COST BASED INDUSTRIAL RECTIFYING SAMPLING INSPECTION

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ABSTRACT

This paper aimed to develop a quality cost model which is considered to be a generalization or extension to the model of Schmidt and Taylor (1973). The new model is built by using a more general formula for p (t), the average proportion defective of a lot where line failure occurred at time t after the start of production of a lot. The proposed formula for p (t) fits all kinds of the time failure distributions not just the uniform time failure distribution. In this paper we assumed that the average proportion defective p (t) depends upon the failure time which varies exponentially, and the number of defective items in a sample of size n has a Poisson distribution with mean n p (t). The expected total cost has been built and a sample plan for rectifying inspection which minimizes the expected total cost is done by determining the optimum sample size, acceptance number and inter inspection period. A real-life data show that the percentage reduction in the total cost by using the proposed model instead of Schmidt's and Taylor's model is about 65.9%.

Keywords: Rectifying sampling inspection, generalized cost model, Percentage reduction, Poisson distribution

1. INTRODUCTION

The quality of products has been the focus of producers in different areas of industrial process control. The control chart is one of the most powerful tools for monitoring industrial process. Shewhart (1931) introduced control charts, which play an important role in statistical quality control to detect out-of-control signals and to improve the quality of product. Burr (1967), Schilling and Nelson (1976), Champ and Woodall (1987), Ryan (1989), Quesenberry (1997), Deming (2000), Woodall et al. (2004), Montgomery (2005), Albers et al. (2006), and several other researchers modified and popularized Shewhart control charts. Dodge (1964) asserted that distinguishing
between sampling inspection schemes and quality control charts was clear since World War II. The usage of acceptance sampling plans may belong to the date of establishing the department of inspection and quality control of Bell laboratories in United States of America in the year of 1924. The minimum cost determinations that satisfy the required quality of the products are the main objectives to industrial engineering and to production engineering as well as to different areas of research sectors.

Several methods for obtaining ordinary sampling plans, Bayesian sampling plans, double sampling and sequential sampling plans which minimize the total quality control cost function have been proposed in the literatures by several researchers, among them Barnett (1972), Guenther (1977, 1981), Bain (1978), Hald (1981), Schilling (1982), Baklizi (2003), and Hassan et al. (2013).

The objective of this paper is to build a quality control cost model used in developing a single sampling plan for rectifying inspection which minimizes the expected total cost by determining the sample size n, acceptance number c and time period, \( \tau \), between successive inspections. This paper derives the expected total cost model under the same assumptions given by Schmidt and Taylor (1973) except that the average proportion defective of the lot, \( p(t) \), is given by,

\[
P(t) = p_0 P(T \leq t \mid T \leq \tau) + p_1 P(t < \tau \mid T < \tau)
\]

The proposed formula of equation (1) fits all kinds of time failure distributions not just the uniform time failure distribution which reduces the formula (1) to the special case

\[
p(t) = \left( t p_0 + (\tau - t)p_1 \right) / \tau
\]

that has been given by equation (4) of Schmidt and Taylor (1973). The proposed model is compared with the model of Schmidt and Taylor (1973) by using a real-life data from an industrial company in Iraq which produces pure corn oil. The results from using the multivariate and a partial enumeration show that the percentage reduction in the expected total cost by using the proposed model instead of Schmidt's and Taylor's model is about 65.9%.

2. DEVELOPMENT OF THE COST MODEL

The main objective of the rectifying inspection is to determine the sample size n, the acceptance number c, and the inter inspection time \( \tau \), that minimizes the following expected total cost T.C of quality control,

\[
T.C = E(\text{inspection cost}) + E(\text{rejection cost}) + E(\text{acceptance cost}) + E(\text{line failure cost})
\]

under the following assumptions:

1. The probability density function \( f(t, \lambda) \), of the time until failure \( T \) is exponential of mean \( 1/\lambda \).
2. The probability mass function \( p(x, n, p) \), of the number of defectives \( X \), in a sample of size \( n \) drawn from a lot of proportion defective \( p \), is a Poisson of mean \( np \).
3. A decision to reject any given lot causes a line shutdown and adjustment at a cost \( C_F \).
4. \( C_1 \), \( C_2 \), and \( C_3 \) are respectively represented the unit cost of inspection, the unit cost of rejection, and the unit cost of acceptance.

