INTEGRATE FAULT TREE ANALYSIS AND FUZZY SETS IN QUANTITATIVE RISK ASSESSMENT

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ABSTRACT

Quantitative risk assessment is the most important step to judge the results of risk estimation in the process of decision making to improve level of safety. Quantitative risk assessment has faced big problem with complexity of engineering systems in term of reliability. In this study, reliability of risk Assessment proposed to solve problem of uncertainty based on fuzzy sets and fault tree analysis to precise values of top event. The results demonstrated that the model proposed is the best to solve problem of uncertainty in quantitative risk assessment.

Keywords: Quantitative risk assessment, Fault tree analysis, Fuzzy set, Reliability.

1. INTRODUCTION

In recent decades the world has witnessed an increase in the number of major accidents and disasters, where considerable efforts are made to control the safety of industrial systems. Risk assessment approach are designed primarily to reduce the existing risk inherent in engineering system to a level considered tolerable and maintain it over time. The method of fault tree analysis used to study the reliability and dependability of complex systems (Ding et al, 2010; Gupta et al, 2007). A study of system reliability, the probabilities are often considered accurate and fully determinable. It is also assumed that all the information on the performance of the reliability of the system and its components is provided (Wang et al, 2009). The Improve of reliability for prolonging the life of the item based on two steps essential, on the one hand, study reliability issues and on the other hand, estimate and reduce the failure rate. Two approaches for risk analysis, which can be qualitatively and quantitatively. The qualitative approach used when there is a source of danger, and when there are no safeguards against exposure of the hazard, and then there is a possibility of loss or injury. In complex engineering systems, there are often safeguarded against exposure of hazards for maximizing the level of safeguards, and minimize the level of risk. The quantitative risk assessment is the approach concerned with this study. Since quantitative risk analysis involves estimation of the
degree or probability of loss, risk analysis is fundamentally intertwined with the concept of probability of occurrence of hazards. This study proposes FTA uses fuzzy set to precise the values of top events which help us to precise the value of risk.

2. FUZZY FAULT TREE ANALYSIS FFTA

Probabilistic safety assessment by FTA was considered as an important tool to evaluate the systems engineering. Boolean algebras are used to mathematically represent the tree diagram and calculate the output of every logic gate (Haimes et al., 2004; Epstein & Rauzy, 2005; Ericson, 2005; Huang, Tonga & Zuo, 2004; N. Limnios, 2007). The occurrence probability of the undesired top event is a function of the reliability data of primary events, which are also known as basic events (IAEA, 2007; Yang, 2007). Fault tree analysis can give two types of results, i.e. qualitative and quantitative results.

2.1 FAULT TREE ANALYSIS

The FTA is composed of a number of symbols to describe events, Boolean gates, and page transfers. Transfer event symbols are pointers to indicate sub-tree branches that are used elsewhere in the tree (Ericson, 2005; Vesely et al., 1981).

2.1.1 BOOLEAN ALGEBRA

Boolean algebra is the algebra of fault events used in a fault tree to mathematically represent the relationship between input and output fault event of a Boolean gate in the tree. This relationship describes a situation where an output of the gate either fails or not (Haimes, 2004; Vesely et al., 1981).

<table>
<thead>
<tr>
<th>Rules</th>
<th>Engineering symbolism</th>
<th>Mathematical Symbolism</th>
</tr>
</thead>
<tbody>
<tr>
<td>Idempotent law</td>
<td>X.X=X</td>
<td>X∩ X=X</td>
</tr>
<tr>
<td></td>
<td>X+X=X</td>
<td>X∪ X=X</td>
</tr>
<tr>
<td>Distributive law</td>
<td>X. (Y+Z)= X.Y+X.Z</td>
<td>X∩(Y∪Z)=(X∩Y)∪(X∩Z)</td>
</tr>
<tr>
<td>Commutative law</td>
<td>X.Y=Y.X</td>
<td>X∩Y=Y∩X</td>
</tr>
<tr>
<td>Absorption law</td>
<td>X.(X+Y)=X</td>
<td>X∩(X∪Y)=X</td>
</tr>
<tr>
<td></td>
<td>X+(X.Y)=X</td>
<td>X∪(X∩Y)=X</td>
</tr>
</tbody>
</table>

2.1.2 FAILURE PROBABILITY CALCULATION

The failure probability of an output event for two or more independent input events combined by a Boolean OR gate calculated using Eq. (1). And by a Boolean AND gate calculated using Eq. (2).

