EFFECT OF LOAD LEVELS ON SIZING AND LOCATION OF CAPACITORS IN DISTRIBUTION SYSTEMS

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ABSTRACT

A distribution system is an interface between the bulk power system and the consumers. Among these systems, radial distributions system is popular because of low cost and simple design. In distribution systems, the voltages at buses decreases proportionally, when moved away from the substation, also the losses increases quadratically. The reason for decrease in voltage and increase in losses is the insufficient amount of reactive power, which can be provided by the shunt capacitors. But the placement of the capacitor with appropriate size is always a challenge. Thus the optimal capacitor placement problem is to determine the location and size of capacitors to be placed in distribution networks in an efficient way to reduce the power losses and improve the voltage profile of the system. For this purpose, in this paper, the load flow of pre-compensated distribution system is carried out using ‘dimension reducing distribution load flow algorithm (DRDLFA)’. On the basis of this load flow the potential locations of compensation are computed. And then, Differential Evolution (DE) Algorithm is used to determine the optimal location and size of the capacitors. The above method is tested on IEEE 69 bus system and compared with other methods like Genetic Algorithm.

Keywords
Dimension reducing power flow algorithm (DRDLFA), Differential Evolution (DE), Electrical Distribution Network, Optimal Capacitor Placement.

1. INTRODUCTION

Capacitors are generally used for reactive power compensation in distribution systems. The purpose of capacitors is to minimize the power and energy losses and to maintain better voltage regulation for load.
buses and to improve system security. The amount of compensation provided with the capacitors that are placed in the distribution network depends upon the location, size and type of the capacitors placed in the system [1]. A lot of research has been made on the location of capacitors in the recent past [2], [3]. All the approaches differ from each other by the way of their problem formulation and the problem solution method employed. Some of the early works could not take into account of capacitor cost. In some approaches the objective function considered was for control of voltage. In some of the techniques, only fixed capacitors are adopted and load changes which are very vital in capacitor location was not considered. Other techniques have considered load changes only in three different levels. A few proposals were schemes for determining the optimal design and control of switched capacitors with non-simultaneous switching [4]. It is also very important to consider the problem solution methods employed to solve the capacitor placement problem, such as gradient search optimization, local variation method, optimization of equal area criteria method for fixed capacitors and dynamic programs [5], [6], [7]. In some proposals different load levels are not considered [8,9,14]. Although these techniques have solved the problem, most of the early works used analytical methods with some kind of heuristics. In doing so, the problem formulation was oversimplified with certain assumptions, which was lacking generality. There is also a problem of local minimal in some of these methods. Furthermore, since the capacitor banks are non-continuous variables, taking them as continuous compensation, by some authors, can cause very high inaccuracy with the obtained results. A differential evolution algorithm (DEA) is an evolutionary computation method that was originally introduced by Storn and Price in 1995 [15, 16]. Furthermore, they developed DEA to be a reliable and versatil function optimizer that is also readily applicable to a wide range of optimization problems [18]. DEA uses rather greedy selection and less stochastic approach to solve optimization problems than other classical EAs. There are also a number of significant advantages when using DEA, which were summarized by Price in [19]. Most of the initial researches were conducted by the differential evolution algorithm inventors (Storn and Price) with several papers [17, 18, 20] which explained the basis of differential evolution algorithm and how the optimization process is carried out. In this respect, it is very suitable to solve the capacitor placement or location problem. IEEE 69 bus distribution system is considered for case study. The test system is a 10 KVA, 69-bus radial distribution feeder consisting of one main branch and seven laterals containing different number of load buses.

2. DISTRIBUTION POWER FLOW

The distribution power flow is different from the transmission power flow due to the radial structures and high R/X ratio of transmission line. Because of this conventional transmission power flow algorithms does not converge for distribution systems. In this dimension reducing power flow [13] is implemented to determine the losses and the voltage profile. The distribution power flow algorithm is the heart of optimal capacitor placement.