In order to accomplish this task, the probability \( B \) that the system is in a failed state after any given inspection is

\[
B = \frac{Q_1}{1 - P(X \leq c) + Q_1}
\]

and hence,
\[ B = \frac{\sum_{n=0}^{\infty} \frac{1 - e^{-\lambda t}}{n!} R(x, np_0) - R(x, np_0)}{1 - R(x, np_0) + \sum_{n=0}^{\infty} \frac{1 - e^{-\lambda t}}{n!} R(x, np_0) - R(x, np_0)} \] (3)

where

\[ R(x, \mu) = \sum_{n=0}^{\infty} e^{-\lambda t} \mu^n / n! \]

And

\[ Q_1 = \frac{\sum_{n=0}^{\infty} e^{-\lambda t} \mu^n / n!}{1 - e^{-\lambda t}} \]

(4)

and the new average proportion defective \( p(t) \) of a lot is given by

\[ p(t) = \frac{(p_1 - p_0)e^{-\lambda t} + p_0 - p_1 e^{-\lambda t}}{1 - e^{-\lambda t}} \] (5)

and hence the expected total cost of the quality control system will be

\[ T. C. = C_1 n + C_2 \left\{ np_1 B + n(1 - B) \left[ \int_0^\infty p(t)f(t)dt + \phi R(x, np_0) \right] \right\} + (\psi t - n) C_3 \left\{ (1 - B) \sum_{n=0}^{\infty} [p(x, np_0) - R(x, np_0)] \right\} \]

(6)

Simplifying the results, we get

\[ T. C. = C_1 n + C_2 \left\{ np_1 B + n(1 - B) \left[ \frac{p_0 - p_1 e^{-\lambda t} + p_0 - \phi R(x, np_0) \frac{1 - e^{-\lambda t}}{1 - e^{-\lambda t}}} \right] \right\} + (\psi t - n) C_3 \left\{ (1 - B) \sum_{n=0}^{\infty} [p(x, np_0) - R(x, np_0)] \right\} \]

(7)

where

\[ S^n = \frac{1 - e^{-\lambda t}}{n!(p_0 - p_1)} \sum_{n=0}^{\infty} (x + 1) [R(x + 1, np_0) - R(x + 1, np_0)] \]
3. APPLICATION

After the expected total cost function is built, it is necessary to apply the proposed model given in equation (6) and compare it withmodel that has been given by Schmidt and Taylor (1973).

In this paper we considered a production facility from an industrial company in Iraq which produces pure corn oil at a constant rate $\psi = 700$ units/hr. The historical data about this production facility show that the time between two successive failures is exponentially distributed with rate $\tau = 0.1667/\text{hr}$. Moreover, the following data concerning the costs are collected by the authors:

- $C_1 = \$0.009/\text{unit}$
- $C_2 = \$0.260/\text{unit}$
- $C_3 = \$0.742/\text{unit}$
- $C_F = \$288/\text{shutdown}$
- $p_0 = 0.004265 \text{ units}$
- $p_1 = 0.05 \text{ units}$
- $\psi = 700 \text{ units/hr}$
- $\lambda = 0.1667/\text{hr}$

A multivariate optimization technique and a partial enumeration with a one-factor-at-a-time was employed on the proposed model and gave the following optimum sampling plan $(c=8 \text{ units}, n=240 \text{ units}, \tau = 2.5 \text{ hours}, \text{T.C.} = \$131.576)$.

But the optimum sampling plan for the model of Schmidt and Taylor (1973) using the same data was

$(c=7 \text{ units}, n=400 \text{ units}, \tau = 8.5 \text{ hours}, \text{T.C.} = \$386.29)$.

To find the percentage reduction $R$ in the expected total cost by using the proposed model instead of Schmidt's and Taylor's model, we get

$$R = \frac{386.29 - 131.576}{386.29} \times 100 = 65.9\%$$

This means that the expected total cost can be reduced by 65.9% if one uses the proposed model instead of the model of Schmidt and Taylor (1973). This is because of using the general formula of $p(t)$ with exponentially distributed time failure instead of uniformly distributed time failures.

CONCLUSIONS

This research paper developed a new cost-based rectifying sampling inspection scheme which is considered to be a generalization of the model of Schmidt and Taylor (1973) which to our knowledge has not been considered before in the literature. The results of the optimum proposed sampling plan were compared with the optimum sampling plan for the model of Schmidt and Taylor (1973) through considering a real-life data. It appears that the proposed model is more flexible and more accurate than the model of Schmidt and Taylor (1973) when the distribution of the time until failure is not uniformly distributed. The results show that the percentage reduction in expected cost is 65.9% when using the proposed model.
REFERENCES