EFFECT OF VISCOSOUS DISSIPATION ON MHD FLOW AND HEAT TRANSFER OF A NON-NEWTONIAN POWER-LAW FLUID PAST A STRETCHING SHEET WITH SUCTION/INJECTION

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ABSTRACT

This paper deals with the MHD effects on convection heat transfer of an electrically conducting, non-Newtonian power-law stretched sheet with surface heat flux by considering the viscous dissipation. The effects of suction/injection at the surface are considered. The resulting governing equations are transformed into non linear ordinary differential equations using appropriate transformation. The set of non linear ordinary differential equations are first linearized by using Quasi-linearization technique and then solved numerically by using implicit finite difference scheme. Then the system of algebraic equations is solved by using Gauss-Seidal iterative method. The solution is found to be dependent on six governing parameters including the magnetic field parameter $M$, the power-law fluid index $n$, the sheet velocity exponent $p$, the suction/blowing parameter $f_w$, Eckert number $Ec$ and the generalized Prandtl number $Pr$. Numerical results are tabulated for skin friction co-efficient and the local Nusselt number. Velocity and Temperature profiles drawn for different controlling parameters reveal the tendency of the solution.

Keywords: MHD, Power-law stretched sheet, suction/injection, Surface heat flux and viscous dissipation.

INTRODUCTION

Most of the fluids such as molten plastics, artificial fibers, drilling of petroleum, blood and polymer solutions are considered as non-Newtonian fluids. In modern technology and in industrial applications, non-Newtonian fluids play an important role. Increasing emergence of non-Newtonian fluids such as molten plastic pulp, emulsions, raw materials in a great variety of industries like petroleum and chemical processes has stimulated a considerable amount of interest in the study of the
behavior of such fluids when in motion. Exact solutions of the equations of motion of non-Newtonian fluids are difficult. The difficulty arises not only due to the non-linearity but also due to increase in the order of differential equations. Many researchers have attempted to find the exact solution of non-Newtonian fluids.

The study of flow and heat transfer problems due to stretching boundary has many practical applications in technological processes, particularly in polymer processing systems involving drawing of fibers and films or thin sheets, etc. Sometimes the polymer sheet is stretched while it is extruded from a die. Usually the sheet is pulled through the viscous liquid with desired characteristics. The moving sheet may introduce a motion in the neighboring fluid or alternatively, the fluid may have an independent forced convection motion which is parallel to that of the sheet. Sakiadis [1] was the first to investigate the flow due to sheet issuing with constant speed from a slit into a fluid at rest. Schowalter [2] has introduced the concept of the boundary layer in the theory of non-Newtonian power-law fluids. Acrivos, Shas and Petersen [3] have investigated the steady laminar flow on non-Newtonian fluids over a plate.

MATHEMATICAL FORMULATION

Let us consider a steady two-dimensional flow of an incompressible, electrically conducting fluid obeying the power-law model past a permeable stretching sheet. The origin is located at the slit through which the sheet is drawn through the fluid medium, the x-axis is chosen along the sheet and y-axis is taken normal to it. This continuous sheet is assumed to move with a velocity according to a power-law form, i.e. \( u = C x^p \), and be subject to a surface heat flux. Also, a magnetic field of strength \( B \) is applied in the positive y-direction, which produces magnetic effect in the x-direction. The magnetic Reynolds number is assumed to be small so that the induced magnetic field is negligible in comparison to the applied electric field and the Hall Effect is neglected.

Under the foregoing assumptions and invoking the usual boundary layer approximations, the problem is governed by the following equations:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

\[
u \frac{\partial u}{\partial x} + \nu \frac{\partial U}{\partial y} = \frac{K}{\rho} \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y}^{n+1} \right) - \frac{\sigma B^2 u}{\rho}
\]  

\[
u \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{K}{\rho \alpha} \frac{\partial u}{\partial y}^{n+1}
\]

Where \( u \) and \( v \) are the velocity components, \( T \) is the temperature, \( B \) is the magnetic field strength, \( K \) is the consistency coefficient, \( n \) is the flow behavior index, \( \rho \) is the density, \( \sigma \) is the electrical conductivity and \( \alpha \) is the thermal diffusivity. The appropriate boundary conditions are given by

\[
u_0(x) = C x^p, \quad v = v_w, \quad \frac{\partial T}{\partial y} = -\frac{q_w}{k} \quad \text{at} \ y = 0, \ x > 0
\]

\[u \rightarrow 0, \quad T \rightarrow T_\infty \text{as} \ y \rightarrow \infty
\]

where \( v_w \) is the surface mass flux and \( q_w \) is the surface heat flux. It should be noted that positive \( p \) indicates that the surface is accelerated while negative \( p \) implies that the surface is decelerated from the slit. Also note that positive \( v_w \) is for fluid injection and negative for fluid suction at the sheet surface.

