\infty}/v} y, \quad \psi = \sqrt{u_{\infty}v f} (\eta), \]  

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with the local Reynolds’s number \( \text{Re}_x = \frac{u_m x}{\nu} \), we get the transformed system of ordinary differential equations as

\[
f'' + ff' - f^3 + \lambda [\theta - N_r y] - M f' = 0
\]

\[
\frac{1}{Pr} \theta'' + f \theta' + Nb \theta' y' + N_t \theta^2 = 0
\]

\[
y' + Lef y' +\frac{N_t}{Nb} \theta' = 0
\]

where the primes indicate differentiation with respect to \( \eta \). In usual notations, \( Pr = \frac{\nu}{\alpha_m} \) is the Prandtl number and \( Le = \frac{\nu}{D_b} \) is the Lewis number. \( Nr = \frac{(\rho_p - \rho_\infty)(\phi_w - \phi_\infty)}{\rho_\infty \beta_f (T_f - T_\infty)(1 - \phi_\infty)} \) is the nanofluid buoyancy ratio. Further, \( Nb = \frac{(\rho c)_p D_b (\phi_w - \phi_\infty)}{(\rho c)_f \nu} \) is the Brownian motion parameter, \( Nt = \frac{(\rho c)_p D_f (T_f - T_\infty)}{(\rho c)_f \nu T_\infty} \) is the thermophoresis parameter, \( Gr_x = g (1 - \phi_\infty) \beta_f (T_f - T_\infty) \) is the local Grashof number and \( \lambda = \frac{Gr_x}{u_m^2 x} \) the mixed convection parameter. Finally \( M = \frac{\sigma B_0^2}{\rho_p f \alpha u_m} \) is the magnetic number.

Boundary conditions (5) in terms of \( f, \theta, \gamma \) become

\[
\eta = 0: f(0) = 0, f'(0) = 0, \theta'(0) = -Bi[1 - \theta(0)], \gamma(0) = 1
\]

\[
\eta \to \infty: f'(\infty) \to 1, \theta(\infty) \to 0, \gamma(\infty) \to 0
\]

where \( Bi = \frac{c}{k} \sqrt{\frac{2\nu}{u_m}} \) is the Biot number. This boundary conditions will be free from the local variable \( x \) when we choose \( h_f = cx^{1/2} \). The Biot number \( Bi \) is a ratio of the internal thermal resistance of the plate to the boundary layer thermal resistance of the hot fluid at the bottom of the surface.

It is important to note that this boundary value problem reduces to the classical problem of flow and heat and mass transfer due to the Blasius problem of flow when \( Nb \) and \( Nt \) are zero. Most nanofluids examined to date, have large values for the Lewis number \( Le > 1 \). For water nanofluids at room temperature with nanoparticles of 1-100nm diameters, the Brownian diffusion coefficient \( D_b \)
ranges from $4 \times 10^{-4}$ to $4 \times 10^{-12}$ $m^{-2}/s$. Furthermore, the ratio of the Brownian diffusivity coefficient to the thermophoresis coefficient for particles with diameters of 1-100nm can be varied in the ranges of 2-0.02 for alumina, and from 2 to 20 for copper nanoparticles, Hence, the variation of the non-dimensional parameters of nanofluids in the present study is considered in the mentioned range.

3. SKIN FRICTION, HEAT AND MASS TRANSFER COEFFICIENTS

The primary objective of this study is to estimate the skin friction coefficient $C_f$, Nusselt number $Nu$ and the nanoparticle Sherwood number $NSh$. These parameters characterize the surface drag, the wall heat and nanoparticles mass transfer respectively. The shearing stress, local heat and local nanoparticle mass from the vertical plate can be obtained from

$$\tau_w = \mu \frac{\partial u}{\partial y} \bigg|_{y=0}, q_w = -k \frac{\partial T}{\partial y} \bigg|_{y=0} \quad \text{and} \quad q_n = -D_B \frac{\partial \phi}{\partial y} \bigg|_{y=0}$$

(12)

The non-dimensional shear stress $C_f = \frac{\tau_w}{\rho_f \mu u_{\infty}^2}$, the Nusselt number $Nu = \frac{q_w x}{k(T_f-T_{\infty})}$ and the nanoparticle Sherwood number $NSh = \frac{q_n x}{D_B (\phi_w - \phi_\infty)}$ are given by

$$C_f (2 Re_x)^{1/2} = \phi^\prime\prime(0),$$

$$Nu_x (Re_x/2)^{-1/2} = -\theta^\prime(0),$$

$$NSh_x (Re_x/2)^{-1/2} = -\gamma^\prime(0)$$

Effect of the various parameters involved in the investigation on these coefficients is discussed in the following section.