\[
P(A_0) = 1 - \prod_{i=1}^{n} \{1 - P(A_i)\} \quad (1)
\]

\[
P(A_0) = \prod_{i=1}^{n} P(A_i) \quad (2)
\]

Where \(P(A_i)\) is the failure probability of the input event \(A_i\), and \(n\) is the number of input events to the Boolean gate.
2.2 FUZZY SET THEORY

2.2.1 Fuzzy Sets
The utility of fuzzy sets lies in their ability to model uncertain or ambiguous data. Fuzzy Sets (FS) is important to observe that there is an intimate connection between Fuzziness and Complexity. Fuzzy sets provide a means to model the uncertainty associated with vagueness, imprecision, and lack of information regarding a problem or a plant, etc (Dubois, 1980, Zadeh, 1978). The soft computing is a useful tool for solving problems in many fields. It is a high-performance language for technical computing (Dubois, 1998). Typical usage includes: Development of algorithms, Data analysis, exploration and visualization, Mathematics and computing, Scientific and technical graphics, Modeling, simulation and prototyping (Cavallo et al, 1996; Canos, 2008; Bouchon et al, 1995).

Fuzzy set theory deals with mathematically model information uncertainties and the theory has been developed and applied in a number of real-world applications (FuzzyTECK, 1995; Chen, 2001; Gebhardt, 1995; Zadeh, 1965).

The value of the membership function \( \mu_A(x) \) represents the membership grade of \( x \) in \( A \). The closer value to 1 is, the stronger degree of membership of \( x \) in \( A \) is. (Bector & Chandra, 2005; Lu et al., 2007).

\[
\mu_A: X \rightarrow [0,1], x \rightarrow \mu_A(x) \in [0,1]
\]  
(3)

The membership function of the number \( A \) can be expressed as follows.

\[
\mu_{A\cup B}(x) = \max(\mu_A(x), \mu_B(x)) = \mu_A(x) \cup \mu_B(x)
\]  
(4)

\[
\mu_{A\cap B}(x) = \min(\mu_A(x), \mu_B(x)) = \mu_A(x) \cap \mu_B(x)
\]  
(5)

\[
\mu_A^c(x) = 1 - \mu_A(x)
\]  
(6)

2.2.2 FUZZY NUMBERS
A fuzzy number is one type of fuzzy sets with normalized membership function. A fuzzy number \( \tilde{A} \) is a subset of real line \( \mathbb{R} \) who membership function \( \mu_{\tilde{A}}(x) \) can be a continuous mapping from \( \mathbb{R} \) into a closed interval \([0,1]\). The membership function \( \mu_{\tilde{A}}(x) \) has the following characteristics (Dubois & Prade, 1978; Wang et al., 2006).

The membership function of the number \( \tilde{A} \) can be expressed as follows.

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\mu_{\tilde{A}}^L(x), & a \leq x \leq b \\
1, & b \leq x \leq c \\
\mu_{\tilde{A}}^R(x), & c \leq x \leq d \\
0, & \text{otherwise}
\end{cases}
\]  
(7)

In a special case when \( b = c \), the trapezoidal fuzzy number into a triangular fuzzy number (Abbasbandy & Hajjari, 2009; Wang et al., 2006).

\[
\mu_{\tilde{A}}^L(x) = \frac{x-a}{b-a}
\]  
(8)

\[
\mu_{\tilde{A}}^R(x) = \frac{d-x}{d-c}
\]  
(9)
2.2.3 Fuzzy inference system FIS

The output level z of each rule is weighted by firing strength w of the rule (Guh et al, 2008; Castillo, 1999a). The final output of the system is weighted average of all the rule output which is given as:

$$\text{Final output} = \frac{\sum_{i=1}^{N} w_i z_i}{\sum_{i=1}^{N} w_i} \quad (10)$$

The Sugeno proposed a fuzzy inference method that guarantees the continuity of the output. Tomohiro Takagi and Michio Sugeno introduced a mathematical tool to build a fuzzy model of a system where fuzzy implication and reasoning are used in their paper in the year of 1985 (Sugeno, 1985). A FIS with five functional blocks described in Figure 2.

3. CASE STUDY

The storage tank is designed to hold a flammable liquid under slight nitrogen positive pressure under controls pressure (PICA-I)(CCPS, 2000). The incident is the top event that will be developed in the fault tree. Construction of the fault tree FT based on the knowledge of the system and initiating events in the hazard operability HAZOP study, the fault tree is constructed manually.

