SQUEEZE FILM LUBRICATION OF FINITE POROUS PARTIAL JOURNAL BEARING WITH MICROPOLAR FLUIDS

MONAYYA MAREPPA
Department of mathematics, Government Degree College,
Yadgir-585202, Karnataka, INDIA

S. SANTOSH
Government First Grade College, Shahapur-585223, Karnataka, INDIA

ABSTRACT
In this paper the general Reynolds type equation of finite porous partial journal bearing lubricated with micropolar fluid is solved numerically by using finite difference technique. The first order non-linear equation for time height relation is solved numerically with the given initial condition. From the numerical results obtained, it is observed that, the effect of micropolar fluid is to increases the film pressure, the load carrying capacity and to lengthen the squeeze film time as compared with Newtonian case. The reduction in load carrying capacity and the response time of porous partial journal bearings can be compensated by the use of lubricants with proper microstructure additives by which the bearing life can be increased.

Key words: Porous, Partial journal bearings, Squeeze films, Micropolar fluids.

http://www.iaeme.com/issue.asp?JType=IJTE&VType=3&IType=2

NOMENCLATURE

\begin{align*}
c & \quad \text{radial clearance} \\
e & \quad \text{eccentricity} \\
h & \quad \text{film thickness } (h = c + e \cos \theta) \\
\bar{h} & \quad \text{non-dimensional film thickness } (= h/c) \\
\bar{h}_0 & \quad \text{minimum film height} \\
H_0 & \quad \text{porous layer thickness} \\
k & \quad \text{permeability of the porous matrix}
\end{align*}
Monayya Mareppa and S. Santosh

\[ l \quad \text{characteristic length of the polar suspension} \quad \left( = \frac{\gamma}{4 \mu} \right) ^{\frac{1}{2}} \]

\[ \bar{T} \quad \text{non-dimensional form of } \quad l \ (= l/c) \]

\[ L \quad \text{bearing length} \]

\[ N \quad \text{coupling number} \quad \left( = \frac{\chi}{\chi + 2 \mu} \right) ^{\frac{1}{2}} \]

\[ p \quad \text{lubricant pressure} \]

\[ \bar{p} \quad \text{non-dimensional pressure} \quad \left( = \frac{pc^2}{\mu R^2 \left( \frac{\partial \varepsilon}{\partial t} \right)} \right) \]

\[ R \quad \text{radius of the journal} \]

\[ t \quad \text{time} \]

\[ u, v, w \quad \text{components of fluid velocity in } x, y \text{ and } z \text{ directions, respectively} \]

\[ v_1, v_2, v_3 \quad \text{microrotational velocity components in the } x, y \text{ and } z \text{ directions} \]

\[ r, \theta, z \quad \text{cylindrical co-ordinates} \]

\[ V \quad \text{squeeze velocity} \quad \frac{\partial h}{\partial t} \left( = c \frac{\partial \varepsilon}{\partial t} \cos \theta \right) \]

\[ W \quad \text{load carrying capacity} \]

\[ \bar{W} \quad \text{non-dimensional load carrying capacity} \quad \left( = \frac{We^2}{\mu LR^3 \left( \frac{\partial \varepsilon}{\partial t} \right)} \right) \]

\[ x, y, z \quad \text{Cartesian co-ordinates} \]

\[ \varepsilon \quad \text{eccentricity ratio} \ (= e/c) \]

\[ \chi \quad \text{spin viscosity} \]

\[ \gamma \quad \text{viscosity co-efficient for micropolar fluids} \]

\[ \mu \quad \text{viscosity co-efficient} \]

\[ \tau \quad \text{dimensionless response time} \]

\[ \psi \quad \text{permeability parameter} \ (= kH/c^3) \]

\[ \theta \quad \text{circumferential co-ordinate} \ (= x/R) \]

\[ \lambda \quad \text{length to diameter ratio} \ (= L/2R) \]

\[ \Delta \quad \text{gradient operator} \]
INTRODUCTION
The squeeze film mechanism is of practical significance in many areas of engineering and is commonly observed in the bearings of automotive engines, aircraft engines, machine tools, turbo chemistry and skeletal joints. The viscous lubricant contained between the two surfaces cannot be instantaneously squeezed out because it takes certain time for these surfaces to come into contact. Since the viscous lubricant has a resistance to extrusion, a pressure is built up during that interval and then the load is supported by the lubricant film.

