INFLATIONARY INVENTORY MODEL UNDER TRADE CREDIT SUBJECT TO SUPPLY UNCERTAINTY

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ABSTRACT

This paper develops a model to determine an optimal ordering policy for non-deteriorating items under inflation, permissible delay of payment and allowable shortage for future supply uncertainty for two suppliers. In this paper we have introduced the aspect of part payment. A part of the purchased cost is to be paid during the permissible delay period. In case of two suppliers, spectral theory is used to derive explicit expression for the transition probabilities of a four state continuous time Markov chain representing the status of the systems. These probabilities are used to compute the exact form of the average cost expression. We use concepts from renewal reward processes to develop average cost objective function. The effect of inflation and time value of money was investigated under the given sets of inflation and discount rates. Optimal solution is obtained using Newton Rapson method in R programming. Finally sensitivity analysis of the varying parameter on the optimal solution is done.

Keywords: Future supply uncertainty, trade credit, Partial payment, inflation, two suppliers.

1. INTRODUCTION

Supply uncertainty can have a drastic impact on firms who fail to protect against it. Supply uncertainty has become a major topic in the field of inventory management in recent years. Supply disruptions can be caused by factors other than major catastrophes. More common incidents such as snow storms, customs delays, fires, strikes, slow shipments, etc. can halt production and/or transportation capability, causing lead time delays that disrupt material flow. Silver (1981) appears to be first author to discuss the need for models that deal with supplier uncertainty. Articles by Parlar and Berkin (1991) consider the supply uncertainty problem, for a class of EOQ model with a single supplier where the availability and unavailability periods constitute an alternating Poisson process.
Kandpal and Tinani (2009) developed inventory model for deteriorating items with future supply uncertainty under inflation and permissible delay in payment for single supplier.

In today’s business transactions it is found that a supplier allows a certain fixed period to settle the account. During this fixed period the supplier charges no interest, but beyond this period interest is charged by the supplier under the terms and conditions agreed upon, since inventories are usually financed through debt or equity. Goyal (1985) has studied an EOQ system with deterministic demands and delay in payments is permissible which was reinvestigated by Chand and Ward (1987). Parlar and Perry (1996) developed inventory model for non-deteriorating items with future supply uncertainty considering demand rate d=1 for two suppliers. Aggarwal and Jaggi (1995) developed a model to determine the optimum order quantity for deteriorating items under a permissible delay in payment. In this paper we have introduced the aspect of part payment. It is common practice that an installment of payments is made during the period of the admitted delay in payment. The part to be paid and the time at which it is to be paid are mutually settled between the supplier and the buyer at the time of purchase of goods.

The effects of inflation are not usually considered when an inventory system is analyzed because most people think that the inflation would not influence the inventory policy to any significant degree. Following Buzacott (1975) and Bierman and Thomas (1977) investigated the inventory decisions under an inflationary condition in a standard EOQ model. Chandra and Bahner (1985) developed models to investigate the effects of inflation and time value of money on optimal order policies. Tripathi et al. (2010) developed an inventory model for non-deteriorating items and time-dependent demand under inflation when delay in payment is permissible.

In this paper it is assumed that the inventory manager may place his order with any one of two suppliers who are randomly available. Here we assume that the decision maker deals with two suppliers who may be ON or OFF. Here there are three states that correspond to the availability of at least one supplier that is states 0, 1 and 2 whereas state 3 denotes the non-availability of either of them. State 0 indicates that supplier 1 and supplier 2 both are available. Here it is assumed that one may place order to either one of the two suppliers or partly to both. State 1 represents that supplier 1 is available but supplier 2 is not available. State 2 represents that supplier 1 is not available but supplier 2 is available.

2. NOTATIONS, ASSUMPTIONS AND MODEL

The inventory model here is developed on the basis of following assumptions.
(a) Demand rate d is deterministic and it is d>1.
(b) We define $X_i$ and $Y_i$ be the random variables corresponding to the length of ON and OFF period respectively for $i^{th}$ supplier where $i=1, 2$. We specifically assume that $X_i \sim \text{exp} (\lambda_i)$ and $Y_i \sim \text{exp} (\mu_i)$. Further $X_i$ and $Y_i$ are independently distributed.
(c) Ordering cost is Rs. $k$/order (d) Holding cost is Rs. $h$/unit/unit time.
(e) Shortage cost is Rs. $\pi$/unit. (f) $R=\text{present value of the nominal inflation rate}$.
(g) $q_i=\text{order upto level } i=0, 1, 2$ (h) $r=\text{reorder upto level}$; $q_i$ and $r$ are decision variables.
(i) $c_0 = \text{Present value of the inflated price of an item Rs./unit}$

\[ c_0 = ce^{(f-r_1)t_1} = ce^{Rt_1} \quad , \quad R = f - r_1 \]