METHOD OF SOLUTION

We shall further transform equations (2) & (3) into a set of partial differential equations amenable to a numerical solution. For this purpose we introduce the variables
\[ \eta = \left( \frac{C^{2-n}}{K/\rho} \right)^{1/(n+1)} x^{(p(2-n)-1)/(n+1)} y \]  \hspace{1cm} (6) \\

\[ \psi = \left( \frac{C^{1-2n}}{K/\rho} \right)^{-1/(n+1)} x^{(p(2n-1)+1)/(n+1)} f \]  \hspace{1cm} (7) \\

\[ \phi = \frac{(T - T_\infty) \text{Re}_x^{1/(n+1)}}{q_w x/k} \]  \hspace{1cm} (8) \\

Where the dimensionless stream function \( f \) satisfies the continuity equation with \( u = \frac{\partial \psi}{\partial y} \) and \( v = -\frac{\partial \psi}{\partial x} \).

Under the transformations (6), (7) and (8), the differential equations (2) and (3) reduce to

\[ \left( f^n f' \right)'' + \frac{p(2n-1)+1}{n+1} ff'' - p(f')^2 - Mf' = 0 \]  \hspace{1cm} (9) \\

\[ \frac{1}{\text{Pr}} \phi'' + \frac{p(2n-1)+1}{n+1} f\phi' + \frac{p(2n-1)-1}{n+1} f'\phi + Ec f^n = 0 \]  \hspace{1cm} (10) \\

With the boundary conditions

\[ \begin{align*}
\left\{ f'(0) &= 1, f(0) = \frac{n+1}{p(2n-1)+1} f_w, \phi'(0) = -1 \\
f'(\infty) &= 0, \phi(\infty) = 0
\end{align*} \]  \hspace{1cm} (11) \\

Where \( M \) is the magnetic field parameter, \( f_w \) is the suction/injection parameter, \( \text{Pr} \) is the generalized Prandtl number for the power law fluid, \( \text{Ec} \) is the Eckert number and primes indicate the differentiation with respective to \( \eta \). The parameters \( M, f_w, \text{Pr} \) and \( \text{Ec} \) are defined as

\[ M = \frac{\sigma B^2 x}{\rho \mu_w} \], \hspace{2cm} f_w = \frac{\nu_w}{u_w} \text{Re}_x^{1/(n+1)}, \]

\[ \text{Pr} = \frac{x \mu_w}{\alpha}, \hspace{2cm} \text{Ec} = -\frac{u_w^2 k}{x q_w c_p} \text{Re}_x^{1/(n+1)} \]

Where \( \text{Re}_x = \frac{\rho u_w^{2-n} x^n}{K} \) is the local Reynolds number. Note here that the magnetic field strength \( B \) should be proportional to \( x \) to the power \((p-1)/2\) to eliminate the dependence of \( M \) on \( x \), i.e. \( B(x) = B_0 x^{(p-1)/2} \) where \( B_0 \) is a constant. Quantities of main interest include the velocity components \( u \) and \( v \), the skin friction coefficient, viscous dissipation and the local Nusselt number. In terms of the new variables, the velocity components can be expressed as \( u = u_w f' \) and \( v = -u_w \text{Re}_x^{-1/(n+1)} \left( \frac{p(2n-1)+1}{n+1} f + \frac{p(2-n)-1}{n+1} \eta f' \right) \).
The wall shear stress is given by
\[
\tau_w = \left[ K \frac{\partial u}{\partial y} \right]_{y=0}^{n-1} - \rho u_n^2 \frac{1}{Re} \left| f'(0) \right|^{n-1} f''(0)
\]

To solve the system of transformed governing equations (9) and (10) with the boundary conditions (11), first equation (9) is linearized using the Quasi linearization technique. Then equation (9) is changed to
\[
n[F]^{n-1} f'' + [f^*]^{n-1} F'' - [F^*]^{n-1} F'' + \frac{p(2n-1)+1}{n+1} [Ff'' + Ff'' - FF'] - p(2Ff' - (F')^2 - Mf') = 0
\]

(12)

Where F is assumed to be a known function and the above equation can be rewritten as
\[
A_0 f'' + A_1 f'' - A_3 f' + A_4 f = A_5 - A_6 [f^*]^{n-1}
\]

(13)

Where
\[
A_0 = n[F^*]^{n-1}, \quad A_1 = nF'', \quad A_2 = \frac{p(2n-1)+1}{n+1} F',
\]
\[
A_3 = 2pF' - M, \quad A_4 = \frac{p(2n-1)+1}{n+1} F'',
\]
\[
A_5 = n[F^*]^{n-1} F'' + \frac{p(2n-1)+1}{n+1} FF'' - p(F')^2,
\]

