HEAT TRANSFER AND FLUID FLOW CHARACTERISTICS OF 
VERTICAL SYMMETRICAL TRIANGULAR FIN ARRAYS

N.G.Narve¹, N.K.Sane² 
LNBCIE & T, Satara, Kolhapur, India 
JSCOE, Hadapsar, Pune, India

ABSTRACT

This paper deals with study of heat transfer and fluid flow characteristics of natural convection heat flow through vertical symmetrical triangular fin arrays. It was studied numerically and its results were compared with equivalent rectangular fin arrays. In the numerical arrangement, spacing between fins was varied. Results were generated for \( S^+ = 0.5 & 0.105 \) and \( Gr_H = 10^6 \) to \( 10^8 \). Average, base Nusselt number and Grashof number were calculated. It was observed that with increase in Grashof number, average and base Nusselt number increases. Similarly average Nusselt number increases with spacing whereas base Nusselt number increases to maximum value with spacing and then decreases [11].

Keywords: Fin arrays, Grashof number, Heat transfer, Natural convection, Spacing.

I INTRODUCTION

Many proposed applications of electronic and thermo electric devices depend upon the feasibility of rejecting waste heat by economical, trouble free methods. For these applications, better utilization of the available heat rejection area may be realized by the proper application of outstanding fins.

Fins are extended surfaces used to improve the overall heat transfer rate when it is limited by low rate between a solid surface and surrounding fluids. Fins provide larger surface area for heat dissipation. Fins are casted or fabricated by pressing, soldering or welding. Fins find application in variety of fields of which some are-
(i) The heads and cylinders of the air cooled engines and compressors
(ii) Electronic components such as power diodes, transistors etc.
(iii) Tubes of various heat exchangers for example condenser tubes of domestic refrigerators, radiators of automobiles.
(iv) Outside surfaces of the cooling and dehumidifying coils in the air conditioning systems.
(v) Direct energy conversion devices.
(vi) Nuclear fuel modules.
(vii) Chemical and Cryogenic equipments.
(viii) Conventional furnaces and gas turbine.

There are different types and shapes of fins used in practice. Fins are used on plane surfaces or cylindrical surfaces. Fins may be of having different cross sections. Depending on cross section we may have rectangular, parabolic or triangular fins.

The heat can be removed effectively if the fluid flow and the resulting flow pattern are capable of removing the heat efficiently. The heat dissipation from fins under natural convection condition depends on the geometry and orientation of finned surface.

The literature survey revealed that the problem of free convection heat transfer from vertical fin arrays has been investigated by a few investigators. Elenbass [1] had done extensive work on channels and parallel plates on experimental and semi-empirical basis. Starnner and Mcmanus [2] presented free convection data for four rectangular arrays in three positions including vertical position for the fin base. Similarly experimental work for vertical fin arrays are carried out [3-8]. Theoretical work on rectangular fin arrays for natural convection is also reported [9,10].

In the present work, the free convection heat transfer from isothermal vertical symmetric triangular fin arrays was analyzed theoretically. It is proved by many investigators that free convection heat transfer from vertical triangular fin array results in the single chimney flow pattern.

The object of this theoretical study is to determine the local, average and base Nusselt number for free convective heat transfer from vertical triangular fin arrays along with stream function and velocity distribution. In order to achieve this objective the set of differential equations governing the fluid flow and heat transfer are to be used. These are derived from fundamental laws. Then its comparison was done with equivalent rectangular fins arrays. In both cases spacing was the variable.

II FORMULATION OF THE PROBLEM

2.1 Statement of the problem

The vertical symmetric triangular fin array to be analyzed is shown in Fig. 1. It consists of large number of vertical triangular fins of height ‘H’ and length ‘L’. The spacing between two adjacent fin flats is ‘S’. Each array has number of fin channels. The
assumption of the entire fin array to be isothermal is supported by fin material having high thermal conductivity, so that the fin flats and fin base are at same temperature $T_b$, where $T_b$ is the temperature of the base. The fin is surrounded by fluid at ambient temperature and all fluid properties are considered as constant. Due to large number of fin channels, the end effects can be neglected.

2.2 Domain of Interest

From Fig.1, it is clear that symmetry is in the Z direction. Due to large number of channels, only a single fin channel needs to be analyzed and due to symmetry, only half the vertical symmetric triangular fin channel ABCDEF is chosen and is the domain of interest. It is surrounded by solid wall viz. fin flat and fin base and planes of opening i.e. at the top, bottom and side of channel.

![Figure 1 Domain-Half Symmetrical Triangular fin duct](image)

2.3 Governing equations

Following are the fundamental equations used for analyzing and solving the problem.

