DISTURBANCE IN GENERALIZED THERMOELASTIC MEDIUM WITH INTERNAL HEAT SOURCE UNDER HYDROSTATIC INITIAL STRESS AND ROTATION

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ABSTRACT

The present investigation is aimed at studying the deformation in isotropic generalized thermoelastic medium with internal heat source under hydrostatic initial stress and rotation. The normal mode analysis is used to obtain the exact expressions for displacement components, force stresses and temperature distribution. The numerical results are given and presented graphically for Lord-Shulman[2] theory of thermoelasticity when mechanical force is applied. The variations of the considered variables through the horizontal distance are illustrated graphically. Comparisons are made in the presence and absence of hydrostatic initial stress and rotation.

Key Words: Thermoelasticity, hydrostatic initial stress, rotation, Temperature distribution, Normal-mode.

Nomenclature

\( \lambda, \mu \): Lame’s constants
\( \rho \): density
\( C^* \): specific heat at constant strain
\( u \): Displacement vector

\( t_{ij} \): stress tensor

\( \tau_0 \): relaxation time

\( t \): time

\( T \): absolute temperature

\( K^* \): thermal conductivity

\( T_0 \): reference temperature chosen so that \(|(T - T_0)/T_0| < 1\)

\( \alpha_i \): coefficient of linear thermal expansion

\( \tilde{\Omega} \): rotation vector

1 INTRODUCTION

The theory of thermoelasticity deals with the effect of mechanical and thermal disturbances in an elastic body. The theory of uncoupled thermoelasticity consists of the heat equation, which is independent of mechanical effects, and the equation of motion, which contains the temperature as a known function. There are two defects in this theory. First is that the mechanical state of body has no effects on the temperature. Second, the heat equation, which is parabolic, implies that the speed of propagation of the temperature is infinite, which contradicts physical experiments. Biot[1] introduced the theory of coupled thermoelasticity to overcome the first shortcoming. The governing equations for this theory are coupled, eliminating the first paradox of the classical theory. However, both theories share the second overcoming since the heat equation for the coupled theory is also parabolic. To overcome this drawback, two generalizations to the coupled theory were introduced.

The first is due to Lord and Shulman[2], who obtained a wave-type heat equation by postulating a new law of heat conduction to replace the classical Fourier’s law. This new law contains the heat flux vector as well as its time derivative. It contains also a new constant that act as a relaxation time. Since the heat equation of this theory is of the wave-type, it automatically ensures finite speeds of propagation for heat and elastic waves. The remaining governing equations for this theory, namely, the equations of motion and constitutive relations, remain the same as those for the coupled and uncoupled theories.

The second generalization to the coupled theory of elasticity is what is known as the theory of thermoelasticity with two relaxation times or the theory of temperature-rate-dependent thermoelasticity. Muller[3], in a review of the thermodynamics of thermoelastic solids, proposed an entropy production inequality, with the help of which he considered restrictions on a class of constitutive equations. A generalization of this inequality was proposed by Green and Laws[4]. Green and Lindsay[5] obtained another version of the constitutive equations. These equations were also obtained independently and more explicitly by Suhubi[6]. This theory contains two constants that act as relaxation times and modify all the equations of the coupled theory, not only the heat equation. The classical Fourier’s law of heat conduction is not violated if the medium under consideration has a center of symmetry.
Some researchers in past have investigated different problem of rotating media. Chand et al. [7] presented an investigation on the distribution of deformation, stresses and magnetic field in a uniformly rotating homogeneous isotropic, thermally and electrically conducting elastic half-space. Many authors (Schoenberg and Censor [8], Clarke and Burdness[9], Destrade[10]) studied the effect of rotation on elastic waves. Ting[11] investigated the interfacial waves in a rotating anisotropic elastic half space. Sharma and his co-workers [12-15] discussed effect of rotation on different type of waves propagating in a thermoelastic medium. Othman[16] investigated plane waves in generalized thermo-elasticity with two relaxation time under the effect of rotation. Othman and Song [17, 18]) presented the effect of rotation in magneto-thermoelastic medium. Ailawalia et al. [19] discussed effect of rotation due to various sources at the interface of elastic half space and generalized thermoelastic half space. Ailawalia and Narah[20] obtained the expressions for displacement, force stress and temperature distribution in a rotating generalized thermoelastic medium due to a moving load.

