NUMERICAL SIMULATION OF FLOW MODELING IN DUCTED AXIAL FAN USING GAUSSIAN QUADRATURE METHOD

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ABSTRACT

The paper presents to develop the numerical simulation of flow model for three dimensional flows and one dimensional flow in Ducted Axial Fan by using the numerical integral procedure of Three Point Gaussian Quadrature methods. Main objective of this paper is to develop the numerical flow model and measure the pressure rise for varying the functional parameters of inlet velocity, whirl velocity, rotor speed and diameter of blade from hub to tip in ducted axial fan by using the code of MATLAB for Gaussian Quadrature techniques. In this main phase of paper, the analogy of three dimensional flow and one dimensional flow of numerical flow modeling have been investigated to optimize the parameter of pressure rise in ducted axial flow fan by using three point Gaussian Quadrature methods.

Keywords: Numerical Integration, Gaussian Quadrature, Pressure rise, Whirl velocity, Pressure Ratio, Flow ratio, Rotor speed, Axial Fan.

1.0 INTRODUCTION

Mining fans and cooling tower fans normally employ axial blades and or required to work under adverse environmental conditions. They have to operate in a narrow band of speed and throttle positions in order to give best performance in terms of pressure rise, high efficiency and also stable condition. Since the range in
which the fan has to operate under stable condition is very narrow, clear knowledge has to be obtained about the whole range of operating conditions if the fan has to be operated using active adaptive control devices. The performance of axial fan can be graphically represented as shown in figure 1.

![Performance curve of Axial Fan](image)

**Fig: 1 Graphical representation of Axial Fan performance curve**

### 2. TEST FACILITY AND INSTRUMENTATION

Experimental setup, fabricated to create stall conditions and to introduce un stalled conditions in an industrial ducted axial fan is shown in figure 2.

![Ducted Axial Fan Rig](image)

**Fig: 2 Ducted Axial Fan Rig**

A 2 HP Variable frequency 3-phase induction electrical drive is coupled to the electrical motor to derive variable speed ranges. Schematic representation of ducted fan setup is shown in figure 3.
The flow enters the test duct through a bell mouth entry of cubic profile. The bell mouth performs two functions: it provides a smooth undisturbed flow into the duct and also serves the purpose of metering the flow rate. The bell mouth is made of fiber reinforced polyester with a smooth internal finish. The motor is positioned inside a 381 mm diameter x 457 mm length of fan casing. The aspect (L/D) ratio of the casing is 1.2. The hub with blades, set at the required angle is mounted on the extended shaft of the electric motor. The fan hub is made of two identical halves. The surface of the hub is made spherical so that the blade root portion with the same contour could be seated perfectly on this, thus avoiding any gap between these two mating parts. An outlet duct identical in every way with that at inlet is used at the downstream of the fan. A flow throttle is placed at the exit, having sufficient movement to present an exit area greater than that of the duct.

3.0 NUMERICAL ANALYSIS

Numerical analysis is the study of algorithms that use numerical approximation for the problems of mathematical analysis. In Numerical algorithm is a step-by-step procedure for calculations. Algorithms are used for calculation, data processing, and reasoning. More precisely, an algorithm is an effective method expressed as a finite list of well-defined instructions for calculating a function. Starting from an initial state and initial input for the instructions describe a computation that, when executed, will proceed through a finite number of well-
defined successive states, eventually producing output and terminating at a final ending state. Mathematical analysis, which mathematicians refer to simply as analysis, is a branch of pure mathematics that includes the theories of differentiation, integration and measure, limits, infinite series, and analytic functions.

3.1 MATLAB

MATLAB is a programming environment for algorithm development, data analysis, visualization, and numerical computation. MATLAB is a wide range of applications, including signal and image processing, communications, control design, test and measurement, financial modeling and analysis, and computational biology. MATLAB contains mathematical, statistical, and engineering functions to support all common engineering and science operations. These functions, developed by experts in mathematics, are the foundation of the MATLAB language. The core math functions use the LAPACK and BLAS linear algebra subroutine libraries and the FFT Discrete Fourier Transform library. Because these processor-dependent libraries are optimized to the different platforms that MATLAB supports, they execute faster than the equivalent C or C++ code.