2.1 Dimension reducing Power Flow Algorithm

The basis for the proposed method is that an n-bus radial distribution network has only n – 1 lines (elements) and the branch currents (powers) can be expressed in terms of bus currents (powers). For an element ‘ij’ connected between nodes ‘i’ and ‘j’ the bus current of node j can be expressed as a linear equation. In terms of branch current.

\[ I_{j} = \sum_{k(j)} I_{j,k(j)} - \frac{V_i}{V_j} \]  \hspace{1cm} (2.1)
$k(j)$ is the set of nodes connected to node $j$. For the slack bus the power is not specified, so it is excluded and the relationship between remaining bus currents and branch currents are derived as a non-singular square matrix.

$$I_{bus} = K X I_{branch} \quad (2.2)$$

$$I_{bus} = [I_{b_1}, I_{b_2}, \ldots, I_{b_n}] \quad (2.3)$$

The matrix $K$ is named element incidence matrix. It is a non-singular square matrix of order $(n-1)$. $I_{bus}$ is the column matrix of size $n-1$. The elemental incidence matrix is constructed in a simple way same like bus incidence matrix. In this matrix $K$ each row is describing the element incidences. The elements are numbered in conventional way i.e. the number. of element ‘$ij$’ is $(j-1)$.

- The diagonal elements of matrix $K$ are one. The variable $j$ is denoting the element number.
  $$K (i,j) = 1 \quad (2.4)$$
- For each ‘$ij$’ the element let $m (j)$ is the set of element numbers connected at its receiving end.
  $$K (i,m(j)) = -1 \quad (2.5)$$
- All the remaining elements are zero. It can be observed that all the elements of matrix $K$ below the main diagonal are zero.
  $$I_{branch} = K^{-1} X I_{bus} \quad (2.6)$$

The relationship between the branch currents and bus currents can be extended to complex branch powers and bus powers. The sending end power and the receiving end powers are not same due to transmission loss. The transmission loss is included as the difference between the sending end/receiving end powers derived. The relationship between branch powers and bus powers is established in same way of bus/branch currents. Multiplying both sides by element incidence matrix $K$.

$$S_{bus} = K[S_{branch}^{sending} - TL_{branch}] \quad (2.7)$$

$$S_{branch} = K^{-1}S_{bus} + TL_{branch} \quad (2.8)$$

The power flow equations are complex multi variable quadratic equations. A new variable $R_{ij}$ is introduced for each element ‘$ij$’ and the equations becomes recursively linear.

$$R_{ij} = V_i (V_i^* - V_j^*) \quad (2.9)$$

The branch power of ‘$ij$’ th element is expressed in terms of $R_{ij}$

$$S_{ij} = P_{ij} + jQ_{ij} = R_{ij} Y_{ij}^* \quad (2.10)$$

$$R_{ij} = S_{ij} Z_{ij}^* \quad (2.11)$$

The dimension reducing power flow method is summarized as following steps.

**Step 1** For the first iteration transmission losses are initialized as zero for each element.

From the bus powers specified the branch powers are determined as per equation $\ (2.7 & 2.8)$.

**Step 2** The variable $R_{ij}$ is determined for each element using equation $(2.9)$.

The bus voltage, branch current and bus current are determined from $R_{ij}$.  

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Step 3  The bus currents are determined from (2.1) and bus powers are calculated. Since the transmission losses are neglected in the first iteration there will be mismatch between the specified powers and calculated powers. The mismatch is a part of the transmission loss. $T_{Lij}^r$ is the transmission loss part for ‘ij’th element for ‘r’th iteration. Transmission loss of each element is the summation of the transmission loss portions of all previous iterations.

$$T_{Lij} = \sum_{r=1}^{r} r T_{Lij}^r$$  \hspace{1cm} (2.14)

Where $r$ is the iteration count

$$T_{Lij}^r = K^{-1} (S_{j}^{spec} - V_j^{-1} I_j^*)$$  \hspace{1cm} (2.15)

$$S_{ji} = S_{ij} - T_{Lij}$$  \hspace{1cm} (2.16)

$$S_{branch}^{sending} = S_{branch}^{receiving} - T_{Lij}$$  \hspace{1cm} (2.17)

It can be concluded that the power flow solution always exists for a distribution system irrespective of the R/X ratio if it is having connectivity from the source (slack bus) to all the nodes. For system having less transmission loss the algorithm will perform faster. The convergence criteria is that during the ‘r’th iteration the mismatch of power should be less than the tolerance value.