The development of initial stresses in the medium is due to many reasons, for example resulting from differences of temperature, process of quenching, shot pinning and cold working, slow process of creep, differential external forces, gravity variations etc. The earth is assumed to be under high initial stresses. It is therefore of much interest to study the influence of these stresses on the propagation of stress waves. Biot[21] showed the acoustic propagation under initial stress which is fundamentally different from that under stress-free state. He has obtained the velocities of longitudinal and transverse waves along the co-ordinate axis only.

The wave propagation in solids under initial stresses has been studied by many authors for various models. The study of reflection and refraction phenomena of plane waves in an unbounded medium under initial stresses is due to Chattopadhyay et al. [22], Sidhu and Singh[23] and Dey et al. [24]. Montanaro[25] investigated the isotropic linear thermoelasticity with a hydrostatic initial stress. Singh et al. [26] and Othman and Song[27] studied the reflection of thermoelastic waves from a free surface under a hydrostatic initial stress in the context of different theories of generalized thermoelasticity. Abd-Alla et al. [28] investigated the influence of gravity field and initial stress on the propagation of Rayleigh waves in an orthotropic thermoelastic medium. Fahmy and Shahat[29] investigated the generation of the thermal stresses in a non-homogeneous rotating anisotropic solid under compressive initial stress. Ailawalia et al. [30] studied deformation in a generalized thermoelastic medium with hydrostatic initial stress. Ailawalia and Budhiraja [31] discussed the effect of hydrostatic initial stress and rotation in Green-Naghdi(Type III) thermoelastic half-space with two temperature. Lotfy[32]have studied the transient disturbance in a half-space under generalized magneto-thermoelasticity with a stable internal heat source.

The present paper is concerned with the investigations related to effect of hydrostatic initial stress and rotation in isotropic generalized thermoelastic medium with internal heat source. The normal mode method is used to obtain the exact expressions for the considered variables. The distributions of the considered variables are represented graphically.

2 FORMATION OF THE PROBLEM

We consider a homogeneous isotropic generalized thermoelastic medium with hydrostatic initial stress of infinite extent rotating uniformly with angular velocity $\tilde{\Omega} = \Omega \hat{n}$, where $\hat{n}$ is a unit vector representing the direction of the axis of rotation. All quantities are considered are functions of the time variable $t$ and of the coordinates $x$ and $y$. The
displacement equation in the rotating frame has two additional terms Schoenberg and Censor[8]: Centripetal acceleration $\ddot{\Omega} \times (\ddot{\Omega} \times \ddot{u})$ due to time varying motion only and Coriolis acceleration $2\ddot{\Omega} \times \ddot{u}$ where $\ddot{u} = (u_1, u_2, 0)$ is the dynamic displacement vector and angular velocity is $\ddot{\Omega} = (0, 0, \Omega)$. These terms do not appear on non-rotating media.

A rectangular cartesian coordinate system $(x, y, t)$ having origin on the surface $y = 0$ and $y$-axis pointing normally into the medium is introduced. We assume the displacement vector as

$$\ddot{u}(x, y, t) = (u_1, u_2, 0)$$

(1)

3 FORMULATION OF THE PROBLEM

To analyze the displacement components, stresses and temperature distribution at the interior of the medium, the continuum is divided into two half spaces defined by