4.0 FLOW MODELING

The aim of the flow modeling is to measure the pressure rise as a function of whirl velocity and rotor speed for different diameter of blade from hub to tip in ducted axial flow fan. In this flow modeling equation helps to optimize the parameter of pressure rise for different whirl velocity in a ducted axial fan.

4.1 Three Dimensional Flow Equation

\[(p + dp)(r + dr)d\theta - p r d\theta - \left( p + \frac{1}{2} dp \right) dr \ d\theta = dm \ \frac{c_\theta^2}{r} \] 
\[
\frac{1}{\rho} \frac{dp_\theta}{dr} = c_x \ \frac{dc_x}{dr} + \frac{c_\theta}{r} \ \frac{d}{dr} \ (r \ c_\theta) \ ]
\[
\text{Stagnation pressure rise} = \frac{1}{\rho} \frac{dp}{dr} + v_a \ \frac{dv_a}{dr} + v_w \ \frac{dv_w}{dr} \]
\[
p(V_w . U) . dr - V_a \ . dV_a - V_w \ . dV_w = dp \]
Stagnation Pressure rise = Total Input Power = 500 Watts as an assumption for this Analytic and simulation studies of ducted Axial Fan. If integrate that equation 4.4, obtain that final form of equation.

\[
500 \left( r_2 - r_1 \right) \left[ \left( \frac{v^2}{2} \right) r_2 - \left( \frac{v^2}{2} \right) r_1 \right] - \left[ \left( \frac{\omega^2}{2} \right) r_2 - \left( \frac{\omega^2}{2} \right) r_1 \right] = \frac{1}{\rho} \left[ dp \right] \quad (4.5)
\]

4.2 One Dimensional Flow Equation

Pressure rise \( dp = \rho \frac{v^2}{r} \, dr \)

Pressure Rise \( = \rho \left( \frac{\psi \times 3.14 \times N}{60} \right)^2 \times \int r \, dr \) \quad (4.6)

5.0 NUMERICAL MODELING

From this flow modeling equation, adopt and continue the procedure of numerical integration scheme. In this numerical integration procedure, MATLAB code has computed for developing the pressure rise in ducted axial fan using three point Gaussian quadrature methods.

**Three Point Gaussian Quadrature Integration Method**

\[
\int_a^b f(x) \, dx = \int_{-1}^1 f(t) \, dt \quad (5.1)
\]

Where the interval \((a,b)\) is changed into \((-1,1)\) by the transformation.

\[
X = \frac{b+a}{2} + \left( \frac{b-a}{2} \right) \, t
\]

Where \( A_1 = A_2 = 0.55555 \)

\( A_3 = 0.8888 \)

\( t_1 = -0.7745 \)

\( t_2 = 0 \)

\( t_3 = 0.77745 \)

Then \( \int_{-1}^1 f(t) \, dt = A_1 f(t_1) + A_2 f(t_2) + A_3 f(t_3) \) \quad (5.2)

5.1 MATLAB CODE FOR THREE DIMENSIONAL FLOWS USING THREE POINT GAUSSIAN QUADRATURE METHODS

Objective

To compute the pressure rise in three dimensional flow equations by using Three Point Gaussian Quadrature Methods.
%!('Gaussian three point quadrature method for Three Dimensional Flow Equations');
 for i=0:1:2;
 t=[-0.7745:0.7745:0.7745];
m=(1.4846+(0.62553*t));
disp(m);
 end
A=0.5555;
B=0.5555;
C=0.8888;
y=A*m(i)+B*m(i)+C*m(i);
disp(y);
r2=input('enter the final radius');
r1=input('enter the initial radius');
res= 500*(r2-r1);
disp(res);
ans=res-y;
disp(ans);
PR=1.048*ans;
fprintf(',Pressure Rise in N/m2=%g',PR)