4. RESULT AND DISCUSSION

The resulting transport Eqns. 8-11 are non-linear, coupled, ordinary differential equations, which possess no closed-form solution. Therefore, these are solved numerically subject to the boundary conditions given by Eqn. 12. The computational domain in the $\eta$ -direction was made up of 196 non-uniform grid points. It is expected that most changes in the dependent variable occur in the region close to the plate where viscous effect dominate. However, small changes in the dependent variables are expected far away from the plate surface. For these reasons, variable step sizes in the $\eta$ -direction are employed. The initial step size $\Delta \eta_i$ and growth factor $K^*$ employed such that $\Delta \eta_{i+1} = K^* \Delta \eta_i$ (where the subscript $i$ indicates the grid location) were $10^{-3}$ and 1.0375 respectively. These values were found (by performing many numerical experimentations) to give accurate and grid-independent solutions. The solution convergence criterion employed in the present work was based on the difference between the values of the dependent variables at the current and
the previous iteration. When this difference reached $10^{-5}$, the solution was assumed converged and the iteration process was terminated.

To have a better understanding of the flow characteristics, numerical results for the velocity, temperature, volume fraction are calculated for different sets of values of the parameters $M, Le, \lambda, Nb, Nt, Nr, Bi$ also, the effect of these parameters on skin fraction, non-dimensional heat and nanoparticle mass coefficients is discussed.

Fig. 2(a)-(c) show the non-dimensional velocity, temperature and volume fraction profiles for various values of magnetic along with varying values of the Lewis number $Le$ for a given $Nt = 0.2, Nb = 0.2, Nr = 0.2, Pr = 1.0, \lambda = 0.4, Bi = 0.01$. The magnetic number $M$ accounts for the additional mass diffusion due to the temperature gradients. It is noticed from Fig. 2 that an increase in the magnetic number resulted in an increase in the velocity while decrease in temperature and nanoparticle volume fraction is noted within the boundary layer. The present analysis shows that the flow field is appreciably influenced by magnetic parameter. It is clear from these figures that an increase in Lewis number $Le$ increased the momentum boundary layer thickness, while reduction in the thermal and nanoparticle volume fraction boundary layer thickness is noted.

The non-dimensional velocity for different values of Biot number $Bi$ with fixed values of the other parameters is plotted in Fig. 3(a). Increased convective heating associated with an increase in $Bi$ is seen to thicken the momentum boundary layer. A reverse trend is seen in the case of nanofluid buoyancy ratio $Nt$. Given that convective heating increases with Biot number simulates the isothermal surface, which is clearly seen from Fig. 3 (b), where, In fact, a high Biot number indicates higher internal thermal resistance of the plate than the boundary layer thermal resistance. As a result, an increase in the Biot number leads to increase of fluid temperature efficiently, these figures confirm this fact also. As the parameter values $Bi$ and $Nt$ increases, the volume fraction increased for the fixed values of the other parameters seen from Fig. 3(c).

Fig. 4 presents the effect of the Brownian motion $Nb$ and thermophoresis $Nt$ parameters on the velocity, temperature and volume fraction. It is observed that the momentum boundary layer thickness increases with the increase in values of $Nb$ but it decreases with increasing values of $Nt$. The nanoparticle volume fraction decreased with increase in $Nb$ and it increased with increasing values of $Nt$. It is also noticed that the nanoparticle volume fraction increased with an increase in $Nb$ in the case of forced convection flow. $Nt > 0$ indicates a cold surface while negative $Nt < 0$ corresponds a hot surface, in case of hot surface, thermophoresis tends to blow the nanoparticle volume fraction away from the surface since a hot surface repels the sub-micron sized particles from it, thereby forming a relative particle-free layer near the surface.