<table>
<thead>
<tr>
<th>Equipment and valves</th>
<th>Instruments</th>
</tr>
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<tbody>
<tr>
<td>FV = Flow control Valve</td>
<td>P = Pressure</td>
</tr>
<tr>
<td>T = Tank</td>
<td>T = Temperature</td>
</tr>
<tr>
<td>P = Pump</td>
<td>L = Level, F = Flow</td>
</tr>
<tr>
<td>PV = Pressure control Valve</td>
<td>I = Indicator</td>
</tr>
<tr>
<td>RV = Relief Valve</td>
<td>C = Controller</td>
</tr>
<tr>
<td>1” = 1 inch size</td>
<td>A = Alarm, H = High,</td>
</tr>
<tr>
<td></td>
<td>L = Low</td>
</tr>
</tbody>
</table>
Figure. 3: Flammable liquid storage tank

Figure. 4 shows fault tree constructed manually. Every event is labeled sequentially with a T for the top event, M for intermediate event, and B for a basic or undeveloped event. The procedure starts at the top event, Tank rupture due to overpressure, and determines the possible events that could lead to this incident (CCPS, 2000).

![Fault tree diagram](image)

Figure. 4: Fault tree analysis for tank rupture due to overpressure using boolean algebra

Tables. 2 shows calculate frequency using boolean algebra and fuzzy inference methods, the results by fuzzy sets are more precise than classical method, and in tables we can see two kinds of results. The first illustrate using same inputs in classical with results more precise, the second is the
results by change the value of Input which influence directly in output. The results show the method to calculate top event as follow:

Table.2: Tank rupture due to overpressure using Boolean algebras and fuzzy inference methods

<table>
<thead>
<tr>
<th>If</th>
<th>And</th>
<th>Then</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tank overpressured = 1.51x10^-2/yr</td>
<td>Pressure relief system failure = 2.5x10^-3</td>
<td>Tank rupture due to overpressure = 2x10^-5/yr</td>
</tr>
</tbody>
</table>

\[
\alpha - Level\ intervals\ of\ frequency\ using\ fuzzy\ inference\ method
\]

<p>| | | | | | | | | | | | |</p>
<table>
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<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>0.0151</td>
<td></td>
<td></td>
<td></td>
<td>0.0025</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>[0.0101 0.0133]</td>
<td></td>
<td></td>
<td></td>
<td>0.0025</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td>[0.0134 0.0201]</td>
<td></td>
<td></td>
<td></td>
<td>0.0025</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>0.0151</td>
<td></td>
<td></td>
<td></td>
<td>[0.002 0.003]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td>0.0101</td>
<td></td>
<td></td>
<td></td>
<td>[0.002 0.003]</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Fault tree analysis after calculating the frequency using fuzzy inference method, the results show that fuzzy sets is very appropriate for precise the value of initiating events for risk assessment.
4. DISCUSSION

From our results above we can say that:

The results of fault tree analysis using fuzzy sets is more powerfull than classical methods, which help us to precise the value of top event and to deal with uncertainty. The Surface Viewer shows a three-dimensional curve that represents the mapping from inputs and outputs, the diagram drawn using three equations essential (7)-(9).From the simulation results, it may be observed that the fuzzy model can be successfully employed to instantaneous determine the exactitude of fault event with only two input specifications.

The two-dimensional diagrams show the relationship between one input and one output, whereas the three dimensional diagrams show the relationship between two inputs and one output and the tables show also the change of outputs by changing the inputs. The two dimension diagram for OR gate happened one steps as line continue, this show the extent of one input directly on output, however the AND gate, the two dimensional diagrams show three steps which means that one input not influence directly on output. The same analysis for three dimensional diagram for both OR and AND gates. Results calculated of the rule output using equation (10).

5. CONCLUSION

In this paper, we have proposed a fuzzy fault tree analysis FFTA for reliability quantitative risk assessmentas new model to solve problem of imprecise and uncertainty of results. The application of this model gavebest results for the decision making. FFTA as new model for reliability quantitative risk assessment based on Fuzzy inference system FIS which considered as the best tool to precise the value of top events of fault tree analysis. The fault tree analysis is constructed using two different calculation approaches, boolean algebras classic and fuzzy inference using sugeno method,
whereas the fuzzy inference system gave results more precise than boolean algebras method, which consider as complementry to solve problems of uncertainty. Fault tree analysis with fuzzy sets is good solution for reliability quantitative risk assessment.

REFERENCES