(I) half space I $| x |< \infty$, $-\infty < y \leq 0$, $| z |< \infty$,

(II) half space II $| x |< \infty$, $0 \leq y < \infty$, $| z |< \infty$,

if we restrict our analysis to the plane strain parallel to $xy$-plane with displacement vector $\ddot{u} = (u_1, u_2, 0)$, then the field equations and constitutive relations for such a medium in the absence of body forces are written as,

$$\frac{\partial t_{11}}{\partial x} + \frac{\partial t_{12}}{\partial y} = \rho(\frac{\partial^2 u_1}{\partial t^2} - 2\Omega \frac{\partial u_2}{\partial t} - \Omega^2 u_1),$$

(2)

$$\frac{\partial t_{21}}{\partial x} + \frac{\partial t_{22}}{\partial y} = \rho(\frac{\partial^2 u_2}{\partial t^2} + 2\Omega \frac{\partial u_1}{\partial t} - \Omega^2 u_2),$$

(3)

where

$$t_{11} = -p + (\lambda + 2\mu) \frac{\partial u_1}{\partial x} + \lambda \frac{\partial u_2}{\partial y} - \nu(1 + \nu) \frac{\partial}{\partial t} T,$$

(4)

$$t_{12} = (\mu - \frac{p}{2}) \frac{\partial u_2}{\partial x} + (\mu + \frac{p}{2}) \frac{\partial u_1}{\partial y},$$

(5)

$$t_{21} = (\mu + \frac{p}{2}) \frac{\partial u_2}{\partial x} + (\mu - \frac{p}{2}) \frac{\partial u_1}{\partial y},$$

(6)
\[ t_{22} = -p + \lambda \frac{\partial u}{\partial x} + (\lambda + 2\mu) \frac{\partial u}{\partial y} - \vartheta (1 + \vartheta_0 \frac{\partial}{\partial t}) T, \quad (7) \]

where \( \vartheta = (3\lambda + 2\mu)\alpha_f \).

Using equations (4)-(7) in equations (2)-(3) we obtain,

\[ (\lambda + 2\mu) \frac{\partial^2 u_1}{\partial x^2} + (\mu - \frac{p}{2}) \frac{\partial^2 u_1}{\partial y^2} + (\lambda + \mu + \frac{p}{2}) \frac{\partial^2 u_2}{\partial x \partial y} - \vartheta (1 + \vartheta_0 \frac{\partial}{\partial t}) \frac{\partial T}{\partial x} \]

\[ = \rho (\frac{\partial^2 u_1}{\partial t^2} - 2\Omega \frac{\partial u_2}{\partial t} - \Omega^2 u_1), \quad (8) \]

\[ (\lambda + \mu + \frac{p}{2}) \frac{\partial^2 u_2}{\partial x \partial y} + (\mu - \frac{p}{2}) \frac{\partial^2 u_2}{\partial x^2} + (\lambda + 2\mu) \frac{\partial^2 u_2}{\partial y^2} - \vartheta (1 + \vartheta_0 \frac{\partial}{\partial t}) \frac{\partial T}{\partial y} \]

\[ = \rho (\frac{\partial^2 u_2}{\partial t^2} + 2\Omega \frac{\partial u_1}{\partial t} - \Omega^2 u_2), \quad (9) \]

The heat conduction equation is given by

\[ K^* (n^* + t_1 \frac{\partial}{\partial t}) (\bar{\frac{\partial T}{\partial x^2}} + \bar{\frac{\partial^2 T}{\partial y^2}}) = \rho C^* (n_1 \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}) T \]

\[ + \vartheta T_0 (n_1 \frac{\partial}{\partial t} + \tau_0 n_0 \frac{\partial^2}{\partial t^2}) (\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y}) - (n_1 + n_0 \tau_0 \frac{\partial}{\partial t}) Q, \quad (10) \]

The use of thermal relaxation times \( \tau_0, \vartheta_0 \) and the parameters \( n^*, n_1 \) and \( n_0 \) helps to make the above mentioned fundamental equations possible for two different theories as:

L-S theory due to internal heat source, when

\[ n^* = n_1 = n_0 = 1, \quad t_1 = \vartheta_0 = 0, \tau_0 > 0. \quad (11) \]

G-L theory due to internal heat source, when

\[ n^* = n_1 = 1, n_0 = 0, \quad t_1 = 0, \vartheta_0 \geq \tau_0 > 0, \quad (12) \]

where \( \vartheta_0, \tau_0 \) are the two relaxation times.

