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MATHEMATICAL ESTIMATION OF FLIGHT CHARACTERISTICS OF CIVIL AIRPLANES

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ABSTRACT

Importance of estimation of numerous characteristics of aircrafts is described. A mathematical method is proposed for estimation of flight characteristics of civil airplanes based on calculation and comparison of indices of operation efficiency and engineering level. Improved method of determination of generalized index of engineering level is described. Predictions of indices are analyzed and conclusions are made about consistency between the considered method and trends of characteristics of medium and long range civil airplanes.

Key words: **Airplane, civil aviation, flight characteristics, technical efficiency, technical level.**

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1. INTRODUCTION

Aircrafts and airplanes in particular are characterized by numerous properties and specifications. Herewith, according to the Mozhaysky formula it is impossible to provide extreme values of all airplane specifications, since at certain R&D level improvement of some airplane specifications leads to deterioration of another specifications. In this regard it is required to estimate the level of combination of airplane specifications so that designers could adopt decisions on reasonability of development and subsequent commissioning of certain project with consideration for its market competitiveness and operators could adopt decisions on reasonability of ordering of certain airplane models aiming at their subsequent efficient operation, thus reducing risks of air companies [1].

Both during development and during operation such problem should be solved under conditions of limited information about the objects of research. Concerning civil airplanes, the most available information is that about characteristics describing their purposes: flight distance, cruise speed, maximum payload or passenger capacity, class of home aerodrome, etc. Such characteristics are also used for estimation of technical efficiency and technical level of airplanes, the relevant theoretical aspects are described in [2–4], and the methods of their determination are described in [5–8]. This promotes decrease in environmental and economic hazard of flight accidents [9] and/or economic risks of air companies [10].

2. MATERIALS AND METHODS

The efficiency is interrelation between the achieved result and used resources, it can be estimated by the ratio of positive effect to negative one. Technical efficiency is determined by major airplane flight specifications and is subdivided into fuel, weight, and target efficiencies.

While estimating fuel efficiency, the positive effect is transportation work defined as the product of maximum payload and flight distance. The negative effect is the weight of fuel consumed during the flight. The weight efficiency is often determined by the ratio of maximum payload to aircraft gross weight.

The target efficiency is the degree of aircraft facility to perform the predefined task. If this is a cargo aircraft, then the target efficiency is transport efficiency. In order to take into account, the most significant positive and negative aircraft properties upon estimation of transport efficiency, Sheinin [3, 4] proposed the index where transportation work and flight speed were considered as the positive effect, and the negative effect was determined by fuel weight and aircraft operating empty weight.

Technical level is a relative characteristic based on comparison of quality indices characterizing technical perfection of estimated products with respective basic performances. Technical perfection is the main constituent of product quality which is developed only on the basis of achievements of science and technology. Technical perfection can be referred to as the main quality constituent created without increase in weight.

In order to compare airplanes with each other, the method of qualitative estimation was developed [2] with adjustments and modification described below.

Payload operating efficiency was selected as the integrated index of technical perfection: $\bar{m}_{\mathrm{pl}} = \frac{m_{\mathrm{pl}}}{m_0}$, where m_{pl} was the payload; m_0 was the airplane takeoff weight calculated on the hypothesis that up till that time its increase was based only on achievements of science and technology.

According to definition of technical level and applying the gradient method including the influence of different payload and takeoff weights of the considered and basic airplane, the general equation was derived for determination of generalized index of technical level:

$$\begin{aligned} W_{\mathrm{TL}} &= \frac{W_{\mathrm{TP}}}{W_{\mathrm{TP.B}}} = \frac{\bar{m}^*_{\mathrm{pl}}}{\bar{m}_{\mathrm{pl.B}}} = 1 - \frac{\Delta \bar{m}^*_{\mathrm{pl}}}{\bar{m}_{\mathrm{pl.B}}} = \\ &= 1 + \frac{1}{\bar{m}_{\mathrm{pl.B}}} \sum_i \frac{\partial \bar{m}_{i\mathrm{B}}}{\partial w_{i\mathrm{B}}} \left(w_i - w_{i\mathrm{B}} \right) - \frac{m_0 - m_{0\mathrm{B}}}{m_{0\mathrm{B}}} + \frac{m_{\mathrm{pl}} - m_{\mathrm{pl.B}}}{m_{\mathrm{pl.B}}} \end{aligned}, \tag{1}$$

where the index "B" here and further was applied to basic airplane (the basic airplane was selected from group of airplanes of similar purpose, similar takeoff weight and operation conditions); $\Delta \bar{m}^*_{\mathrm{pl}}$ was the variation of relative payload weight of the considered airplane with regard to basic one related only with those variations which were obtained due to variation of specific parameters (for instance, aerodynamic quality, specific fuel consumption per hour, specific weight of engines); $\bar{m}_{i\mathrm{B}} = \frac{m_{i\mathrm{B}}}{m_0}$ was the relative weight of the i-th constituent of takeoff weight of the basic airplane; w_i was the value of the i-th characteristic.