Porous bearings contain the porous filled with lubricating oil so that the bearing requires no further lubrication during the whole life of the machine. Self-lubricated bearings or oil retaining bearings exhibit this feature. Self-lubricating porous bearings have the advantage of high production rate because, short sintering time is required. Graphite is added to enhance the self-lubricating property of the bearings. Porous metal bearings are widely used in home appliances, small motors, instruments and construction equipment’s because of their low cost and good bearing qualities. The analytical study of porous bearings with hydrodynamic conditions was first made by Morgan and Cameron [1]. There have been numerous studies of various types of porous bearings in literature viz; Slider bearings [2-3], Journal bearings [4-6], Squeeze film bearings [7-12]. An extensive study of porous bearings has been made during the last few decades [13-15]. Recently, the studies of porous bearings are focused on Newtonian lubricants. However, the use of non-Newtonian fluids as lubricants is of growing interest in recent times. The pulsating or reciprocating loads on bearings and bearing surfaces are produced in several machine components. Due to this the oil film breaks down and relatively high friction and wear are to be expected. When the conditions are favorable an oil film is maintained between the contacting surfaces when the relative motion is momentarily zero. When the load is relived or reversed the lubricant film can recover its thickness before the next cycle if the bearing has been designed to permit this build up. Such phenomenon is observed in reciprocating machines in which the bearings are subjected to fluctuating dynamic loads. When the bearings are subjected to reciprocating loads the lubricants may become contaminated with dirt and metal particles then the lubricant behaves as a fluid suspension. The classical Newtonian theory will not predict the accurate flow behavior of fluid suspensions especially when the clearance in the bearing is comparable with average size of the lubricant additives. The Eringen’s [16] micro continuum theory of micro polar fluid accounts for the polar effects. Several investigators used this theory for the study of different bearings systems [17-18].

On the basis of microcontinuum theory, the present study the squeeze film characteristics of finite partial porous journal bearings lubricated with micropolar fluids. The modified Reynolds equation is solved numerically by using finite difference technique. The load carrying capacity and time-height relation are compared with the classical Newtonian case.

MATHEMATICAL FORMULATION OF THE PROBLEM
The physical configuration of the problem under consideration is shown in the figure 1. The journal of radius R approaches the porous bearing surface at a circumferential section, θ with velocity, \( V \left( \frac{\partial h}{\partial t} \right) \) The film thickness \( h \) is a function of \( \theta \) and is given by \( h=c+e \cos \theta \)
Where ‘c’ is radial clearance and ‘e’ is the eccentricity of the journal centre. The lubricant in the film region and also in the porous region is assumed to be Eringen’s [19] micropolar fluid.

![Diagram of journal bearing](image)

**Figure 1.** Physical configuration of a finite partial porous journal bearing.

The constitutive equations for micropolar fluids proposed by Eringen [19] simplify considerably under the usual assumptions of hydrodynamic lubrication. The resulting equations under steady–state conditions are

Conservation of linear momentum:

\[
\left( \mu + \chi \right) \frac{\partial^2 u}{\partial y^2} + \chi \frac{\partial v}{\partial y} - \frac{\partial p}{\partial x} = 0
\]

(2)

\[
\left( \mu + \chi \right) \frac{\partial^2 w}{\partial y^2} - \chi \frac{\partial v_1}{\partial y} - \frac{\partial p}{\partial y} = 0
\]

(3)

Conservation of angular momentum:

\[
\gamma \frac{\partial^2 v_1}{\partial y^2} - 2 \chi v_1 + \chi \frac{\partial w}{\partial y} = 0
\]

(4)

\[
\gamma \frac{\partial^2 v_3}{\partial y^2} - 2 \chi v_3 - \chi \frac{\partial u}{\partial y} = 0
\]

(5)
Conservation of mass:
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]  
(6)

Where \((u, v, w)\) are the velocity components of the lubricant in the \(x, \ y\) and \(z\) directions, respectively, and \((v_1, v_2, v_3)\) are micro rotational velocity components, \(\chi\) is the spin viscosity and \(\gamma\) is the viscosity coefficient for micropolar fluids and \(\mu\) is the Newtonian viscosity coefficient.