(j) Time dependent part of the backorder cost is Rs. $\hat{\pi}$/unit/time.
(k) Purchase cost is Rs. $c$/unit. (l) $f=\text{inflation rate}$ (m) $t_1=\text{time period with inflation}$
(n) $T_i$ is the time allowed by $i^{th}$ supplier where $i=1, 2$ at which $\alpha_i (0<\alpha_i<1)$ fraction of total amount has to be paid to the $i^{th}$ supplier where $i=1, 2$.
(o) $T_i (T_i > T_{\hat{\pi}})$ is the time at which remaining amount has to be cleared.
(p) \( T_{00} \) is the expected cycle time. \( T_{ji} \) and \( T_i \) are known constants and \( T_{00} \) is a decision variable.  
(q) \( r_1 \) := Discount rate representing the time value of money.  
(r) \( Ie_i \) := Interest rate earned when purchase made from \( i^{th} \) supplier where \( i=1, 2 \)  
\( Ic_i \) := Interest rate charged by \( i^{th} \) supplier where \( i=1, 2 \).  
(s) \( U_i \) and \( V_i \) are indicator variables for \( i^{th} \) supplier where \( i=1, 2 \)  
\( U_i = 0 \) if part payment is done at \( T_{1i} \)  
\( V_i = 0 \) if the balanced amount is cleared at \( T_i \)  
\( =1 \) otherwise  
\( =1 \) otherwise 

In this paper, we assume that supplier allows a fixed period \( T_{1i} \) during which \( \alpha_i \) fraction of total amount has to be paid and at time \( T_i \) remaining amount has to be cleared, hence up to time period \( T_{1i} \) no interest is charged for \( \alpha_i \) fraction, but beyond that period, interest will be charged upon not doing promised payment of \( \alpha_i \) fraction. Similarly for \( (1- \alpha_i) \) fraction no interest will be charged up to time period \( T_i \) but beyond that period interest will be charged. However, customer can sell the goods and earn interest on the sales revenue during the period of admissible delay.

For inflation rate \( f \), the continuous time inflation factor for the time period \( t_{1} \) is \( e^{ft_{1}} \) which means that an item that costs Rs. \( c \) at time \( t_{1}=0 \), will cost \( ce^{ft_{1}} \) at time \( t_{1} \). For discount rate \( r_1 \), representing the time value of money, the present value factor of an amount at time \( t_{1} \) is \( e^{-rt_{1}} \). Hence the present value of the inflated amount \( ce^{ft_{1}} \) (net inflation factor) is \( ce^{ft_{1}} e^{-rt_{1}} \). For an item with initial price \( c \) (Rs./unit) at time \( t_{1}=0 \) the present value of the inflated price of an item is given by \( c_0 = ce^{f(0)} = ce^{Rt} \), \( R = f - r_1 \) in which \( c \) is inflated through time \( t_{1} \) to \( c e^{ft_{1}} \), \( e^{-rt_{1}} \) is the factor deflating the future worth to its present value and \( R \) is the present value of the inflation rate.

Interest earned and interest charged is as follows.

(i) Interest earned on the entire amount up to time period \( T_{1i} \) is \( dce^{Rt} T_{00} T_{1i} Ie_i \)

(ii) Interest earned on (1-\( \alpha_i \)) fraction during the period \( (T_i - T_{1i}) \) is \( (1- \alpha_i) dce^{Rt} (T_i - T_{1i}) T_{00} Ie_i \)

(iii) If part payment is not done at \( T_{1i} \) then interest will be earned over \( \alpha_i \) fraction for period \( (T_i - T_{1i}) \) but interest will also be charged for \( \alpha_i \) fraction for \( (T_i - T_{1i}) \) period.

Interest earned= \( dce^{Rt} \alpha_i (T_i - T_{1i}) T_{00} Ie_i \)

Interest charged= \( dce^{Rt} \alpha_i (T_i - T_{1i}) T_{00} Ic_i \)

To discourage not doing promised payment, we assume that \( Ic_i \) is quite larger than \( Ie_i \)

(iv) Interest earned over the amount \( dce^{Rt} T_{00} T_{1i} Ie_i \) over the period \( (T_i - T_{1i}) \) is \( dce^{Rt} T_{00} T_{1i} Ie_i (T_i - T_{1i}) Ie_i \)

(v) If the remaining amount is not cleared at \( T_i \) then interest will be earned for the period \( (T_{00} - T_i) \) for \( (1- \alpha_i) \) fraction simultaneously interest will be charged on the same amount for the same period.

