CHARACTERIZATION OF MEASUREMENT ERROR AND UNCERTAINTY IN WORKING STANDARDS

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ABSTRACT

Measurement uncertainty and error in metrology of the basic proposition is frequently used measurement tester’s important concept. It has a direct bearing on the reliability of measurement results and the exact same value transfer. Quality statements for calibrated measuring instruments give the systematic error and the uncertainty of measurement at the moment of calibration. In contrast to this, the statement of conformity for verified measuring instruments indicates whether or not the error limits on verification specified by law are met. The conformity assessment is based on the result of a previous calibration. Calibration uncertainty thus becomes an uncertainty of conformity decision. In this study five years’ test and verification data of working standards such as non-automatic weighing instrument, weight measures, and volumetric measures were analyzed. Regression analysis has also been done to understand and predict time variance of error. This study also investigates how to introduce the concept of uncertainty in the criteria of conformity in type approval and verification. For that, calibration data of non-automatic weighing instruments were used. It was observed from the study that the uncertainty in type approval is different from the uncertainty in the performance of measuring instruments, the error grows with ageing of instruments and also there is instrument-to-instrument difference in error contribution. This study also addresses the differences, common bases and the relationship between calibration and verification. In particular, the relationships between legally prescribed error limits and uncertainty and the uncertainty contribution of verified measuring instruments are discussed.

Index terms: Legal metrology, Type Evaluation and Verification, Measurement Uncertainty, Non-Automatic Weighing Instrument, Weight Measures, Volumetric Measures.
1. INTRODUCTION

The correctness of measurements and measuring instruments is one of the most important prerequisites for the assurance of the quality and quantity of products and services, and the accuracy of the instruments must be consistent with their intended use.

In compliance with the ISO 9000 standard series and the ISO/IEC 17025 standard, traceability of measuring and test equipment to the realization of SI units must be guaranteed by an unbroken chain of comparison measurements to allow the necessary statements about their metrological quality [C. F. Dietrich, 1991]. The most important actions to ensure the correct indication of measuring instruments are:

- **in industrial metrology**: regular calibration of the measuring instruments according to the implemented quality systems; and
- **in legal metrology**: periodic verification or conformity testing of the measuring instruments according to legal regulations.

Both actions are closely related and are mostly based on the same measuring procedures. Historically, however, these actions have been established with separate rules and metrological infrastructures and activities. Verification has become a principal part of legal metrology systems and calibration is widely used in quality assurance and industrial metrology - accreditation bodies prefer calibration as a primary action to provide proof of the correctness of the indication of measuring instruments.

The need to ensure international consistency of trade and regulatory measurements, and to “resolve internationally the technical and administrative problems raised by the use of measuring instruments”, led to the establishment in 1955 of a second metrology treaty organization, the International Organization of Legal Metrology (OIML). In the past, various interests as well as regional and historical differences led to differing units and systems. As cross-border trade increased in significance, pressure grew for harmonization; this resulted in the introduction of the SI system which not only became the legal basis for official dealings and commercial transactions, but also gained in importance in the non-regulatory field of industrial metrology. An efficient metrological infrastructure is the basis of all modern industrial societies and from this point of view, legal metrology was the pioneer of uniform measurement.

2. MEASURES AND PROCEDURES

In order to reach the objectives of legal metrology, both preventive and repressive measures are needed. Preventive measures are taken before the instruments are placed on the market or put into use and include pattern approval and verification. Market surveillance is an example of a repressive measure, and involves inspection of the instrument at the supplier’s, owner’s or user’s premises. The competent body has to examine at least one instrument, to ensure compliance with the legal requirements. Approval tests and calibrations are carried out, and the results show whether the given requirements are met. It is particularly important to determine whether the maximum permissible error [MPE’s] at rated or foreseeable in situ operating conditions are likely to be met. The sample instrument is also subjected to quality tests which should guarantee its reliability in use.