The flow of micropolar lubricants in a porous matrix governed by the modified Darcy law, which account for the polar effects is given by [20]

\[
q^* = \frac{-k}{(\mu + \chi)} \nabla p^*
\]  
(7)

Where \(q^* = (u^*, v^*, w^*)\) is the modified Darcy velocity vector, with

\[
\begin{align*}
  u^* &= \frac{-k}{(\mu + \chi)} \frac{\partial p^*}{\partial x}, \\
  v^* &= \frac{-k}{(\mu + \chi)} \frac{\partial p^*}{\partial y}, \\
  w^* &= \frac{-k}{(\mu + \chi)} \frac{\partial p^*}{\partial z}
\end{align*}
\]  
(8)

\(k\) is the permeability of the porous matrix and \(p^*\) is the pressure in the porous region.

Due to continuity of fluid in the porous matrix, \(p^*\) satisfies the Laplace Equation.

\[
\frac{\partial^2 p^*}{\partial x^2} + \frac{\partial^2 p^*}{\partial y^2} + \frac{\partial^2 p^*}{\partial z^2} = 0
\]  
(9)

The relevant boundary conditions are

(a) at the bearing surface \((y=0)\)

\[
\begin{align*}
  u &= 0, \ v = v^*, \ w = 0 \\
  v_1 &= 0, \ v_3 &= 0
\end{align*}
\]  
(10a)

(b) at the journal surface \((y=h)\)

\[
\begin{align*}
  u &= 0, \ v = \frac{\partial h}{\partial t}, \ w = 0 \\
  v_1 &= 0, \ v_3 &= 0
\end{align*}
\]  
(11a)
SOLUTION OF THE PROBLEM

The generalized Reynolds equation is given by [21].

\[
\frac{\partial}{\partial x} \left[ f(N,l,h) + \frac{12\mu k H_0}{(\mu + \chi)} \right] \frac{\partial p}{\partial x} + \frac{\partial}{\partial z} \left[ f(N,l,h) + \frac{12\mu k H_0}{(\mu + \chi)} \right] \frac{\partial p}{\partial z} = 12\mu \frac{\partial h}{\partial t} \tag{12}
\]

Where

\[f(N,l,h) = h^3 + 12l^2 h - 6Nh^2 \coth \left( \frac{Nh}{2l} \right)\]

\[\frac{\partial h}{\partial t} = c \frac{\partial \epsilon}{\partial t} \cos \theta\]

Introducing the non-dimensional scheme into equation (19)

\[\theta = \frac{x}{R}, \quad \zeta = \frac{z}{L}, \quad \bar{T} = \frac{l}{c}, \quad \bar{h} = \frac{h}{c} = 1 + \epsilon \cos \theta, \quad \bar{k} = \frac{k}{c^2}, \quad \bar{H}_0 = \frac{H_0}{c}\]

\[\bar{p} = \frac{p c^2}{\mu R^2 \left( \frac{d\epsilon}{dt} \right)}, \quad \psi = \frac{kH_0}{c^3}, \quad N = \left( \frac{\chi}{\chi + 2\mu} \right)^{\frac{1}{2}}, \quad \lambda = \frac{L}{2R}\]

The modified Reynolds equation (12) can be written in a non-dimensional form as

\[\frac{\partial}{\partial \theta} \left[ \bar{f}(N,\bar{T},\bar{h}) + 12\psi \left( \frac{1-N^2}{1+N^2} \right) \bar{p} \right] + \left( \frac{1}{4\lambda^2} \right) \times \frac{\partial}{\partial \zeta} \left[ \bar{f}(N,\bar{T},\bar{h}) + 12\psi \left( \frac{1-N^2}{1+N^2} \right) \bar{p} \right] = 12\cos \theta \tag{13}\]

where

\[\bar{f}(N,\bar{T},\bar{h}) = \bar{h}^3 + 12\bar{T}^2 \bar{h} - 6N\bar{T} \bar{h}^2 \coth \left( \frac{N\bar{h}}{2\bar{T}} \right)\]

