Multiple Cyclic passage approach has been used to develop Barkhausen criteria in Laser Induced Microwave Oscillator (LIMO) and having been applied to study the transient and steady state behaviour of LIMO for first time. A novel structure of LIMO has been introduced for further improving its spectral purity. Equation for Microwave oscillation Envelope in case of single loop Optoelectronic Oscillator is determined from Barkhausen criteria. A detailed study is performed on the stability of oscillator.

1. INTRODUCTION

No body has ever gone through the subject of oscillation without paying tribute to Galileo (1564-1642), who noticed that a swinging lamp in the cathedral of Pisa had a constant time of swing, and so the pendulum clock was born and the physics of oscillations was conceived. Oscillators are the devices which convert energy from a continuous source to a periodically varying signal. There are many forms of oscillators to generate time varying waveforms such as mechanical (such as pendulum), electromagnetic (such as LC and cavity based) and atomic (like laser and maser). Lack of adequate frequency stability and spectral purity of the valve and solid state oscillators is detrimental to the applications where very high stability and spectral purity are required. These limitations of electronic oscillators are due to ohmic and dispersive losses in the various circuit elements of the oscillators. For over fifty years attempts have been made to overcome this barrier by using high Q tank circuits. This has resulted in the development of electromechanical (quartz resonator), electromagnetic (dielectric resonator) and electro-acoustic varieties of oscillators.

One of the effective ways to increase the Q value is to increase the delay of the feedback loop \[ Q = 2\pi f_0 \tau_d \]. The only way to increase this delay to large value is to use optical method. At present, optical generation of RF signal is usually done with help of beating of two lasers or the techniques of sideband locking of two commercial laser diodes with the help of injection locking or optical phase locking and then
heterodyning. For high spectral purity over large frequency range a new type of oscillator called Optoelectronic Oscillator (OEO) (Figure 1) based on the use of photonic delay line was invented in 1994 by Yao and Maleki [1,2,3] of the NASA Jet Propulsion Laboratory.

In this paper we use the cyclic passage theory in OEO to calculate the oscillation amplitude and Barkhausen criteria. We also show the oscillation build up process in case of single loop OEO. For this oscillator we also check the stability criteria.

2. HOW AN OEO WORKS

The basic OEO configuration is shown in Figure-1. An OEO is fundamentally similar to an Armstrong / van der Pol oscillator (Figure -2) [4,5,11]. Photons functionally replace electrons and an electro optic modulator replacing the function of the grid and a photo detector replacing the function of the anode. The energy storage function of the tuned circuit is replaced with a long optical fibre. The operation of the Optoelectronic Oscillator is based on converting the continuous light wave energy from the laser to microwave signal and hence the name optoelectronic oscillator (OEO). In contradistinction to a vacuum tube oscillator an OEO has a very high stability and a narrow spectral width. The oscillation loop consists of the following: (1) A wideband integrated optics LiNbO$_3$ Mach-Zehender modulator seeded by a continuous wave semiconductor laser with optical power $P$, (2) A thermally controlled optical fibre (say 4 km) performs a delay of 20 microseconds on the RF signal carried by the optical signal and (3) RF amplifier and RF narrowband filter. Once the feedback loop has the loop gain larger than unity, the OEO generates equally spaced peaks or modes (Figure-3) similar to that of a Fabry-Perot resonator, provided there is no electrical Bandpass RF filter in the loop. The mode spacing is determined by $c/nL$, where $c$ is the light speed, $n$ is the fibre refractive index, and $L$ is the loop length. It is to be noted that larger the loop length ($L$) the lower is mode spacing.

The power-spectral density of the each mode is found to be [7,8,10]

$$S_{nrf}(f) = \frac{\rho_{n}G_{n}^{2}/P_{inc}}{(\delta/2\tau)^{2}+(2\pi f \cdot \tau)^{2}}$$

(1)
Where \( f \) is the frequency offset from the oscillation frequency, \( \tau \) is the loop delay time, \( \rho_n \) is the total noise density which is sum of the detector noise density including thermal and shot noise and the laser’s relative intensity noise density. The quality factor \( Q \) of the oscillator is

\[
Q = \frac{f_{osc}}{\Delta f_{FWHM}} = Q_D \frac{\tau}{\rho_n G_A^2 / P_{osc}} \tag{2}
\]

Where \( Q_D = 2\pi f_{osc} \) \( \tau \) is the quality factor of the optical delay line and it has larger value with longer delay line. For example, 1-km optical delay line length can produce \( Q_D \approx 10^6 \) at 30 GHz.