2.2 Distribution power flow Software Development

After studying and rewriting the power flow equations, a new solution methodology has been developed to determine the voltage profile and power losses in radial distribution system. The algorithm for Distribution Power Flow summarized as follow.

Step 1: Assume base MVA, base KV, slack bus voltage, and initial transmission losses
Step 2: Read the data.
Step 3: Form the bus incidence matrix $K$.
Step 4: Determine the inverse of bus incidence matrix $K^{-1}$.
Step 5: Form the complex power matrix ‘$S$’ for the remaining buses (from 2 to n) from the data.
Step 6: Store the specified bus powers in a new matrix $S_{spec} = S$.
Step 7: Find out the branch power using the equation (2.8).
Step 8: Determine the impedance matrix from the data and express in a per unit impedance matrix.
Step 9: Find out nodal voltage at each nodes using the equation (2.12).
Step 10: Find out the branch and bus currents for the network using the equations (2.2 to 2.16).
Step 11: Find out the calculated bus power for all nodes. (2.7)
Step 12: Find out the transmission losses using equation (2.17) and add it to specified bus and repeat for ‘r’ iterations till convergence.
2.2. Problem Statement

The general capacitor placement problem can be formulated as a constrained optimization problem.

\[
\min f(x,u) \\
\text{Subject to} \\
F(x,u) = 0 \\
G(x,u) \leq 0
\]

where \( f(x,u) \) is the objective function. The state variable ‘\( x \)’ represents the state of the distribution system (bus voltages) and the capacitor location and values are represented by the variable ‘\( u \)’.

\( F(x,u) \) --- represents the set of equality constraints (Power flow equations)

\( G(x,u) \) --- presents the set of inequality constraints (Voltage and reactive power limits) of the problem.

2.3 Assumptions

The following assumptions were considered while formulating the problem: The system is balanced.

- All the loads vary in a conforming manner.
- The forecasted active and reactive powers provided by the load duration curve represent fundamental-frequency powers. Additional powers at harmonic frequencies are negligible.
- Loads at bus are partitioned into linear loads.
- Loads are represented as constant power sink.
- Lines are modeled as a resistance in series with reactance \((r + jx)\)

3. DIFFERENTIAL EVOLUTION

Differential evolution (DE) is a population-based stochastic optimization algorithm for real-valued optimization problems. In DE each design variable is represented in the chromosome by a real number. The DE algorithm is simple and requires only three control parameters: weight factor (\( F \)), crossover rates (CR), and population size (NP). The initial population is randomly generated by uniformly distributed random numbers using the upper and lower limitation of each design variable. Then the objective function values of all the individuals of population are calculated to find out the best individual \( x_{\text{best,G}} \) of current generation, where G is the index of generation. Three main steps of DE, mutation, crossover, and selection were performed sequentially and were repeated during the optimization cycle.

Mutation:

For each individual vector \( x_{i,G} \) in the population, mutation operation was used to generate mutated vectors in DE according to the following scheme equation:

\[
v_{i,G+1} = x_{\text{best,G}} + F(x_{r1,G} - x_{r2,G}), i = 1,2,3...,NP
\]

In the Eq. 15, vector indices \( r1 \) and \( r2 \) are distinct and different population index and they are randomly selected. The selected two vectors, \( x_{r1,G} \) and \( x_{r2,G} \) are used as differential variation for mutation. The vector \( x_{\text{best,G}} \) is the best solution of current generation. And \( v_{i,G+1} \) are the best target vector and mutation vector of current generation. Weight factor \( F \) is the real value between 0 to 1 and it controls the amplification of the differential variation between the two random vectors. There are different mutation mechanisms available.
for DE, as shown Table 1, which may be applied in optimization search process. The individual vectors \( x_{r1,G}, x_{r2,G}, x_{r3,G}, x_{r4,G}, x_{r5,G} \), are randomly selected from current generation and these random number are different from each other. So the population size must be greater than the number of randomly selected ion if choosing Rand/2/exp mechanism of DE mutation, the NP should be bigger than 5 to allow mutation.