For convenience, we shall use the following non-dimensional variables
\[ \{x', y'\} = \frac{\omega}{c_i} \{x, y\}, \quad \{u_1, u_2\} = \frac{\rho c_i \omega^2}{\partial T_0} \{u_1, u_2\}, \quad T' = \frac{T}{T_0}, \quad t_j' = \frac{t_j}{\partial T_0}, \quad \Omega' = \frac{\Omega}{\omega}, \]

\[ i' = \omega^* t, \quad t_1' = \omega^* t_1, \quad \tau_0' = \omega^* \tau_0, \quad \nu_0' = \omega^* \nu_0, \quad p' = \frac{p}{\partial T_0}, \quad Q_0' = \frac{1}{\lambda \omega} Q_0, \]  

(13)

where

\[ c_i^2 = \frac{\lambda + 2 \mu}{\rho}, \quad \omega^* = \frac{\rho C^* c_i^2}{K_1}. \]

Using the expression relating displacement components \( u_1(x, y, t) \) and \( u_2(x, y, t) \) to the scalar potential functions \( \varphi(x, y, t) \) and \( \psi(x, y, t) \) in dimensionless form:

\[ u_1 = \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial y}, \quad u_2 = \frac{\partial \varphi}{\partial y} + \frac{\partial \psi}{\partial x}. \]  

(14)

Equations (8)-(10), with the help of equations (13) and (14) may be recast into dimensionless form after suppressing the primes as:

\[ (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial t^2} + \Omega^2 \varphi - (1 + \nu_0 \frac{\partial}{\partial t}) T + 2 \Omega \frac{\partial}{\partial t} \psi = 0, \]

(15)

\[ (a_2 \frac{\partial^2}{\partial x^2} + a_2 \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial t^2}) \psi - 2 \Omega \frac{\partial}{\partial t} \psi = 0, \]

(16)

\[ (n^* + t_1 \frac{\partial}{\partial t}) (\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial x^2}) T = (n_1 \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}) T \]

\[ + a_4 (n_1 \frac{\partial}{\partial t} + \tau_0 n_0 \frac{\partial^2}{\partial t^2}) (\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial x^2}) \varphi - a_4 (n_1 + n_0 \tau_0 \frac{\partial}{\partial t}) Q. \]

(17)

4 NORMAL MODE ANALYSIS

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form

\[ [\varphi, \psi, T, t_j](x, y, t) = [\varphi, \psi, T, t_j](y) e^{(\alpha x + \beta y)}, \]

(18)

\[ Q = Q_0 e^{(\alpha x + \beta y)}, Q = Q_0. \]  

(19)

where \([\varphi, \psi, T, t_j]\) are the magnitude of the functions, \( \omega \) is the complex time constant and \( \alpha \)
is the wave number in $x$-direction and $Q_0$ is the magnitude of stable internal heat source.

Using (18) and (19) in equations (15)-(17) we obtain,

$$[\nabla^2 - a_8] \phi + a_{11} \nabla - a_{10} T = 0,$$  \hspace{1cm} (20)

$$a_{11} \phi - [a_2 \nabla^2 - a_9] \nabla = 0,$$  \hspace{1cm} (21)

$$[a_3 \nabla^2 - a_7 a^2] \phi - [a_5 \nabla^2 - a_3 a^2 - a_6] T = \varepsilon Q_0,$$  \hspace{1cm} (22)

where

$$a_1 = (\lambda + \mu + \frac{\nu T_0 p}{2} \frac{1}{\rho c_i^2}), \quad a_2 = (\mu - \frac{\nu T_0 p}{2} \frac{1}{\rho c_i^2}), \quad a_3 = \frac{\nu^2 T_0}{\rho k^2 \omega^2}, \quad a_4 = \frac{\lambda c_i^2}{\omega K},$$