For reasons of efficiency, verification usually only requires a single measurement (observation) to be carried out. It is therefore important that the spread or dispersion of measured values is determined during the type approval tests. This determination of so-called apriori characteristic values forms the justification for the evaluation of the uncertainty of measurement on the subsequent verifications [K.D. Sommer et.al., 2002].

European harmonization allows the manufacturer to carry out conformity assessment on new instruments as an alternative to verification by a verification body. This leads to the need to
harmonize the measuring and testing methods, including determination of the measurement uncertainties and accounting for them in conformity assessments.

3. OBJECTIVES

The aims of this research were:
- to study the effect of ageing on measurement error
- to study the limits of maximum permissible errors on verification (MPEV) and uncertainty

3.1 Effect of ageing on measurement error

3.1.1 Maximum permissible errors on verification and in service

In many economies with developed legal metrology systems, two kinds of error limits have been defined:
- the maximum permissible errors (MPE) on verification; and
- the maximum permissible errors (MPES) in service.

The latter is normally twice the first. MPES on verification equal “MPE on testing” that are valid at the time of verification. For the measuring instrument user, the MPE in service are the error limits that are legally relevant. The values of the error limits are related to the intended use of the respective kind of instrument and determined by the state of the art of measurement technology [M. Buzoianu, 2000].

The different instrument standards used in legal metrology are reference standards, secondary standards, working standards and commercial standards. Verification and testing of a particular standard is done with respect to immediate superior standard, for example working standard is compared with secondary standard, which is preserved at the State Legal Metrology Laboratory [Gupta S. 1984]. Therefore working standards were taken in this study.

3.1.2 Working Standard

The working standards in legal metrology are used as the standards to calibrate commercial standards. The maximum permissible errors taken for such working standards are maximum permissible error on service (MPES) and will depend on the type of working standards. In legal metrology laboratory the working standards of different centers are compared with secondary standards as is shown in Fig. 1.

Fig. 1. Example of calibration by comparison method
The verification and test data of working standards from the legal metrology laboratory for five consecutive years was collected for the following working standards:

i. Non-Automatic Weighing Instrument(s)
ii. Weights Measures
iii. Volumetric measures

Twenty working standards of each category has been chosen.

<table>
<thead>
<tr>
<th>Working Standards</th>
<th>Denomination of measures</th>
<th>No. of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAWI</td>
<td>--</td>
<td>20</td>
</tr>
<tr>
<td>Weight measures</td>
<td>20000, 10000, 5000, 2000, 1000, 500, 200, 100, 50, 20, 1, .500, .200, .100, .050, .020, .010, .005, .002, .001</td>
<td>20</td>
</tr>
<tr>
<td>Volumetric measures</td>
<td>10000, 5000, 2000, 1000, 500, 200, 100, 50, 20, 10</td>
<td>20</td>
</tr>
</tbody>
</table>

3.1.2.1. Non-Automatic Weighing Instrument(s)

The Non-automatic Weighing Instrument(s) used in the study was an electronic balance. The electronic load cell type balance uses the principle that a force applied to an elastic element produces a measurable deflection. The elastic elements used are specially shaped and designed. The block shapes can be cylindrical, rectangular ring proving frame, parallelogram-cut proving frame and octagonal-cut proving frame. The design aims are to obtain a linear output relationship between the applied force and the measured deflection and to make the measurement insensitive to forces which are not applied directly along the sensing axis. Displacement transducer, strain-gauge, is used to measure the deflection of the elastic elements. This instrument is to be recalibrated from time to time otherwise it will lead to significant measurement errors in the form of a bias on all readings [Earnest O. Doebelin].