As the permeability parameter \(\psi \to 0\), equation (13) reduces to the corresponding solid case. For the \(180^\circ\) partial porous journal bearing, the boundary conditions for the fluid film pressure are

\[\bar{p} = 0 \quad \text{at} \quad \theta = \frac{\pi}{2}, \frac{3\pi}{2}\]

\[\text{and} \quad \bar{p} = 0 \quad \text{at} \quad \zeta = \pm \frac{1}{2}\tag{14}\]

The modified Reynolds equation will be solved numerically by using a finite difference scheme. The film domain under consideration is divided by grid spacing shown in figure2. In finite increment format, the terms of equation (13) can be expressed as
Figure 2 Grid point notation for film domain

\[
\frac{\partial}{\partial \theta} \left[ \bar{f} + 12 \psi \left( \frac{1-N^2}{1+N^2} \right) \frac{\partial \bar{P}}{\partial \theta} \right] = 0
\]

\[
\frac{1}{\Delta \theta} \left[ \left( \bar{f}_{i,j+\frac{1}{2}} + 12 \psi \left( \frac{1-N^2}{1+N^2} \right) \left( \frac{\bar{P}_{i,j+1} - \bar{P}_{i,j}}{\Delta \theta} \right) - \left( \bar{f}_{i,j-\frac{1}{2}} + 12 \psi \left( \frac{1-N^2}{1+N^2} \right) \left( \frac{\bar{P}_{i,j} - \bar{P}_{i,j-1}}{\Delta \theta} \right) \right. \right. \right]
\]

(15)

\[
\frac{1}{4\lambda^2} \times \frac{\partial}{\partial \Sigma} \left[ \left( \bar{f} + 12 \psi \left( \frac{1-N^2}{1+N^2} \right) \frac{\partial \bar{P}}{\partial \Sigma} \right) = 0 \right]
\]

\[
\frac{1}{4\lambda^2} \times \left[ \left( \bar{f}_{i,j+\frac{1}{2}} + 12 \psi \left( \frac{1-N^2}{1+N^2} \right) \left( \frac{\bar{P}_{i,j+1} - \bar{P}_{i,j}}{\Delta \theta} \right) - \left( \bar{f}_{i,j-\frac{1}{2}} + 12 \psi \left( \frac{1-N^2}{1+N^2} \right) \left( \frac{\bar{P}_{i,j} - \bar{P}_{i,j-1}}{\Delta \theta} \right) \right. \right. \right]
\]

(16)

Substituting these expressions (15) and (16) into the Reynolds equation (13) we get

\[
\bar{P}_{i,j} = C_1 \bar{P}_{i+1,j} + C_2 \bar{P}_{i-1,j} + C_3 \bar{P}_{i,j+1} + C_4 \bar{P}_{i,j-1} + C_5
\]

(17)

where

\[
C_0 = 4 \lambda^2 r^2 \left\{ \left( \bar{f}_{i+\frac{1}{2},j} + 12 \psi \left( \frac{1-N^2}{1+N^2} \right) \right) + \left( \bar{f}_{i-\frac{1}{2},j} + 12 \psi \left( \frac{1-N^2}{1-N^2} \right) \right) \right\}
\]

\[
+ \left( \bar{f}_{i,j+\frac{1}{2}} + 12 \psi \left( \frac{1-N^2}{1-N^2} \right) \right) + \left( \bar{f}_{i,j-\frac{1}{2}} + 12 \psi \left( \frac{1-N^2}{1-N^2} \right) \right)
\]
\[ C_1 = 4\lambda^2 r^2 \left( \tilde{f}_{i+1/2,j} + 12\psi \left( \frac{1-N^2}{1-N^2} \right) \right) / C_0, \]
\[ C_2 = 4\lambda^2 r^2 \left( \tilde{f}_{i-1/2,j} + 12\psi \left( \frac{1-N^2}{1-N^2} \right) \right) / C_0, \]
\[ C_3 = \left( \tilde{f}_{i,j+1/2} + 12\psi \left( \frac{1-N^2}{1-N^2} \right) \right) / C_0, \]
\[ C_4 = \left( \tilde{f}_{i,j-1/2} + 12\psi \left( \frac{1-N^2}{1-N^2} \right) \right) / C_0, \]
\[ C_5 = -48\lambda^2 \cos \theta \Delta \xi^2 / C_0, \quad r = \Delta \xi / \Delta \theta. \]

The pressure, $\bar{p}$, is calculated by using the numerical method with grid spacing of $\Delta \theta = 9^0$ and $\Delta \xi = 0.05$.