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Mathematical equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best / 1 / exp</td>
<td>( v_{i,G+1} = x_{best,G} + F(x_{1,G} - x_{2,G}) )</td>
</tr>
<tr>
<td>Rand / 1 / exp</td>
<td>( v_{i,G+1} = x_{1,G} + F(x_{1,G} - x_{2,G}) )</td>
</tr>
<tr>
<td>Rand-to-Best</td>
<td>( v_{i,G+1} = x_{1,G} + F(x_{1,G} - x_{2,G}) )</td>
</tr>
<tr>
<td>Best / 2 / exp</td>
<td>( v_{i,G+1} = x_{best,G} + F(x_{1,G} + x_{2,G} - x_{3,G} - x_{4,G}) )</td>
</tr>
<tr>
<td>Rand / 2 / exp</td>
<td>( v_{i,G+1} = x_{1,G} + F(x_{1,G} + x_{2,G} - x_{3,G} - x_{4,G}) )</td>
</tr>
</tbody>
</table>

Crossover:
In the crossover operator, the trial vector \( u_{i,G+1} \) is generated by choosing some arts of mutation vector, \( v_{i,G+1} \) and other parts come from the target vector \( x_{i,G} \). The crossover operator of DE is shown in Fig. 1

![Fig.1 The Schematic diagram of crossover operation](image)

Where \( Cr \) represents the crossover probability and \( j \) is the design variable component number. If random number \( R \) is larger than \( Cr \) value, the component of mutation vector, \( v_{i,G+1} \) will be chose to the trial vector. Otherwise, the component of target vector is selected to the trial vector. The mutation and crossover operators are used to diversify the search area of optimization problems.

Selection operator:
After the mutation and crossover operator, all trial vectors \( u_{i,G+1} \) have found. The trial vector \( u_{i,G+1} \) are compared with the individual vector \( x_{i,G} \) for selection into the next generation. The selection operator is listed in the following description:

\[
x_{i,G+1} = u_{i,G+1}, \text{ if } f(u_{i,G+1}) > f(x_{i,G})
\]
\[
x_{i,G+1} = x_{i,G}, \text{ if } f(u_{i,G+1}) \leq f(x_{i,G}), i = 1, 2, ... NP \quad (18)
\]
If the objective function value of trial vector is better than the value of individual vector, the trial vector will be chosen as the new individual vector $x_{i,G+1}$ of next generation. On the contrary, the original individual vector $x_{i,G}$ will be kept as the individual vector $x_{i,G+1}$ in next generation. The optimization loop of DE runs iteratively until the stop criteria are met. There are three stop criteria used in the program. The first criterion is maximum number of optimization generation. The second criterion is maximum number of consecutive generations that no better global optimum is found in the whole process. If the improvement of objective function between two consecutive generations is less than the threshold set by program, it will be considered as fitting convergence requirement. The last stop criterion is conformed if the accumulated number of generations fitted convergence requirement is greater than maximum counter set by the program. The flowchart of DE is shown in Fig. 2. The flowchart of differential evolution

![Flowchart of DE](image.png)

4. DE IMPLEMENTATION

Algorithm to find capacitor sizes using DE:

The basic procedure of DE is summarized as follows.
Step 1: Randomly initialize the population of individual for DE.
Step 2: Evaluate the objective values of all individuals, and determine the best individual.
Step 3: Perform mutation operation for each individual according to Eq. 15 in order to obtain each individual’s corresponding mutant vector.
Step 4: Perform crossover operation between each individual and its corresponding mutant vector in order to obtain each individual’s trial vector.
Step 5: Evaluate the objective values of the trial vectors.
Step 6: Perform selection operation between each individual and its corresponding trial vector according to Eq.16 so as to generate the new individual for the next generation.
Step 7: Determine the best individual of the current new population with the best Objective value then updates best individual and its objective value.
Step 8: If a stopping criterion is met, then output gives its bests and its objective value, otherwise go back to step 3.