Using implicit finite difference formulae, the equations (13) and (10) are transformed to
\[
B_0[i] f[i+2] + B_1[i] f[i+1] + B_2[i] f[i] + B_3[i] f[i-1] = B_4[i]
\]

(14)

\[
D_1[i] \phi[i+1] + D_2[i] \phi[i] + D_3[i] \phi[i-1] + D_4[i] = 0
\]

(15)

where
\[
B_0[i] = 2A_0[i], \quad B_1[i] = 2hA_2[i] - 6A_0[i] - h^2A_3[i],
\]
\[
B_2[i] = 6A_0[i] - 4hA_2[i] + 2h^2A_4[i], \quad B_3[i] = 2hA_2[i] + h^2A_4[i] - 2A_0[i],
\]
\[
B_4[i] = 2h^2 \{ A_2[i] - A_1[i] \} [F'(i)]^{n-1},
\]
\[
D_1[i] = hC_1[i] + 2, \quad D_2[i] = 2h^2C_2[i] - 4, \quad D_3[i] = 2 - hC_1[i],
\]
\[
C_1[i] = \frac{p(2n-1)+1}{n+1} Pr f
\]
\[
C_2[i] = \frac{p(2-n)+1}{n+1} Pr f'
\]

Here ‘h’ represents the mesh size in \( \eta \) direction. The system of equations (14) & (15) are solved under the boundary conditions (11) by Gauss-Seidel iteration method and computations were carried out by using C programming. The numerical solutions of \( f \) are considered as \((n+1)\)th order iterative solutions and \( F \) are the \( n \)th order iterative solutions. After each cycle of iteration the convergence check is performed, and the process is terminated when \( \left| F - f \right| < 10^{-4} \).
SKIN FRICTION

The skin friction coefficient is defined as

\[ C_f = \frac{\tau_w}{\rho u_s^2} = 2 \text{Re}_s^{\frac{1}{(n+1)}} |f^*|^n f^*(0) \]

Or

\[ C_f \text{Re}_s^{\frac{1}{(n+1)}} = 2 |f^*(0)|^{n-1} f^*(0) \]  \hspace{1cm} (16)

HEAT TRANSFER

The local heat transfer coefficient is

\[ h = \frac{q_w}{T_w - T_m} = \frac{k \text{Re}_s^{\frac{1}{(n+1)}}}{\phi(0)} \]

The local Nusselt number is given by

\[ Nu_x = \frac{hx}{k} = \frac{\text{Re}_s^{\frac{1}{(n+1)}}}{\phi(0)} \]

Or

\[ Nu_x \text{Re}_s^{\frac{1}{(n+1)}} = \frac{1}{\phi(0)} \]  \hspace{1cm} (17)

RESULTS AND DISCUSSIONS

The effects of various parameters on the skin friction coefficient \(- f^*(0)\) and the Nusselt number \(1/\phi(0)\) are displayed in the tables. The values for \(- f^*(0)\) are tabulated for various values of \(n\) and \(M\) in Table 1. It can be seen from the table that the value of \(- f^*(0)\) increases as magnetic field parameter \(M\) increases and decreases as power law fluid index \(n\) increases. Table 2 shows the results of heat transfer obtained for a Newtonian fluid in the absence of magnetic field i.e. \(n = 1\) and \(M = 0\). It is obvious from the table that the Nusselt number \(1/\phi(0)\) increases with the increase of Prandtl number \(Pr\) and velocity exponent \(p\). Also the effect of viscous dissipation is to reduce the value of Nusselt number \(1/\phi(0)\). Table 3 lists the calculations for the flow and heat transfer characteristics, including the sheet surface temperature \(\phi(0)\), the skin-friction co-efficient \(C_f \text{Re}_s^{\frac{1}{(n+1)}}\) and the Nusselt number \(Nu_x \text{Re}_s^{\frac{1}{(n+1)}}\) for various values of \(n\), \(M\) and \(f_w\) with \(Pr = 5\) and \(p = 0.5\). It is apparent from this table that the sheet surface temperature increases with increasing the magnetic field parameter \(M\), but it decreases with increasing the suction/injection parameter \(f_w\). With all other parameters fixed the magnitude of skin-friction coefficient increases with increasing the magnetic field parameter due to the fact that the magnetic field retards the fluid motion and thus increases this coefficient. To impose suction is to increase the skin-friction coefficient, but fluid injection decreases it. Also, the local Nusselt number is decreased as a result of the applied magnetic field. The effect of suction \((f_w > 0)\) is found to increase the Nusselt number, whereas injection has the opposite effect. More detailed discussions about the influence of various governing parameters on the local Nusselt number are presented latter by making use of figs. 14-18.
REFERENCES