$$a_5 = (n + t_1 \omega), \quad a_6 = (n \omega + \tau_0 \omega^2), \quad a_7 = a_2 (n_1 \omega + n_0 \tau_0 \omega^2), \quad a_8 = (a^2 + \omega^2 - \Omega^2),$$

$$a_9 = (a^2 + \omega^2 - \Omega^2), \quad a_{10} = (1 + \nu^2 \omega), \quad a_{11} = 2 \Omega \omega, \quad a_{12} = \frac{\lambda}{\rho c_i^2}, \quad a_{13} = (\mu + \frac{\nu T_0 p}{2} \frac{1}{\rho c_i^2}).$$

(23)

Eliminating $\nabla$ and $T$ from equations (20)-(22), we obtain,

$$[\nabla^2 + \lambda_1 \nabla^4 + \lambda_2 \nabla^2 + \lambda_3] (\phi (y)) = \lambda a_9 a_{10} Q_0,$$  \hspace{1cm} (24)

where, $\nabla = \frac{d}{dy},$

$$\lambda_1 = -\frac{a_9}{a_2} + \frac{a_6}{a_5} + a^2 + a_8, \quad \lambda_2 = \frac{a^2 \eta_1 + \eta_2 + a_6 (a_2 a_8 + a_9)}{a_2 a_5},$$

$$\lambda_3 = -\frac{\eta_2 (a^2 a_5 + a_6)}{a_2 a_5}, \quad \eta_1 = (a_2 a_8 + a_9), \quad \eta_2 = (a_8 a_9 + a_{11}), \quad \varepsilon = a_4 (n_1 + n_0 \tau_0 \omega).$$

(25)

The solution of equation (24) is given by:

$$\phi (y) = \sum_{j=1}^{3} R_j (a, \omega) e^{-j\gamma y} + \sum_{j=1}^{3} S_j (a, \omega) e^{j\gamma y} + f_1,$$

In a similar way, we get

$$\nabla (y) = \sum_{j=1}^{3} R_j (a, \omega) e^{-j\gamma y} + \sum_{j=1}^{3} S_j (a, \omega) e^{j\gamma y} + f_2,$$  \hspace{1cm} (26)
\[ \bar{T}(y) = \sum_{j=1}^{3} \bar{R}^j (a, \omega) e^{-k_j y} + \sum_{j=1}^{3} \bar{S}^j (a, \omega) e^{k_j y} + f_3, \]  

(27)

where
\[ f_1 = \frac{\varepsilon a_\omega a_{10} Q_0}{\lambda_3}, \quad f_2 = \frac{\varepsilon Q_0 n_3}{\lambda_3}, \quad f_3 = -\frac{\varepsilon a_{10} a_{11} Q_0}{\lambda_3}, \]  

(28)

and \( R_j(a, \omega), \bar{R}_j(a, \omega) \) and \( \bar{R}_j'(a, \omega) \) are some parameters depending on \( a \) and \( \omega \). \( k_j^2 \) \( (j = 1, 2, 3) \) are the roots of the characteristic equation (24).

Using equations (25)-(27) into equations (20)-(22), we get the following relations:

\[ \bar{\phi}(y) = \sum_{j=1}^{3} \bar{a}_j R_j(a, \omega) e^{-k_j y} + \sum_{j=1}^{3} \bar{b}_j S_j (a, \omega) e^{k_j y} + f_2, \]  

(29)

\[ \bar{T}(y) = \sum_{j=1}^{3} \bar{b}_j^2 R_j(a, \omega) e^{-k_j y} + \sum_{j=1}^{3} \bar{b}_j S_j (a, \omega) e^{k_j y} + f_3, \]  

(30)

where
\[ \bar{a}_j = \frac{a_{11}}{a_2 k_j^2 - a_9}, \quad \bar{b}_j = \frac{k_j^2 - a_9 + a_1 a_j}{a_{10}}, \quad j = 1, 2, 3. \]