Continuity equation:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]  

--------[1]
Momentum equation

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + g \beta (T - T_a) \] ----[2]

\[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + v \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \] ----[3]

\[ u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + v \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \] ----[4]

Energy Equation

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = k \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \] ----[5]

Substituting \( w = 0 \), following equations are simplified,

\[ \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \] ----[6]

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + g \beta (T - T_a) \] ----[7]

\[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + v \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \] ----[8]

\[ \frac{\partial p}{\partial z} = 0 \] ----[9]

Pressure term is eliminated in above equation by cross differentiation and subtracting (7) from equation (8) and equations are non-dimensionised by using characteristic dimension ‘H’

\[ X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad Z = \frac{z}{H} \]

\[ \theta = \frac{T - T_a}{T_b - T_a}, \quad U = \frac{uH}{\theta}, \quad V = \frac{vH}{\theta}, \quad Pr = \frac{C_p \mu}{k} \]

Dimensionless vorticity is defined by -

\[ U = \frac{\partial \psi}{\partial v}, \quad V = -\frac{\partial \psi}{\partial x} \text{ and } \omega = \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} \]

The basic governing equation can be represented in the following way-

Energy equation:

\[ U \frac{\partial \theta}{\partial x} + V \frac{\partial \theta}{\partial y} = -\frac{1}{\text{Pr}} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) \] ----[10]

Vorticity Transport Equation Equation
\[ U \frac{\partial \omega}{\partial x} + V \frac{\partial \omega}{\partial y} = \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} + \frac{\partial^3 \omega}{\partial z \partial y} - \text{GrH} \frac{\partial \theta}{\partial y} \]  

-----[11]

\[ \omega = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \]  

-----[12]

Above set of (10) to (12) governing equations are partial differential equations and their simultaneous analytical solution is not possible. So finite difference technique is used to solve these equations. The second order terms are replaced by central differences while non-linear convective terms are replaced by upwind difference procedure. These form three algebraic equations with three unknowns at each nodal point in the grids.

2.4 Fixing of Boundary Conditions

The array is having opening from the top, bottom and side, therefore no definite boundary conditions can be assumed at these open surfaces. Therefore an attempt has been made to accommodate these open boundaries by extending them, a certain distance away from channel. At these extended top, bottom and side boundary surfaces, ambient conditions can be assumed. This approach has been made previously many investigators.

In the bottom region, below the channel, incoming flow of air is assumed at ambient temperatures. Above the temperature at the side entrance of channel has been assumed to be equal to ambient value.

2.5 Calculation of local and average Nusselt number

The expression for local Nusselt number can be obtained as follows:-

The heat transfer coefficient at the fin base surface is given by

\[ h_b = k \cdot \frac{\partial \theta}{\partial y} \text{ at } Y=0 \]

The local Nusselt number for the fin base is defined

\[ Nu_b = h_b \cdot H/k = H \cdot \frac{\partial \theta}{\partial y} \text{ at } Y=0 \]

Similarly the local Nusselt number for the fin flat is

\[ Nu = - \frac{\partial \theta}{\partial z} \text{ at } Z=0 \]

The temperature gradient \( \frac{\partial \theta}{\partial y} \) at \( Y=0 \) and \( - \frac{\partial \theta}{\partial z} \) at \( Z=0 \) are obtained by using a five point numerical differentiation based on Taylor’s expansion series[8]. Then average Nusselt number for the entire fin array is obtained by numerically integrating the local Nusselt number over the surface of the fin array.

III RESULTS AND DISCUSSIONS

The numerical analysis using computational technique is applied to symmetrical triangular and equivalent rectangular fine arrays. The solutions are obtained for different values of dimensionless parameters of S+ and GrH for constant L+. The results are generated for following values of parameters-

L+ - 0.5, and S+ - 0.105, 0.5
GrH -10^6 to 10^8 and Pr- 0.7
The results are obtained in terms of distribution of stream function, U and V velocities within the domain of interest. These gives the nature of fluid flow and heat transfer characteristic The heat transfer rate for the fin arrays is effectively studied in average and base Nusselt number.

### 3.1 FLUID FLOW CHARACTERISTICS

Fluid flow characteristics describe the flow pattern of air in terms of stream line contours and the movement of air in X and Y direction expressed in U and V velocity.

#### 3.1.1 Stream Function

Stream lines describe the actual flow of the air inside the entire domain. They form closed contours satisfying the continuity equation. The stream lines shows the flow lines entering the fin channel from the bottom and side of the array developing vertical component of velocity as they approach in the base surface and then resulting in the outgoing main flow from the top of the channel.