3. THEORETICAL DESCRIPTION

We begin with the assumption that despite the presence of the strong nonlinearity due to Mach-Zehender (MZ) modulator within the loop there is an existence of stationary amplitude of the microwave oscillation of the form:

\[
v_m(t) = V \sin(\omega t + \beta) \tag{3}
\]

Here \( V \) is the amplitude of oscillation with a frequency \( \omega \) and initial phase of \( \beta \). As a result of this the output power of the MZ modulator [6, 7, 8] can be expressed as

\[
P(t) = \frac{1}{2} \alpha P_0 \left[ 1 - \eta \sin \left( \frac{V_m(t) + V_b}{V_f} \right) \right] \tag{4}
\]

where \( \alpha \) is the fraction of insertion loss of the modulator, \( V_f \) is the half-wave voltage, \( V_b \) is the bias voltage, \( P_0 \) is the input optical power, \( V_m(t) \) is the input RF voltage to the MZ modulator and \( \eta \) determines the extinction ratio of the modulator. Therefore,
the output voltage of the photo detector when the output of the MZ modulator shines on it

\[ V_0(t) = \rho R P(t - \tau) \]  

Here \( \rho \) is the sensitivity of the photo detector and \( R \) is the output impedance of the photo detector. Using the above relations it not difficult to show that

\[ V_0(t) = V_{ph} \left[ 1 - \eta \sin \left( \frac{\pi V_{ph}}{V} \right) \right] \left[ J_0 \left( \frac{\pi V}{V} \right) + \sum_{m=1}^{\infty} J_{2m} \left( \frac{\pi V}{V} \right) \cos \left( 2m \omega (t - \tau) + 2m \beta \right) \right] \]

\[ -2 \eta \cos \left( \frac{\pi V_{ph}}{V} \right) \sum_{m=0}^{\infty} J_{2m+1} \left( \frac{\pi V}{V} \right) \sin \left( (2m+1) \omega (t - \tau) + (2m+1) \beta \right) \]

Here \( V_{ph} = \frac{\rho R P_0}{2} \)

It is known that highest component of the spectrum attenuates out the smaller components and only the highest component is sustained. Moreover, with the growth of oscillation amplitude, the effective Q value of the tuned RF tuned circuit becomes narrower and automatically rejects the other modes.

Thus the output of the MZ modulator is seen to be

\[ V_0(t) = -2 \eta V_{ph} \cos \left( \frac{\pi V_{ph}}{V} \right) J_1 \left( \frac{\pi V(t - \tau)}{V} \right) \sin \left( \omega (t - \tau) + \beta \right) \]

\[ = N(V(t - \tau)) \sin(\omega (t - \tau)) ; \text{Take } \beta = 0 \]

\[ = \frac{N(V(t - \tau))}{V} \exp(-\tau) v_{in}(t) \]

Here  

\[ N(V(t - \tau)) = -2 \eta V_{ph} \cos \left( \frac{\pi V_{ph}}{V} \right) J_1 \left( \frac{\pi V(t - \tau)}{V} \right) \]  

(6)

4. BARKHAUSEN CRITERION THRU MULTIPLE CYCLIC PASSAGE APPROACH

Referring to the Figure- 4 and remember that \( v_{in}(t) \) is the input to the modulating grid of the MZ modulator. Also that transfer function of the RF tuned amplifier is denoted as \( G(s) \). On the first passage of the input signal \( v_{in}(t) \) to the electrical output port \( v_{01}(t) \) can be written as

\[ v_{01}(t) = \left[ \frac{N(V(t - \tau))}{V} G(s) e^{-\tau t} \right] v_{in}(t) = B(s) v_{in}(t) \]

The RF input to MZ modulator after first passage is therefore

\[ v_{in1}(t) = B(s) \cdot G v_{in}(t) \]

Here \( G \) is the gain of the RF amplifier. In this way it is easy to show that the electrical output after the second passage is given by

\[ v_{02}(t) = \left[ B(s) \right]^2 G v_{in}(t) \]

Similarly after third passage the electrical output is
\[ v_{io}(t) = \left[ \beta(s) \right]^3 G^2 v_{in}(t) \]

Therefore, after a large number of cyclic passages through the loop, the net electrical output can be written as

\[ v_0(t) = \beta(s) v_{in}(t) + \left[ \beta(s) \right]^2 G v_{in}(t) + \left[ \beta(s) \right]^3 G^2 v_{in}(t) + \cdots \]

\[ = \frac{\beta(s)}{1 - \beta(s)G} v_{in}(t) \]

Therefore, Barkhausen criterion for oscillation \[13\] is expressed as

\[ \beta(s) \cdot G = 1 \quad (8) \]