Interest earned= \( (1- \alpha_i) dce^{Rt} (T_{00} - T_i) T_{00} Ie_i \)

Interest charged= \( (1- \alpha_i) dce^{Rt} (T_{00} - T_i) T_{00} Ic_i \)
Total interest earned and charged is as follows

\[
dce^{RT_i}T_{00}T_{11}Ie_i + \left(1 - \alpha_i\right)dce^{RT_i}(T_i - T_{11})T_{00}Ie_i + \{dce^{RT_i}\alpha_i(T_i - T_{11})T_{00}Ie_i
\]

\[
- dce^{RT_i}\alpha_i(T_i - T_{11})T_{00}T_{11}Ie_i(T_i - T_{11})Ie_i + V_i(1 - \alpha_i)dce^{RT_i}(T_{00} - T_i)T_{00}Ie_i + dce^{RT_i}T_{00}T_{1i}Ie_i(T_i - T_{1i})Ie_i(T_{00} - T_i)Ie_i
\]

\[
+dce^{RT_i}T_{00}T_{1i}Ie_i(T_{00} - T_i)Ie_i + (1 - \alpha_i)dce^{RT_i}(T_{00} - T_i)Ie_i(T_{00} - T_i)Ie_i
\]

\[
+ \{dce^{RT_i}\alpha_i(T_{00} - T_i)Ie_i(T_i - T_{1i})Ie_i - dce^{RT_i}\alpha_i(T_{00} - T_i)T_{00}Ie_i\}
\]

\[
- (1 - \alpha_i)dce^{RT_i}(T_{00} - T_i)T_{00}Ie_i\]

The policy we have chosen is denoted by \((q_0, q_1, q_2, r)\). An order is placed for \(q_i\) units whenever inventory drops to the reorder point \(r\) and the state is found to be \(i=0, 1, 2\). When both suppliers are available, \(q_0\) is the total ordered from either one or both suppliers. If the process is found in state \(3\) that is both the suppliers are not available nothing can be ordered in which case the buffer stock of \(r\) units is reduced. If the process stays in state \(3\) for longer time then the shortages start accumulating at rate of \(d\) units/time. When the process leaves state \(3\) and supplier becomes available, enough units are ordered to increase the inventory to \(q_i + r\) units where \(i=0, 1, 2\). The cycle of this process starts when the inventory goes up to a level of \(q_0 + r\) units. Once the cycle is identified, we construct the average cost objective function as a ratio of the expected cost per cycle to the expected cycle length. i.e. 

\[
Ac(q_0, q_1, q_2, r) = \frac{C_{00}}{T_{00}}
\]

where, \(C_{00} = \text{E (cost per cycle)}\) and \(T_{00} = \text{E (length of a cycle)}\). Analysis of the average cost function requires the exact determination of the transition probabilities \(P_{ij}(t)\), \(i, j = 0, 1, 2, 3\) for the four state CTMC. The solution is provided in the lemma (refer Parlar and Perry [1996]).

\[
A(q_i, r) = \text{cost of ordering} + \text{cost of holding inventory during a single interval that starts with an inventory of } q_i + r \text{ units and ends with } r \text{ units.}
\]

\[
A(q_i, r) = k + \frac{1}{2} \frac{hR_i^2}{d} + \frac{hrq_i e^{R_i}}{d} i = 0, 1, 2
\]

Lemma 3.1: \(C_{i0} = P_{i0} \left( \frac{q_i}{d} \right) A(q_i, r) + \sum_{j=0}^{i} P_{ij} \left( \frac{q_i}{d} \right) \left[ A(q_j, r) + C_{j0} \right] \quad i = 0, 1, 2\)

and \(C_{30} = \bar{C} + \sum_{i=1}^{2} \rho_i C_{i0}\) Where \(\rho_i = \frac{\mu_i}{\delta}\) with \(\delta = \mu_1 + \mu_2\) and

\[
\bar{C} = \frac{-\delta}{\delta^2} \left[ e^{R_2} \left( \delta r - d \right) + \left( \pi d h d + \hat{\alpha} \right) - \mu \delta \right]
\]

(refer Parlar and Perry [1996]).
Theorem 3.2: The Average cost objective function for two suppliers under permissible delay in payments allowing partial payment is given by

\[
Ac = \frac{C_{00}}{T_{00}}
\]