5 APPLICATIONS

The boundary conditions at the interface \( y = 0 \) subjected to an arbitrary normal force \( P_1 \) are

\[ (i) \quad t_{22} (x, 0^+, t) - t_{22} (x, 0^-, t) = -P_1 e^{|\omega t + \omega x|}, \quad (ii) \quad t_{21} (x, 0^+) - t_{21} (x, 0^-) = 0, \]

\[ (iii) \quad u_1 (x, 0^+) = u_1 (x, 0^-), \quad (iv) \quad u_2 (x, 0^+) = u_2 (x, 0^-), \]

\[ (v) \quad T(x, 0^+) = T(x, 0^-), \quad (vi) \quad \frac{\partial T}{\partial y} (x, 0^+) = \frac{\partial T}{\partial y} (x, 0^-). \]  

(31)

where \( P_1 \) is the magnitude of mechanical force. Using equations (13) and (6)-(7) on the non-dimensional boundary conditions and then using (25) and (29)-(30), we get the expressions of displacement, force stress and temperature distributions for isotropic generalized thermoelastic medium as,

\[ u_1 = \sum_{j=1}^{3} c_j R_j(a, \omega) e^{-k_j y} + \sum_{j=1}^{3} d_j S_j (a, \omega) e^{k_j y} + taf_1, \]  

(32)
where

\[ c_j = (\alpha + k, a^*_j), \quad d_j = (\alpha - k, a^*_j), \quad s_j = (-k_j + taa^*_j), \quad r_j = (k_j + taa^*_j), \]

\[ l_j = (k_j^2 - a_{12}a^2 - a_{10}b_j^*) + taa^*_j(a_{12} - 1), \quad f_j = (k_j^2 - a_{12}a^2 - a_{10}b_j^*) + taa^*_j(1 - a_{12}), \]

\[ n_j = taa^*_j(a_{13} - a_2) - a^*(a^2a_{13} + a_2k_j^2), \quad m_j = taa^*_j(a_{13} - a_2) - a^*(a^2a_{13} + a_2k_j^2), \]

Invoking the boundary conditions (31) at the surface \( y = 0 \), we obtain a system of six equations, and applying the inverse of matrix method, we obtain the values of six constants \( R_j \) and \( S_j, j = 1,2,3 \), as

\[ R_1 = \frac{\Delta_1}{\Delta}, \quad R_2 = \frac{\Delta_2}{\Delta}, \quad R_3 = \frac{\Delta_3}{\Delta}, \quad S_1 = \frac{\Delta_4}{\Delta}, \quad S_2 = \frac{\Delta_5}{\Delta}, \quad S_3 = \frac{\Delta_6}{\Delta}. \]

where \( \Delta, \Delta_i, i = 1,2,3,..,6 \) are defined in appendix A.

6 PARTICULAR CASES

6.1 Isotropic generalized thermoelastic medium with internal heat source under hydrostatic initial stress

Neglecting angular velocity \( \bar{\Omega} = 0 \) in equations (32)- (36), we obtain the corresponding expressions of displacement, stress and temperature distribution in isotropic generalized thermoelastic medium with internal heat source under hydrostatic initial stress.

6.2 Isotropic generalized thermoelastic medium with internal heat source under rotation

Letting \( p \to 0 \), in the system of equations (32)- (36), we obtain the components of displacements, force stress and temperature distribution in isotropic generalized thermoelastic medium with internal heat source under rotation.
6.3 Isotropic generalized thermoelastic medium with internal heat source

Letting \( p \to 0 \) in 6.1, we obtain the expressions for displacement, force stress and temperature distribution in isotropic generalized thermoelastic medium with internal heat source.

For all the cases discussed above the components of displacement, stresses and temperature distribution for the region \(-\infty < y \leq 0\), are obtained by inserting \( R_1 = R_2 = R_3 = 0 \) in Eqs.(32)-(36).

Similarly for the region \( 0 \leq y < \infty \), the components are obtained by inserting \( S_1 = S_2 = S_3 = 0 \) in Eqs.(32)-(36).