It is to be noted that \( \beta(s) \) is a function of frequency as well as the instantaneous amplitude of oscillations. From the above relation one can find the steady state amplitude of oscillation by using the final value theorem \((s = 0)\). Thus the steady state oscillator amplitude \( V_{osc} \) is to be obtained from the following relation

\[ \frac{N(V_{osc})}{V_{osc}} = \frac{1}{G} \]

That is

\[-2\eta V_{ph} \cos \left( \frac{\pi V_b}{V_x} \right) J_1 \left( \frac{\pi V_{osc}}{V_x} \right) = \frac{V_{osc}}{G} \]

From the above relation for condition of oscillation the following condition is to be satisfied,

\[ \frac{\pi}{2} < \frac{\pi V_b}{V_x} < 3\pi \text{, that is } \frac{1}{2} < \frac{V_b}{V_x} < \frac{3}{2} \]

Let us assume \( V_b = V_x \); \( \eta = 1 \) then

\[ 2J_1 \left( \frac{\pi V_{osc}}{V_x} \right) = \frac{V_{osc}}{V_x} \frac{V_x}{V_{ph}} \frac{1}{G} \]

This expression can be approximately written as

\[ 1 - \frac{1}{2} \left( \frac{\pi V_{osc}}{2V_x} \right)^2 + \frac{1}{12} \left( \frac{\pi V_{osc}}{2V_x} \right)^4 = \frac{1}{\pi} \frac{V_x}{V_{ph}} \frac{1}{\pi G} \]

If one neglects the higher order term and taking \( V_{ph} = \frac{V_x}{\pi} \) for oscillation threshold \([1-3]\) then

\[ V_{osc} = \frac{2\sqrt{2V_x}}{\pi} \sqrt{1 - \frac{V_x}{V_{ph} \pi G}} = \frac{2\sqrt{2V_x}}{\pi} \sqrt{1 - \frac{1}{G}} \quad (9) \]

If we assume \( V_{ph} = \pi \) then \( V_{osc} = 2\sqrt{2} \sqrt{1 - \frac{1}{G}} \quad (9a) \)
5. EQUATION FOR MICROWAVE OSCILLATION ENVELOPE: BARKHAUSEN CRITERION

Barkhausen criterion for oscillation is expressed as

\[ \beta(s) \cdot G = 1 \]  \hspace{1cm} (8)

Where \( \beta(s) = \left[ \frac{N(V(t-\tau))}{V} \right] G(s) \cdot e^{-\tau s} \) and \( N(V(t-\tau)) \) is given by Eqn. (6).

Now if we assume the gain of the RF 1st order tuned amplifier in the form of

\[ G(s) = \frac{1}{\left( \frac{s}{\omega_b} + \omega_b \right)Q + 1} \]  \hspace{1cm} (8a)

The complex frequency \( s \) can be defined as

\[ s = j\omega = \frac{1}{v(t)} \frac{dv}{dt} = \frac{1}{V} \frac{dV}{dt} + j\omega_1 + j\frac{d\theta}{dt} \]  \hspace{1cm} (10)

We can also write \( e^{-\tau s} \) as

\[ e^{-\tau s} = e^{-\left( \frac{1}{V} \frac{dV}{dt} + j\omega_1 + j\frac{d\theta}{dt} \right)} \]  \hspace{1cm} (11)

Using equation (6) & (8-11) and equating real part we can write the time varying amplitude equation with delay as

\[ \left[ \frac{2Q}{\omega_b} + G\tau N(V) \right] \frac{dV}{dt} + \left[ 1 - G \left( \frac{N(V(t-\tau))}{V} \right) \right] V = 0 \quad (12) \]

\[ \left[ \frac{1}{G} + \frac{\omega_1}{2Q} \tau N(V) \right] \frac{dV}{dt} = \frac{\omega_1}{2Q} \left( \frac{N(V(t-\tau))}{V} - V \frac{1}{G} \right) \]

Equating imaginary part we can write the time varying phase equation as

\[ \left[ \frac{Q}{\omega_b} + \frac{1}{\omega_b^2} + G\tau N(V) \right] \frac{d\theta}{dt} + Q \left( \frac{\omega_1}{\omega_b} \frac{dV}{dt} - \omega_1 \right) = 0 \quad (13) \]

Simplifying Equation (12) we can write

\[ \frac{1}{G} \frac{dV}{dt} = \frac{\omega_1}{2Q} \left( \frac{N(V(t-\tau))}{V} - V \frac{1}{G} \right) \]

\[ \frac{dV}{dt} = \frac{\omega_1}{2Q} \left[ G2j_1(V(t-\tau)) - V \right] \quad (12a) \]

From the Figure 6 we can see that the Oscillator take more time for attaining steady state value for finite time delay. For greater delay time required more.