And

\[
T_{00} = \frac{q_{00}}{d} + p_{01}T_{10} + p_{02}T_{20} + p_{03}(\bar{T} + \rho_{1}T_{10} + \rho_{2}T_{20})
\]

\[
\rho_{1} = \frac{\sum_{i=1}^{2} \rho_{i}}{2}, \quad \rho_{2} = \frac{\sum_{i=1}^{2} \rho_{i}}{2}
\]

**Proof:** Proof follows using Renewal reward theorem (RRT). The optimal solution for \( q_{0}, q_{1}, q_{2} \) and \( r \) is obtained by using Newton Rapson method in R programming.

4. NUMERICAL ANALYSIS

There are sixteen different patterns of payments, some of them we consider here.
1. \( UI=0 \) and \( VI=0 \) where \( i=1, 2 \) that is promise of doing part payment at time \( T_{1} \) and clearing the remaining amount at time \( T_{1} \) both are satisfied, the time period given by \( i^{th} \) supplier where \( i=1, 2 \).
2. \( U_i = 0 \) and \( V_i = 1 \) where \( i = 1, 2 \) that is promise of doing part payment at time \( T_i \) is satisfied but remaining amount is not cleared at time \( T_i \), the time period given by \( i^{th} \) supplier.

3. \( U_i = 1 \) and \( V_i = 0 \) where \( i = 1, 2 \) that is promise of doing part payment at time \( T_i \) is not satisfied but all the amount is cleared at time \( T_i \), the time period given by \( i^{th} \) supplier.

In this section we verify the results by a numerical example. We assume that 
\[ k = \text{Rs. 5/order}, \ c = \text{Rs.1/unit}, \ d = \text{20/units}, \ h = \text{Rs. 5/unit/time}, \ \pi = \text{Rs.350/unit}, \ \pi^\hat{=} = \text{Rs.25/unit/time}, \ \alpha_1 = 0.5, \ \alpha_2 = 0.6, \ I_{c1} = 0.11, \ I_{c2} = 0.13, \ I_{e1} = 0.02, \ I_{e2} = 0.04, \ T_{11} = 0.6, \ T_{12} = 0.8, \ T_{1} = 0.9, \ T_2 = 1.1, \ R = 0.05, \ t_1 = 6, \ \lambda_1 = 0.58, \ \lambda_2 = 0.45, \ \mu_1 = 3.4, \ \mu_2 = 2.5. \]

The last four parameters indicate that the expected lengths of the ON and OFF periods for first and second supplier are 
\[ 1/\lambda_1 = 1.72413794, \ 1/\lambda_2 = 2.2222, \ 1/\mu_1 = .2941176 \] and 
\[ 1/\mu_2 = .4 \] respectively. The long run probabilities are obtained as 
\[ p_0 = 0.7239588, \ p_1 = 0.1303126, \ p_2 = 0.1234989 \] and 
\[ p_3 = 0.02222979. \]

The optimal solution for the above numerical example based on the three patterns of payment is obtained as 
\[ (U_1, U_2, V_1, V_2) \]
\[
(U_1, U_2, V_1, V_2) \quad q_0 \quad q_1 \quad q_2 \quad r \quad Ac \\
(0, 0, 0, 0) \quad 2.8044 \quad 28.8243 \quad 28.0350 \quad 0.71827 \quad 8.28389 \\
(0, 0, 1, 1) \quad 2.55527 \quad 28.5755 \quad 27.715 \quad 0.64861 \quad 8.38186 \\
(1, 1, 0, 0) \quad 2.8538 \quad 28.7990 \quad 28.0153 \quad 0.73958 \quad 7.14469 \\
\]

From this we conclude that the cost is minimum if part payment is not done at \( T_i \) but account is cleared at \( T_i \) and the cost is maximum if part payment is done at \( T_i \) but account is not cleared at \( T_i \), this implies that we encourage the small businessmen to do the business by allowing partial payment and simultaneously we want to discourage them for not clearing the account at the end of credit period.

5. SENSITIVITY ANALYSIS

We study below in the Sensitivity analysis, the effect of change in the parameter on the following three patterns of payment.

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate \( R \) keeping other parameter values fixed where \( U_i = 0 \) and \( V_i = 0 \), \( i = 1, 2 \). Inflation rate \( R \) is assumed to take values 0.05, 0.1, 0.15, 0.2. We resolve the problem to find optimal values of \( q_0, q_1, q_2, r \) and \( AC \).