7 NUMERICAL RESULTS

In order to illustrate our theoretical results obtain in proceeding section, we now consider a numerical example for which computational results are given. The results depict the variations of displacements, force stress, temperature distribution and tangential couple stress in the context of the L-S theory. For this purpose, the values of physical constants are taken as Sharma[33]

\[
\lambda = 8.2 \times 10^{10} \text{ Nm}^{-2}, \quad \mu = 4.2 \times 10^{10} \text{ Nm}^{-2}, \quad \rho = 8.950 \times 10^{3} \text{ kgm}^{-3},
\]

\[
K^* = 1.13 \times 10^{2} \text{ calm}^{-1} \text{s}^{-1} \text{K}^{-1}, \quad \alpha_r = 1.0 \times 10^{-8} \text{K}, \quad T_0 = 300 \text{K},
\]

\[
\omega^* = 4.347 \times 10^{13} \text{ sec}^{-1}.
\]

The computations are carried out for the value of non-dimensional time \( t = 0.1 \) in the range \( 0 \leq x \leq 10 \) and on the surface \( y = 1.0 \). The numerical values for displacements \((u_1, u_2)\), force stresses \((t_{21}, t_{22})\) and temperature distribution \(T\) are shown in figures (1)-(5) for mechanical force with \( P_1 = 1.0, \quad \omega = \omega_0 + \zeta \zeta, \quad \omega_0 = 2.3, \quad \zeta = 0.1, \quad a = 2.1 \) and \( \Omega = 0.5 \) for an

(a) Isotropic generalized thermoelastic medium with internal heat source under hydrostatic initial stress and rotation(IGTHR) by solid line.

(b) Isotropic generalized thermoelastic medium with internal heat source under hydrostatic initial stress(IGTHWR) by solid line with centered symbol (*).

(c) Isotropic generalized thermoelastic medium with internal heat source under rotation(IGTWHWR) by dashed line.

(d) Isotropic generalized thermoelastic medium with internal heat source(IGTWHWR) by dashed line with centered symbol(*).

These graphical results represent the solutions obtained for the generalized theory with
one relaxation time (L-S-theory) by taking $\tau_0 = 0.02$.

8 DISCUSSIONS

Figure 1 depicts the variations of tangential displacement $u_1$ with distance $x$. The values of $u_1$ for IGTHWR increases sharply in the range $0 \leq x \leq 2.0$, then oscillate for the remaining values of $x$. The variations of tangential displacement $u_1$ for (IGTWH, IGTWHR) appears to be inverted image of each other varying only in magnitude.

Figure 2 depicts the variations of normal displacement $u_2$ with distance $x$. The pattern observed for IGTWHR and IGTWHWR are opposite in nature near the point of application of source. The value of $u_2$ for IGTHWR decreases, then follow an oscillatory pattern with decreasing magnitude. It is also noticed that IGTHR show small variations about origin.
The variations of temperature distribution $T$ with distance $x$ is depicted in figure 3. The variations of temperature distribution $T$ for IGTHR and IGHWR show similar patterns with different degree of sharpness. i.e. the values for IGTHR and IGHWR increases and decreases alternately with distance $x$. The value of temperature distribution $T$ for IGTWHWR lie in a very short range. Further temperature distribution $T$ shows small variations near to zero value in the whole range for IGTWHR.
The variations of tangential force stress $t_{21}$ with distance $x$ is depicted in figure 4. The behaviour of variations for IGTHR and IGTHWR are opposite in nature in range $0 \leq x \leq 1.8$, in remaining range both show oscillatory behaviour with significant difference in their magnitude of oscillation, whereas IGTHR and IGTHWR show opposite oscillatory patterns in entire range which shows the impact of rotation.