The load carrying capacity of the bearing, $W$, generated by the film pressure is obtained by

\[ W = -LR \int_{\theta=\pi/2}^{\theta=3\pi/2} \int_{z=-\Delta/2}^{z=\Delta/2} p \cos \theta d\theta dz \]  \hspace{1cm} (18)

The non-dimensional load carrying capacity, $\bar{W}$ of the $180^0$ porous partial journal bearing is obtained in the form

\[ \bar{W} = \frac{Wc^2}{\mu LR^3} = -\int_{\theta=\pi/2}^{\theta=3\pi/2} \int_{\tau=-\Delta/2}^{\tau=\Delta/2} \bar{p} \cos \theta d\theta d\xi \]  \hspace{1cm} (19)

\[ \approx \sum_{i=0}^{M} \sum_{j=0}^{N} \bar{P}_{i,j} \cos \theta_{i,j} \Delta \theta \Delta \xi \]

\[ = g(\epsilon, \bar{T}, N, \psi) \]  \hspace{1cm} (20)

where $M+1$ and $N+1$ are the grid point numbers in the $x$ and $z$ directions respectively.

Time-height relation is calculated by considering the time taken by the journal to move from $\epsilon = 0$ to $\epsilon = \epsilon_1$ can be obtained from equation (20)

\[ \frac{d\epsilon}{d\tau} = \frac{1}{g(\epsilon, \bar{T}, N, \psi)} \]  \hspace{1cm} (21)

Where $\tau = \frac{Wc^2}{\mu LR^3} t$ is the non-dimensional response time.
The first order non-linear differential equation (21) is solved numerically by using the fourth order Runge-Kutta method with the initial conditions $\varepsilon = 0$ to $\tau = 0$.

**RESULTS AND DISCUSSIONS**

To solve squeeze film pressure in the equation (17) the mesh of the film domain has 20 equal intervals along the bearing length and circumference. The co-efficient matrix of the system of algebraic equations is of pentadiagonal form. These equations have been solved by using Scilab tools.

The squeeze film lubrication characteristic of a finite partial porous journal bearings lubricated with micropolar fluids are obtained on the basis of various non-dimensional parameters such as the coupling number, $N = \left(\frac{\lambda}{\chi + 2\mu}\right)^{1/2}$ which characterizes the coupling between the Newtonian and microrotational viscosities, the parameter, $T = \frac{l}{c}$ in which $T$ has the dimension of length and may be considered as chain length of microstructures additives. The parameter $T$, characterizes the interaction of the bearing geometry with the lubricant properties. In the limiting case as $T \to 0$ the effect of microstructures becomes negligible. The effect of permeability is observed through the non-dimensional permeability parameter, $\psi = \frac{kH_0}{c^3}$ and it is to be noted that as $\psi \to 0$ the problem reduces to the corresponding solid case and as $T, N \to 0$ it reduces to the corresponding Newtonian case.

**Squeeze Film Pressure**

The variation of non-dimensional squeeze film pressure $\bar{p}$ for different values of $T$ with $N = 0.6, \lambda = 0.75, \varepsilon = 0.2$ and $\psi = 0.01$ is shown in fig.3. It is observed that $\bar{p}$ increases for increasing values of $T$. Increases in $\bar{p}$ is more pronounced for larger value of $T$. Figure 4. Shows the variation of non-dimensional film pressure $\bar{p}$, for different values of $N$ with $T = 0.2, \lambda = 0.75, \varepsilon = 0.2$ and $\psi = 0.01$. It is observed that $\bar{p}$ increases for increasing value of $N$. Increases in $\bar{p}$ is more pronounced for larger value of $T$. The effect of permeability $\psi$ on the variation of $\bar{p}$ is shown in fig.5. for $N = 0.6, \lambda = 0.75, \varepsilon = 0.2$ and $T = 0.2$. It is observed that the increasing values of permeability parameter $\psi$ decreases $\bar{p}$.
Monayya Mareppa and S. Santosh