3. RESULTS AND DISCUSSION

While determining partial derivatives $\frac{\partial \bar{m}_{iB}}{\partial w_{iB}}$ in Eq. (1) aiming at improvement, the most simple characteristics should be selected which do not require for implementation of R&D achievements, provided that without such implementation the increase in the generalized index of technical level is compensated by the two latter right-side terms in Eq. (1) containing m_0 and m_{pl}.

According to the aforementioned, let us apply the following improvements of the most important characteristics of airplane estimation:

– in order to increase flight distance with maximum payload weight L: increase in fuel weight;

– in order to increase cruise speed V: increase of available thrust of propulsion unit;

– in order to reduce the required runway length (RW) l_{rw}: decrease in load on wing due to increase of wing surface area.

Let us determine consecutively the respective partial derivatives. In order to determine the influence of flight distance on the index of technical level, let us find $\frac{\partial \bar{m}_{m.B}}{\partial L_B}$. Let us apply the known equation of relative fuel weight:

$$\bar{m}_m = 1 - \exp\left(-\frac{Lc_{sp}}{KV}\right) \approx \frac{Lc_{sp}}{KV},$$

where c_{sp} is the specific fuel consumption per hour; K is the airplane aerodynamic quality.

Then $\frac{\partial \bar{m}_m}{\partial L} = \frac{c_{sp}}{KV}$. Hence, $\frac{\partial \bar{m}_{m.B}}{\partial L_B} = \frac{\bar{m}_{m.B}}{L_B}$.

In order to determine the influence of cruise speed on the index of technical level, let us find $\frac{\partial \bar{m}_{PU.B}}{\partial V_B}$. Under the condition of stationary flight, the thrust P equals to drag component:

$$P = c_{xa}\frac{\rho V^2}{2}S \tag{2}$$

where c_{xa} is the coefficient of drag component; ρ is the air density; S is the reference area.

The statistic equation for determination of relative weight of propulsion unit is known:

$$\bar{m}_{PU} = k_{PU}\gamma_{eng}\bar{P} \tag{3}$$

where k_{PU} is the coefficient showing what fold is the weight of propulsion unit with regard to the weight of engines; γ_{en} is the specific weight of engines.

Let us present the start thrust with consideration for Eq. (2) as follows:

$$\bar{P}_0 = \frac{P_0}{m_0 g} = \frac{kP}{m_0 g} = kc_{xa}\frac{\rho V^2}{2m_0 g}S, \tag{4}$$

where k is the coefficient showing what fold is the thrust during stationary flight with regard to the start thrust.

Let us rewrite Eq. (3) with consideration for Eq. (4):

$$\bar{m}_{PU} = k_{PU}\gamma_{eng}kc_{xa}\frac{\rho V^2}{2m_0 g}S. \text{ Then, } \frac{\partial \bar{m}_{PU}}{\partial V} = k_{PU}\gamma_{eng}kc_{xa}\frac{\rho V}{m_0 g}S\cdot$$

It means that $\frac{\partial \bar{m}_{PU.B}}{\partial V_B} = 2\frac{\bar{m}_{SU.B}}{V_B}$. In order to determine the influence of the required runway length on the index of technical level, let us find the partial derivative $\frac{\partial \bar{m}_{w.B}}{\partial l_{\mathrm{RW.B}}}$. Analysis of the equations for prediction of ground and air operating distances of takeoff and landing demonstrates that the lengths of these segments are inversely proportional to specific load on wing and at steady takeoff weight to wing surface area S_w. Hence, it is possible to write as follows:

$$l_{RW} = \frac{k_l}{S_w} \tag{5}$$

where k_l is the coefficient of proportionality independent of S_w.

For cargo airplanes the relative weight of wing structure as a function of its surface area is written as follows:

$$\bar{m}_w = k_m S_w \tag{6}$$

where k_m is the coefficient of proportionality independent of wing surface area S_w.