Figure 5 depicts the variations of normal force stress $t_{22}$ with distance $x$. It is interesting to observe from figure 5, that the behaviour of variations of normal force stress $t_{22}$ with reference to $x$ is same i.e. oscillatory for (IGTHR, IGTHWR) with difference in their magnitude. The value of IGTHWR increases in the range $0 \leq x \leq 2.0$, $4 \leq x \leq 5$, $7 \leq x \leq 8$ and decreases in the remaining range. The pattern observed for IGTHWR and IGTWHWR are opposite in nature with fluctuating values which clearly reveals the effect of hydrostatic initial stress.
9 CONCLUSION

Appreciable effect of hydrostatic initial stress and rotation is observed on the components of displacement, force stress and temperature distribution. The variations of tangential displacement $u_1$ and normal displacement $u_2$ are similar in nature with difference in magnitude. The normal mode analysis used in this article is applicable to wide range of problems in different branches. This method gives exact solutions without any assumed restrictions on either the temperature or stress distributions. This problem, though theoretical, may be used in engineering, seismology and geophysics.

10 APPENDIX A

$$\Delta = D_1 G_2 + D_2 G_1, \quad \Delta_1 = G_2 G_3 + G_4 D_2, \quad \Delta_2 = G_2 G_5 + G_6 D_2, \quad \Delta_3 = G_2 G_7 + G_8 D_2,$$

$$\Delta_4 = D_1 L_4 - L_2 G_1, \quad \Delta_5 = D_1 L_3 - L_4 G_1, \quad \Delta_6 = D_1 L_5 - L_6 G_1,$$

where

$$D_1 = l_1(n_2 c_3 - c_2 n_3) - l_2 (n_1 c_3 - n_3 c_1) + l_3 (n_2 c_2 - n_2 c_1), \quad D_2 = f_1 (m_2 d_3 - m_3 d_2) - f_2 (m_4 d_3 - m_3 d_4)$$

$$+ f_3 (m_1 d_2 - m_2 d_1), \quad G_1 = s_b b_2 (k_3 - k_2) + s_b b_3 (k_1 - k_3) + s_b b_2 (k_2 - k_1),$$
\[ G_2 = r_b b_3 (k_3 - k_2) + r_b b_3 (k_1 - k_3) + r_b b_2 (k_2 - k_1), \quad G_3 = n_c n_c - n_c n_c \]
\[ G_4 = taf_1 b_1 (k_3 - k_1) - f_3 (b_1 s_1 k_2 - s_1 b_1 k_2), \]
\[ G_5 = n_c n_c - n_c n_c - l_1 (N_2 c_3 + taf_1 n_1) + l_1 (taf_1 n_1 + N_2 c_1), \quad G_6 = taf_2 b_2 (k_3 - k_1) \]
\[ G_7 = l_1 (taf_1 n_2 + c_2 N_2) - l_2 (taf_1 n_1 + c_2 N_2) + N_1 (n_c n_c - n_c n_c), \]
\[ G_8 = taf_2 b_2 (k_3 - k_1) - f_3 (b_1 s_1 k_2 - s_1 b_1 k_2), \quad L_2 = N_1 (m_2 d_3 - m_2 d_2) + f_2 (N_2 d_3 + taf_1 m_3) - f_3 (taf_1 m_2 + d_2 N_2), \]
\[ L_3 = taf_1 b_1 (k_3 - k_1) + f_3 (b_1 s_1 k_2 - s_1 b_1 k_2), \quad L_4 = f_3 (m_1 taf_1 + N_2 d_1) \]
\[ L_5 = N_1 (m_3 d_3 - m_3 d_1) + f_4 (m_3 taf_1 + N_1 d_2) + f_3 (b_1 s_1 k_2 - s_1 b_1 k_2), \]
\[ L_6 = N_1 (m_1 d_2 - d_1 m_2) + f_4 (m_1 taf_1 + N_1 d_2) + f_3 (b_1 s_1 k_2 - s_1 b_1 k_2), \]
\[ N_1 = -P_1 - a_10 f_3 - a_1^2 a_12 f_1 + pe^{(\omega t + int)}, \quad N_2 = a_1^2 a_13 f_2. \]

REFERENCES