MATLAB RESULTS FOR GAUSSIAN THREE POINT QUADRATURE METHOD

1.0001  1.4846  1.9691

1.0001  1.4846  1.9691

1.0001  1.4846  1.9691

2.9689

enter the final radius 0.186
enter the initial radius 0.08255
51.7250

48.7561

Pressure Rise in N/m² = 51.0964
5.2 MATLAB CODE FOR ONE DIMENSIONAL FLOW EQUATION USING THREE POINT GAUSSIAN QUADRATURE METHOD

Objective
To compute the pressure rise in one dimensional flow equation by using Three Point Gaussian Quadrature Method

%('Gauss three point quadrature rule for One Dimensional Flow Equation');
for i=0:1:2;
    t=[-0.7745:0.7745:0.7745];
    m=(0.007171+(0.00281*t));
    disp(m);
end
A=0.5555;
B=0.5555;
C=0.8888;
y=A*m(i)+B*m(i)+C*m(i);
disp(y);
PR=1.048*325.89*y;
fprintf(',Pressure Rise in N/m2=%g',PR)

MATLAB RESULTS FOR GAUSSIAN THREE POINT QUADRATURE

<table>
<thead>
<tr>
<th>0.0050</th>
<th>0.0072</th>
<th>0.0093</th>
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Pressure Rise in N/m² = 4.89777

5.3 CALCULATION OF ERROR PERCENTAGE

Case 1: Three Dimensional Flows
Theoretical value of pressure rise for three dimensional flow of ducted Axial Fan = 51.5308 N/m²
Numerical Modeling of pressure rise for three dimensional flow of ducted axial fan using Three Point Gaussian Quadrature method = 51.0964 N/m²
Error Percentage = \[
\frac{\text{Theoretical Value} - \text{Numerical modeling Value}}{\text{Theoretical Value}} \quad (5.3)
\]
Error Percentage \( = \frac{51.5308 - 51.0964}{51.5308} \times 100 \)

\textbf{Error Percentage (\% ) = 0.843}

\textbf{Case 2: One Dimensional Flow}

Theoretical value of pressure rise for one dimensional flow of ducted
Axial fan = 5.034 N/m\(^2\)

Numerical Modeling of pressure rise for one dimensional flow of ducted
Axial fan using Three Point Gaussian Quadrature method = 4.89777 N/m\(^2\)

\text{Error Percentage} = \frac{\text{Theoretical Value} - \text{Numerical modeling Value}}{\text{Theoretical Value}} \quad (5.4)

Error Percentage \( = \frac{5.034 - 4.89777}{5.034} \times 100 \)

\textbf{Error Percentage (\% ) = 2.71}

\textbf{6.0 CONCLUSION}

In this paper, an attempt has been made to develop the numerical simulation of three dimensional flows and one dimensional flow code using MATLAB in Three Point Gaussian Quadrature techniques for ducted axial fan. It is useful to design the operating condition of axial fan to measure the parameters of pressure rise as a function of pressure ratio, rotor speed, and diameter of blade from hub to tip in ducted axial fan. Further, this work can be extended by working on the flow simulation characteristic study in control system algorithm. The results so far discussed, indicate that numerical simulation of flow modeling using three point Gaussian quadrature method for ducted axial fan is very promising.

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\textbf{NOMENCLATURE}

\begin{align*}
c_x & = \text{Axial velocity in m/s} \\
c_\theta & = \text{Whirl velocity in m/s} \\
r_2 & = \text{Radius of blade tip in m} \\
r_1 & = \text{Radius of blade hub in m} \\
N & = \text{Tip speed of the blades in rpm} \\
v_a & = \text{Axial velocity in m/s} \\
dp & = P_2 - P_1 = \text{Pressure rise in N/m}^2
\end{align*}
\begin{align*}
d &= \text{Diameter of the blade in m} \\
\rho_{\text{air}} &= \text{Density of air in kg/m}^3 \\
v_w &= \text{Whirl velocity in m/s} \\
\eta &= \text{Efficiency of fan}
\end{align*}

REFERENCES


AUTHORS

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