Figure 3. Non-dimensional film pressure $\bar{P}$ for different values of $\bar{T}$ with $N = 0.6, \lambda = 0.75, \sigma = 0.2$ and $\nu = 0.01$

Figure 4. Non-dimensional film pressure $\bar{P}$ for different values of $\bar{N}$ with $\bar{T} = 0.2, \lambda = 0.75, \sigma = 0.2$ and $\nu = 0.01$
Squeeze Film Lubrication of Finite Porous Partial Journal Bearing with Micropolar Fluids

Figure 5: Non-dimensional film pressure $p$ for different values of $\psi$ with $N = 0.6, \lambda = 0.75, \varepsilon = 0.2$ and $\bar{T} = 0.2$

Load carrying capacity

The variation of non-dimensional load carrying capacity $\bar{W}$ with $\varepsilon$ for different values of $\bar{T}$ with $N = 0.4$ and $\lambda = 0.75$ for the two values of $\psi = 0.01, 0.05$ is depicted in the figure 6. It is observed that the increasing values of $\bar{T}$ increases $\bar{W}$ as compared to corresponding Newtonian case ($\bar{T} \to 0$). The variation of non-dimensional load carrying capacity $\bar{W}$ with $\varepsilon$ for different value of $N$ with $\bar{T} = 0.2$ and $\lambda = 0.75$ for the two values of $\psi = 0.01, 0.05$ is depicted in the figure 7. It is observed that the increasing values of $N$ increases $\bar{W}$ for both the values of $\psi$. 

http://www.iaeme.com/IJTE/.asp  
editor@iaeme.com
Minimum squeeze film height

The response time of the squeeze film is one of the significant factor in the design of bearings. The response time is the time that will elapse for a squeeze film reduces to some minimum permissible height. The variation of the non-dimensional minimum film height \( \eta_{\text{th}} = (1 - \varepsilon) \) with the non-dimensional time \( \tau \) as a function of \( T \) with \( N = 0.4 \) and \( \lambda = 0.75 \) for two values of \( \psi = 0.01, 0.05 \) is shown in the figure.8. It is
observed that, the response time increases for increasing values of $T$ as compared to Newtonian case. The variation of the non-dimensional minimum film height $h_0$ with $\tau$ for different values of $N$ with $T=0.2$ and $\lambda=0.75$ for two values of $\psi=0.01,0.05$ is depicted in figure 9. It is observed that, the response time increases for increasing values of $N$ as compared with larger values of $N$. 

![Graph showing variation of non-dimensional minimum film height $h_0$ versus $\tau$ for different values of $N$ with $T=0.2$ and $\lambda=0.75$.](image)

![Graph showing variation of non-dimensional minimum film height $h_0$ versus $\tau$ for different values of $N$ with $T=0.2$ and $\lambda=0.75$.](image)
CONCLUSIONS

The effect of micropolar on the squeeze film lubrication of finite porous journal bearings is studied by using the Eringen’s micropolar fluid theory. The finite modified Reynolds type equation is obtained for the problem under consideration and is solved numerically by using finite difference technique with grid spacing of $\Delta \theta = 0^\circ$ and $\Delta z = 0.05$. From the results obtained, the following conclusions are drawn.

1) The effect of micropolar is to increases the squeeze film pressure and the load carrying capacity as compared to the corresponding Newtonian case.

2) The squeeze film time is lengthened for the micropolar lubricants as compared to the corresponding Newtonian case.

3) The longer the bearing length is, the more the micropolar effect on the load carrying capacity.

ACKNOWLEDGEMENTS

The authors sincerely acknowledge the financial support by the U.G.C. New Delhi, India, under DRS-project.

REFERENCE


Squeeze Film Lubrication of Finite Porous Partial Journal Bearing with Micropolar Fluids