Variation of non-dimensional velocity and temperature against the similarity variable $\eta$, is shown respectively in Fig. 5, for a few set of values of $\lambda$ and $Pr$ with fixed values of other parameters. As the parameter $\lambda$ and $Pr$ increase, the velocity increased whereas temperature of nanofluid decreased.
Fig. 2(a): Effect of M and Le on velocity $f'(\eta)$

Fig. 2(b): Effect of M and Le on temperature $\theta(\eta)$
Fig. 2(c): Effect of $M$ and $Le$ on volume fraction $\gamma(\eta)$

Fig. 3(a): Effect of $Bi$ and $Nr$ on the velocity $f'(\eta)$
Fig. 3(b): Effect of $Bi$ and $Nr$ on the temperature $\theta(\eta)$

Fig. 3(c): Effect of $Bi$ and $Nr$ on the volume fraction $\gamma(\eta)$
**Fig. 4(a):** Effect of $N_b$ and $N_t$ on the volume velocity $f'(\eta)$

**Fig. 4(b):** Effect of $N_b$ and $N_t$ on the temperature $\theta(\eta)$
Fig. 4(c): Effect of $Nb$ and $Nt$ on the volume fraction $\gamma(\eta)$

Fig. 5(a): Effect of $\lambda$ and $Pr$ on Velocity $f'(\eta)$
5. CONCLUSION

In this paper, the effect of Magnetic parameter on mixed convection flow along a vertical plate in a nanofluid is analyzed under the convective boundary conditions. Using the similarity variables, the governing equations are transformed into a set of non-dimensional parabolic equations. These equations are solved numerically using the Runge-Kutta Fourth order method. The numerical results are obtained for a wide range of values of the physical parameters. To ascertain the convergence of the numerical method adopted. The nanoparticle is considered in the analysis. The skin friction, heat and nanoparticle mass coefficients are obtained for a physically realistic values of governing parameters. The results are analyzed thoroughly for different values of $M$, $Bi$, and $\lambda$ on the flow, thermal and solutal field. The major conclusion is that the magnetic effect enhanced the skin friction, heat and nanoparticle mass in the medium.

NOMENCLATURE

$Bi$ Biot number
$c$ constant
$D_b$ Brownian diffusion coefficient
$D_r$ Thermophoretic diffusion coefficient
$f$ Dimensionless steam function
$g$ Gravitational acceleration
$Gr$ Local Grashof number
$h_f$  Convective heat transfer coefficient

$\tau$  Ratio between the effective heat capacity of the nano-particle material and heat capacity of the fluid

$k$  Thermal conductivity of the nanofluid

$Le$  Lewis number

$Nb$  Brownian motion parameter

$Nr$  Nanoparticles buoyancy ratio

$Nt$  Thermophoresis parameter

$Nu_x$  Local Nusselt number

$Pr$  Prandtl number

$q_{tn}$  Nanoparticle mass flux at the wall

$q_w$  Heat flux at the wall

$Re_x$  Local Reynolds number

$NSh_t$  Local nanoparticle Sherwood number

$M$  Magnetic number

$T$  Temperature

$T_f$  Temperature of the hot fluid

$T_\infty$  Ambient temperature

$u_\infty$  Characteristic velocity

$u, v$  Velocity components in x and y direction

$x, y$  Coordinates along and normal to the plate

$\alpha_m$  Thermal diffusivity

$\eta$  Similarity variable

$\gamma$  Dimensionless volume fraction

$\lambda$  Mixed convection parameter

$\theta$  Dimensionless temperature

$\phi$  Nanoparticle volume fraction

$\phi_w$  Nanoparticle volume fraction at the wall

$\phi_m$  Nanoparticle volume fraction at large values of $y$(ambient)

$\mu$  Dynamic viscosity of the base fluid

$v$  Kinematic viscosity

$\rho$  Density of the fluid

$\rho_f$  Density of the base fluid

$\rho_p$  Nanoparticle mass density

$(\rho c)_f$  Heat capacity of the fluid

$(\rho c)_p$  Effective heat capacity of the nanoparticle material

$\tau_w$  Wall shear stress

$\psi$  Stream function
Subscripts

\( W \) Wall condition
\( \infty \) Ambient condition
\( C \) Concentration
\( T \) Temperature

REFERENCES


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