Figure 7
If we put $\frac{d\theta}{dt} = 0$ in equation (13) then we get the frequency of oscillation of the OEO as

$$\omega = \omega_0 \quad (14)$$

When time delay is small then the system converges toward its stable fixed point, but only after some oscillatory transients. When the time delay is further increased, the amplitude is modulated, and the modulation period is twice the delay time. (Shown in Figure-8)

6. **STABILITY TEST**

We can check the stability of an oscillator system by adding some small perturbation in the steady state value of the oscillation amplitude. If the perturbation dies with time and the oscillator come back to its steady state value then the system is stable. Now we check the stability of the optoelectronic oscillator with specific delay. Now we consider the transient amplitude relation of the OEO given by equation (12a) (which is nothing but a delay deferential equation)

$$\frac{dV}{dt} = \frac{\omega_0}{2Q}[G2J \{V(t - \tau) - V\}]$$

If we consider the normalized time then the above equation becomes

$$\frac{dV}{dt} = [-V + 2GJ \{V(t - \tau)\}] \quad (15)$$

Now we add some perturbation on the system

i.e. $V(t) = V_{osc} + x(t)$

$$V(t - \tau) = V_{osc} + x(t - \tau) \quad (16)$$

Then from equation (15), (16) we can write

$$\frac{dx(t)}{dt} = [-x(t) + \beta x(t - \tau)] \quad (17)$$

Where $\beta = 2GJ \{V_{osc}\}$

Now we solve the above equation with assuming initial value like

$$x(t - \tau) \rightarrow x_0 \quad at \ t = \tau$$

![Figure-8 Amplitude modulated waveform and the corresponding FFT spectrum for time delay 10sec &](image)

![Figure 9 Solution of Equation 18](image)
And assuming
\[ y(t) = x(t)e^t \] i.e. \( x(t) = y(t)e^{-t} \)
and \( x(t - \tau) = y(t - \tau)e^{-(t-\tau)} \)

So the relation (17) converts into
\[
\frac{dy(t)}{dt} = \beta y(t - \tau)e^t = \beta_1 y(t - \tau)
\]

Where, \( \beta e^t = \beta_1 \). Now the solution of the above equation is of the form [12]
\[
y(t) = y_0 \sum_{j=0}^{N} \frac{(t - j\tau)^j}{j!} (\beta_1)^j \quad x(t) = x_0 \sum_{j=0}^{N} \frac{(t - j\tau)^j}{j!} (\beta e^\tau)^j e^{-t} \quad (18)
\]

Solution of the above equation with three different normalized delay values are shown in figure-9. We notice from the figure-9, that the perturbation becomes zero after sometime, so we can say that the system is stable with specific delay.

7. INJECTION LOCKED OPTOELECTRONIC OSCILLATOR

The simple description of the scheme is shown in the Figure 10, once the multiple modes of OEO is injected to the electronic oscillator, the mode closest to the frequency of the oscillator injections lock the oscillator. The rest of the modes are rejected the locked oscillator. For locking it is necessary that the locking range must be smaller than half of the OEO-mode separation, called free special range (FSR). The locking ranges

\[
\Delta f = \frac{f_0}{2Q} \sqrt{\frac{P_{in}}{P_0}}
\]

When \( f_0 \) is the frequency oscillation frequency, \( Q \) is the quality faster the oscillators tank circuit, \( P_{in} \) and \( P_0 \) are respectively injected and output power. Hard-self oscillation has to be avoided.

Therefore, the injection power range for stable injection locking can be expressed as

\[
P_{in} < P_{out} \left( \frac{FSR}{2Q} \sqrt{\frac{2}{f_0}} \right)^2
\]

Figure 10- Simplified Structure of Injection Locked Optoelectronic Oscillator

Figure 11- Modified Laser Induced Microwave Oscillator
8. Conclusion

A novel structure of Laser Induced Microwave Oscillator (LIMO) is suggested in Figure 11. It incorporates an additional RF oscillator which is fed by the RF amplifier and its output port feeds to MZM. It further purifies the output of the RF amplifier before being fed to MZM. Thus this modified structure improves the spectral purity.

ACKNOWLEDGMENT

Authors are thankful to the management of Sir J.C. Bose School of Engineering for carrying out the work at Sir J.C Bose Creativity Centre of Supreme Knowledge Foundation Group of Institutions, Mankundu, Hooghly.

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