Let us express S_w from Eq. (5) and substitute it into Eq. (6): $\bar{m}_w = k_m \frac{k_l}{l_{RW}}$. Then,

$$\frac{\partial \bar{m}_w}{\partial l_{RW}} = -\frac{k_m k_l}{l_{RW}^2}. \text{ Hence, } \frac{\partial \bar{m}_{w.B}}{\partial l_{RW.B}} = -\frac{\bar{m}_{w.B}}{l_{RW.B}}.$$

On the basis of the aforementioned the mathematical description of estimation of flight characteristics of civil airplanes is as follows:

$$\begin{cases} W_F = \dfrac{m_F}{m_{pl} L} \\ W_w = \dfrac{m_{pl}}{m_0} \\ W_{tr} = \dfrac{m_{pl} V_{fl} L}{m_f m_{oew}} \\ W_{TL} = 1 + \dfrac{\bar{m}_{f.B}}{\bar{m}_{pl.B}}\left(\dfrac{L - L_B}{L_B}\right) + 2\dfrac{\bar{m}_{PU.B}}{\bar{m}_{pl.B}}\left(\dfrac{V - V_B}{V_B}\right) - \dfrac{\bar{m}_{w.B}}{\bar{m}_{pl.B}}\left(\dfrac{l_{RW} - l_{RW.B}}{l_{RW.B}}\right) - \\ \quad - \dfrac{m_0 - m_{0B}}{m_{0B}} + \dfrac{m_{pl} - m_{pl.B}}{m_{pl.B}} \end{cases},$$

where W_f is the fuel efficiency; W_w is the weight efficiency; W_{tr} is the target (transport) efficiency; W_{TL} is the generalized index of technical level; m_f is the fuel weight required for the flight; L is the flight distance at maximum payload; m_0 is the takeoff weight; m_{pl} is the maximum payload; m_{oew} is the airplane operating empty weight; V_{fl} is the flight speed (accounts for time loss for engine start-up and warm-up, taxying, takeoff and ascending, descending and landing); V is the flight cruise speed; l_{RW} is the required runway distance; $\bar{m}_{pl.B} = \frac{m_{pl.B}}{m_{0B}}$, $\bar{m}_{f.B} = \frac{m_{f.B}}{m_{0B}}$, $\bar{m}_{PU.B} = \frac{m_{PU.B}}{m_{0B}}$, $\bar{m}_{w.B} = \frac{m_{w.B}}{m_{0B}}$ are the relative weights of maximum payload, fuel, propulsion unit, and wing structure of basic airplane, respectively.

The described method was applied for estimation of flight characteristics of medium and long range civil airplanes of various generation of leading world manufacturers (first of all Boeing and Airbus). The specifications of the considered airplanes were obtained on the websites of the manufacturers [11–15]. The detailed predictions of technical efficiency and technical level are presented in [7, 8]. The following conclusions can be derived from their analysis.

1) Fuel efficiency increases which can be attributed mainly to improved specifications of propulsion units in terms of specific fuel consumption, since each subsequent generation of airplanes is equipped with more perfect engines. This conclusion is confirmed by approximately equal fuel efficiency for one generation of airplanes of similar purpose. It should be considered

that the fuel efficiency includes transportation work which is determined by the product of maximum payload and flight distance. Both these values increase continuously with the increase in maximum takeoff weight, which also improves fuel efficiency.

2) Weight efficiency is nearly constant which probably can be attributed to features of payload.

3) Target efficiency increases constantly which can be attributed first of all to increase in transportation work. Less intensive growth of target efficiency in comparison with fuel efficiency indicates the absence of improvement of airplane technical perfection determined in this case by airplane operating empty weight.

4) Technical level demonstrates future improvement which is determined by known achievements in the field of engine manufacturing, aviation materials, control systems, etc. The rates of improvement of technical level of current generation of medium range airplanes (Boeing 737 MAX and Airbus 320 NEO) are higher in comparison with the previous generation (737 NG and 320 CEO). Fuel efficiency of current airplane generation is higher mainly due to more efficient propulsion units which, however, are characterized by higher weight and dimensions, which finally does not provide increase in technical perfection.

4. CONCLUSION

The described method of estimation of flight characteristics is reliable, since it is based only on physical and practically proved statistic correlations, the results obtained on its basis agree with the trends of technological level in time due to application of advanced R&D results in aviation, which at least evidences its consistency. Herewith, the above conclusions can be achieved only on the basis of combined indices of technical efficiency and technical level, since the increase in indices of technical efficiency does not permit to make conclusion about improvement of specifications in combination, that is, about improvement of airplane technical perfection. Technical efficiency can estimate only individual properties (each specified index takes into account not all characteristics determining the considered properties), and technical level makes it possible to estimate the achieved level of combination of aircraft specifications and implemented innovations [16].

The efficiency indices are based mainly on one principle: the parameters determining positive effect are directly proportional and those determining negative effect are inversely proportional, which does not permit to consider for the degree of interrelation between the parameters. The equation for calculation of generalized index of technical level was derived on the basis of Mozhaysky formula (equation of weight balance) and known interrelations between the considered characteristics and mass input for their achievement.

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