Fig. 2 to Fig.5 shows the stream line contours in the entire fin domain at the vertical section of (z=50) for \( Gr_H \) at \( 10^6 \) to \( 10^7 \) for symmetrical triangular and rectangular fin arrays.

It is observed that \( Gr_H \) has a strong influence on the nature of flow in the domain of interest. Overall observation is that the flow lines tend to concentrate at the heated edges. At low Grashof number that is at \( 10^6 \) to \( 10^7 \) stream lines moves away from the vertical base towards vertex of fin flat in case of triangular fin arrays. At these Grashof number, recirculation is formed above the fin array. This negative loop is very weak in the rectangular fin arrays or almost absent. This is due to higher buoyancy towards the vertical base. In this case stream line due not tend to move away from the vertical base but are almost parallel to it giving more uniform flow.

It is observed that values of stream functions are always higher for symmetrical triangular fins than rectangular fins. This seems to be one of the reasons for the better effectiveness of this geometry. It is also seen that when fins spacing is increased the negative loop or recirculation weakens or becomes completely absent.

![Figure 2 Stream line contours for Triangular fin array](image1.png)

![Figure 3 Stream line contours for Rectangular fin array](image2.png)
3.1.2 Distribution of velocity

Velocity distribution describes the type of fluid motion present in the X and Y directions. It also indicates the formation of the boundary layer near the heated surface.

X- component of velocity (U)-

Fig. 6 to Fig.7 shows the U- velocity distribution for symmetrical triangular and rectangular fin arrays. It is seen that U- Velocity increases as the distance X from the bottom of fin increased. In Y direction U- Velocity clearly indicates the formation of boundary layer and its growth adjacent to fin base. It is also observed that as the distance from fin flat increases in Z direction and is maximum at the center of the channel (Z=5). The trend for U- Velocity is almost similar for both arrangements.

Y- Component of velocity (V)-

Fig. 6 to Fig.7 shows the V velocity distribution for symmetrical triangular and rectangular fin arrays. It is seen that as one approaches the fin base in Y direction, the magnitude of velocity decreases because some fluid moves upward due to heating. Same trend is observed in both the arrangements.
3.2 HEAT TRANSFER CHARACTERISTICS

In the following the variation of average and base Nusselt number with the Grashof number and fin spacing is discussed.

3.2.1 Average Nusselt number

The heat transfer rate from the fin array can be calculated by knowing the values of average Nusselt number. Average Nusselt number is found to vary with $S+$ and $Gr_H$.

(a) Effect of Grashof number:

Fig. 7.17 shows the variation of $Nu_a$ with $Gr_H$ for different spacing. It is observed that there is a marked increase in $Nu_a$ with increase in $Gr_H$. The value of $Nu_a$ is higher for symmetrical triangular fin arrays than rectangular fins. Similar trend has been observed for all the spacing.

(b) Effect of Spacing:

Fig. 9 shows variation of $Nu_a$ with $S+$ for one particular Grashof number. This value of $Gr_H$ has been obtained during experiment. As expected, $Nu_a$ increases with spacing because with increased spacing, fluid flow through the fin channel more freely without any interference. The values of $Nu_a$ are higher for symmetrical triangular fins than rectangular fin for the given spacing.
3.2.2 Base Nusselt number

Base Nusselt number is found to vary with Grashof number and fin spacing.

(a) Effect of spacing:

Fig 10 shows variation of Nu_b with S+ for assumed value of Gr_H. The peak value of curve indicates optimum fin spacing. This plot confirm the postulate that the peak value of Nu_b is obtained for the spacing at which transition just starts from single chimney flow pattern to some other disturbed flow.
NOMENCLATURE

A Area, $m^2$
Gr Grashof number
g Acceleration due to gravity, $m/s^2$
h_a Average heat transfer coefficient, $m^2K$
h_b Base heat transfer coefficient, $W/mK$
k Thermal conductivity of the air, $W/mK$
L Length of fins, $m$
$L^+$ L/H, length to height ratio
$Nu_a$ Average Nusselt number
$Nu_b$ Base Nusselt number
Pr Prandtl number
S S/H, spacing to height ratio
S Spacing between the fins
X,Y,Z Cartesian coordinates

Greek Symbols

$\beta$ Volumetric expansion coefficient, $K^{-1}$
$\mu$ Dynamic viscosity of air, $N\cdot s/m^2$
$\nu$ Kinematic viscosity of air, $m^2/s$
$\rho$ Density of air, $kg/m^3$

Superscripts/Subscripts

a Average value, ambient
b Base value
H Height of fins, $m$
m Mean film

REFERENCES