<table>
<thead>
<tr>
<th>( R )</th>
<th>( q_0 )</th>
<th>( q_1 )</th>
<th>( q_2 )</th>
<th>( r )</th>
<th>( Ac )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>2.8044</td>
<td>28.8243</td>
<td>28.0350</td>
<td>0.71827</td>
<td>8.28389</td>
</tr>
<tr>
<td>0.1</td>
<td>2.38808</td>
<td>27.72073</td>
<td>26.7435</td>
<td>0.67913</td>
<td>9.80586</td>
</tr>
<tr>
<td>0.15</td>
<td>2.0325</td>
<td>26.8227</td>
<td>25.6677</td>
<td>0.63246</td>
<td>12.9493</td>
</tr>
<tr>
<td>0.2</td>
<td>1.72991</td>
<td>26.0919</td>
<td>24.7733</td>
<td>0.58148</td>
<td>15.9251</td>
</tr>
</tbody>
</table>

We see that as inflation rate \( R \) increases value \( q_0, q_1, q_2 \) and the value of reorder quantity \( r \) decreases and hence average cost increases.
(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate $R$ keeping other parameter values fixed where $U_i=0$ and $V_i=1, i=1, 2$. Inflation rate $R$ is assumed to take values 0.05, 0.1, 0.15, 0.2. We resolve the problem to find optimal values of $q_0, q_1, q_2, r$ and $AC$.

### Table 5.2: Sensitivity Analysis Table by varying the parameter values of $R$
When patterns of payment is $(U_1=0, U_2=0, V_1=1, V_2=1)$

<table>
<thead>
<tr>
<th>$R$</th>
<th>$q_0$</th>
<th>$q_1$</th>
<th>$q_2$</th>
<th>$r$</th>
<th>$AC$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>2.55527</td>
<td>28.5755</td>
<td>27.715</td>
<td>0.64861</td>
<td>8.38186</td>
</tr>
<tr>
<td>0.1</td>
<td>2.20818</td>
<td>27.5466</td>
<td>26.5099</td>
<td>0.6217</td>
<td>10.5985</td>
</tr>
<tr>
<td>0.15</td>
<td>1.904439</td>
<td>26.70043</td>
<td>25.4976</td>
<td>0.58679</td>
<td>13.63506</td>
</tr>
<tr>
<td>0.2</td>
<td>1.63989</td>
<td>26.00519</td>
<td>24.649</td>
<td>0.5464</td>
<td>16.6286</td>
</tr>
</tbody>
</table>

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate $R$ keeping other parameter values fixed where $U_i=1$ and $V_i=0, i=1, 2$. Inflation rate $R$ is assumed to take values 0.05, 0.1, 0.15, 0.2. We resolve the problem to find optimal values of $q_0, q_1, q_2, r$ and $AC$. The optimal values of $q_0, q_1, q_2, r, AC$ and $R$ are plotted in Fig.5.3.

### Table 5.3: Sensitivity Analysis Table by varying the parameter values of $R$
When patterns of payment is $(U_1=1, U_2=1, V_1=0, V_2=0)$

<table>
<thead>
<tr>
<th>$R$</th>
<th>$q_0$</th>
<th>$q_1$</th>
<th>$q_2$</th>
<th>$r$</th>
<th>$AC$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>2.8538</td>
<td>28.799</td>
<td>28.0153</td>
<td>0.73958</td>
<td>7.14469</td>
</tr>
<tr>
<td>0.1</td>
<td>2.43058</td>
<td>27.6947</td>
<td>26.7234</td>
<td>0.70022</td>
<td>9.45932</td>
</tr>
<tr>
<td>0.15</td>
<td>2.0687</td>
<td>26.79694</td>
<td>25.6477</td>
<td>0.6527</td>
<td>12.49431</td>
</tr>
<tr>
<td>0.2</td>
<td>1.76056</td>
<td>26.0669</td>
<td>24.75399</td>
<td>0.60075</td>
<td>15.48683</td>
</tr>
</tbody>
</table>

We see that as inflation rate $R$ increases value $q_0, q_1, q_2$ and the value of reorder quantity $r$ decreases and hence average cost increases.

From the above sensitivity analysis we conclude that cost is minimum if part payment is not done at $T_{1i}$ but account is cleared at $T_i$ and the cost is maximum if part payment is done at $T_{1i}$ but account is not cleared at $T_i$, this implies that we encourage the small businessmen to do the business by allowing partial payment and simultaneously we want to discourage them for not clearing the account at the end of credit period.

REFERENCES