The Cost function can be represented mathematically as:

\[
\begin{align*}
\text{Min} & \quad S = K_E \sum_{i=1}^{L} T_i P_i + \sum_{k=1}^{ncap} K_C Q_{ck} \\
& \quad \text{(19)}
\end{align*}
\]

Where \( S \) is the cost of losses in \$/year, 
\( K_E \) is a factor to convert energy losses to dollars, 
\( K_C \) is the Cost of Capacitor/KVAR, 
\( P_i \) is the peak power loss at any load level \( i \), 
\( T_i \) is the time duration for \( i \)th load level, and 
\( Q_{ck} \) is the size of the capacitor in KVAR at node \( k \),
\( ncap \) is the no of candidate locations for capacitor placement.
\( L \) is the number of load level

The main constraints for capacitor placement have to comply with the load flow constraints. In addition, all voltage magnitudes of load (PQ) buses should be within the lower and upper limits. Power Factor (PF) should be greater than the minimum. There may be a maximum power factor limit. 
\( V_{min} \leq V \leq V_{max} \) and \( PF_{min} \leq PF \leq PF_{max} \) for all PQ buses.

5. CASE STUDY AND RESULTS

In order to test the proposed method, an IEEE 69 bus system has taken. Table 2 specifies the load levels and load duration time data for the given system. Table 3 specifies the minimum per-unit bus voltage, cost of energy losses during all load levels for the bare system.

Table 4 shows the capacitor placement locations and sizes. One bank is considered as 100KVAR, thus 5 banks (500KVAR) of switched capacitors need to be placed at bus #10. Two banks (200KVAR) of switched capacitors need to be placed at bus #16. Two banks (200KVAR) of switched capacitors need to be placed at bus #20. Ten banks (1000KVAR) of switched capacitors need to be placed at bus #60 and nine banks (900KVAR) of switched capacitors need to be placed at bus #61.

Table 5 shows the system conditions when capacitor placement is implemented as per the optimal solution. As can be seen from the table, the required voltage regulation at the small, medium and the peak load levels has been obtained. In addition, real power loss reductions at different load levels have been achieved. The table also specifies the minimum per-unit bus voltage, real power losses in KW.

System conditions are shown in Table 6 and the cost of energy losses during all load levels and the total savings are also shown in the table. The results obtained are better than that of [11] & [22].

Prior to capacitor installation, a load flow program based on dimension reducing power flow method is run to obtain the present system conditions. The proposed solution methodologies have been implemented in MATLAB 7.10.0. The solution algorithm based on DE algorithm and tested on IEEE 69 Bus System in Fig.3 which has been designed to find the optimal solution for this problem. In this system all the loads are assumed to be linear. Computer programs have been written for these algorithms based on the respective procedures highlighted earlier. The parameters are defined as shown below:

\[
\begin{align*}
G_{max} &= 800 \\
F &= 0.8 \\
CR &= 0.8 \\
NP &= 100
\end{align*}
\]
Again, the parameters are set empirically by trial and error procedure. Parameters that have resulted in the best solution were chosen. A Differential Evolution based on steady-state replacement usually converges faster than the one designed based on generational replacement. Due to this, steady-state replacement method requires less number of generations before it converges to the optimal solution.

![IEEE 69 bus system](image)

**Table 2: Load level and Load Duration time**

<table>
<thead>
<tr>
<th>Load Level</th>
<th>0.5 (Light)</th>
<th>1.0 (Medium)</th>
<th>1.6 (Peak)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Duration (h)</td>
<td>2000</td>
<td>5260</td>
<td>1500</td>
</tr>
</tbody>
</table>

**Table 3: Cost of Energy Loss and Minimum System Voltage for bare system**

<table>
<thead>
<tr>
<th>Load Level</th>
<th>0.5 (Light)</th>
<th>1.0 (Medium)</th>
<th>1.6 (Peak)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min System Voltage (V)</td>
<td>0.95668</td>
<td>0.90919</td>
<td>0.84449</td>
</tr>
<tr>
<td>Energy Loss Cost</td>
<td>$6192</td>
<td>$70997</td>
<td>$58716</td>
</tr>
</tbody>
</table>

Total Energy Loss Cost = $135,905
Max System voltage = 1.0 P.U

**Table 4: Optimal capacitor placement location and size**

<table>
<thead>
<tr>
<th>Optimal Location</th>
<th>Control Setting (KVAR)</th>
<th>Optimal Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>500</td>
<td>-</td>
</tr>
<tr>
<td>16</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>20</td>
<td>200</td>
<td>-</td>
</tr>
<tr>
<td>60</td>
<td>1000</td>
<td>700</td>
</tr>
<tr>
<td>61</td>
<td>900</td>
<td>500</td>
</tr>
</tbody>
</table>


### Table 5: System conditions without and with capacitors placement for IEEE 69 Bus System

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 (Light)</td>
<td>Minimum bus voltage(pu)</td>
<td>0.95668</td>
<td>0.9643</td>
<td>0.9613</td>
<td>0.96220</td>
</tr>
<tr>
<td></td>
<td>Real Power Loss(KW)</td>
<td>51.60</td>
<td>36.548</td>
<td>40.30</td>
<td>40.48</td>
</tr>
<tr>
<td>1.0 (Medium)</td>
<td>Minimum bus voltage(pu)</td>
<td>0.90919</td>
<td>0.9296</td>
<td>0.9298</td>
<td>0.93693</td>
</tr>
<tr>
<td></td>
<td>Real Power Loss(KW)</td>
<td>224.96</td>
<td>147.96</td>
<td>147.61</td>
<td>156.62</td>
</tr>
<tr>
<td>1.6 (Peak)</td>
<td>Minimum bus voltage(pu)</td>
<td>0.84449</td>
<td>0.8826</td>
<td>0.8819</td>
<td>0.90014</td>
</tr>
<tr>
<td></td>
<td>Real Power Loss(KW)</td>
<td>652.40</td>
<td>409.18</td>
<td>418.59</td>
<td>460.45</td>
</tr>
</tbody>
</table>

### Table 6: Comparison of the results without and with capacitors placement for IEEE 69 Bus System

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Losses cost($/year)</td>
<td>1,35,905</td>
<td>87,907</td>
<td>89,095</td>
<td>95,727</td>
</tr>
<tr>
<td>Total KVAR required</td>
<td>0</td>
<td>2,800</td>
<td>2,400</td>
<td>3,100</td>
</tr>
<tr>
<td>Total cost of the capacitor ($/ KVAR)</td>
<td>0</td>
<td>8,400</td>
<td>7,200</td>
<td>9,300</td>
</tr>
<tr>
<td>Installation cost of the capacitors($/ KVAR)</td>
<td>0</td>
<td>5,000</td>
<td>6,000</td>
<td>3,000</td>
</tr>
<tr>
<td>Total cost($/year)</td>
<td>1,35,905</td>
<td>1,01,307</td>
<td>1,02,295</td>
<td>1,08,027</td>
</tr>
<tr>
<td>Total Annual Savings ($/year)</td>
<td>-</td>
<td>34,598</td>
<td>33,610</td>
<td>27,878</td>
</tr>
</tbody>
</table>
Table 7: Comparison of the savings for IEEE 69 Bus System

6. CONCLUSION

This study presents DE method for Multi-objective programming to solve the IEEE 69 Bus Problem regarding Capacitor placement in the distribution system. The determined optimal location has reduced the system energy losses, total KVAR required, consequently increased the net savings even though there is some increase in total locations of the capacitors. From Tables 5&6 it can be observed that the results obtained using DE are compared and found to be better than the results obtained in the work under ref. [11and 21] regarding net savings by placing the capacitors optimally with achievement of better Voltage Profile and better Voltage Regulation. The optimal placement and KVAR rating of shunt capacitor banks had been best determined for the studied distribution network using the proposed ‘dimension reducing distribution load flow algorithm (DRDLFA)’ and Differential Evolution.

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