Test procedure during verification and inspection are:

a. Evaluation of error

At a certain load L, the indicated value, I is noted. Additional weights of say 1/10 e are successively added until the indication of the instrument is increased unambiguously by one scale interval (I+e). The additional load ΔL added to the load receptor gives indication P, by using the formula:

\[ P = I + \frac{1}{2} e \Delta L \]

The error is:

\[ E = P - L = I + \frac{1}{2} e - L - \Delta L \leq mpe \]

b. Weights

The standard weights used for verification of an instrument shall not have an error greater than 1/3 of the maximum permissible error of the instrument for the applied load.
3.1.2.1 Weighing tests
Apply test loads from zero up to and including maximum and similarly remove that test loads back to zero. The test loads selected shall include maximum, minimum and values at or near those at which the maximum permissible error changes. When loading or unloading, the weights shall be progressively increased or decreased. If the instrument is provided with an automatic zero-setting device, it shall remain in operation during test. Error is calculated using the formula 1.

3.1.2.1.2 Eccentricity Test
Large weights should be used in preference to several small weights. The load shall be applied centrally in the segment if several weights are used. The location of the load shall be marked on a sketch in the report. The automatic zero-setting device shall not remain in operation during the test.

In the case of instruments with a load receptor having more than four points of supports, the load shall be applied over each support on an area of the same order of magnitude as the fraction of 1/n of the surface area of the load receptor, where n is the number of points of support.

3.1.2.2 Weights
Weights starting from 20kg to 1mg, (23 different weights within the range of 20kg to 1mg) have been utilized for the study. Each of these are compared with the secondary standard weights and the difference is found out. The difference so obtained is the error. Corrections have been done if the error exceeds maximum permissible error (MPE).

3.1.2.3 Volumetric measures
Volumetric measures starting from 10 litres to 10 millilitres (10 different volumetric measures within 10 litre to 10ml) have been utilized for the study. The various measures are compared with secondary standards and the error in each case were found out. If it exceeds MPE corrective measures has been taken.

3.1.3 Analysis
The five years verification and test data of working standards such as non-automatic weighing instruments, weights and volumetric measures were used for year-wise and instrument-wise analysis.

Descriptive statistics such as mean, standard deviation and coefficient of variations has been found out. ANOVA tests, multiple comparison test and regression analysis on the data has been carried out [M. Buzoianu, 2000].

3.2. Limits of Maximum permissible errors on verification and Uncertainty

3.2.1 Uncertainty of measurement
According to the VIM [20] measurement uncertainty is a “parameter, associated with the results of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand”. Measurement uncertainty is usually made up of many components, some of which may be determined from the statistical distribution of the results of series of measurements and which can be characterized by experimental standard deviations [M. T. Clarkson, et. al., 2002]. The other components, which can also be characterized by standard deviations, are evaluated from assumed probability distributions based on experience or other information [Masayoshi, 1998].

Contributions to the measurement uncertainty are: the standards used, the measuring and test equipment used, the measuring methods, the environmental conditions, susceptibility to interference,
the state of the object to be measured or calibrated and the person performing the measurement or calibration [T. Lammi, 2001]. The *Guide to the Expression of Uncertainty in Measurement (GUM)* [16] and document [17] give detailed information on the determination of measurement uncertainties and a summary of the contributions.

### 3.2.2 Calculation of the measurement uncertainty

Basically, the determination of the measurement uncertainty refers to the calibration inherent in conformity verification. Therefore the procedures given in the GUM [16] and in [17] are applicable:

(a) Defining the objective

As a rule, the basic objective in legal metrology is the determination of the expanded measurement uncertainty \((k = 2)\), for the difference between the measuring instrument under test and the standard.

(b) Drawing up a model function

The model function expresses in mathematical terms the dependence of the measurand (output quantity) \(Y\) on the input quantities \(X_i\) according to the following equation:

\[
Y = f(X_1, X_2, ..., X_N) \quad \text{……..} \quad (2)
\]

In most cases it will be a group of analytical expressions which include corrections and correction factors for systematic effects (2). Where a direct comparison is being made between the indications shown by the instrument under test and the standard, the basic equation may be simple:

\[
Y = X_1 - X_2 \quad \text{……..} \quad (3)
\]

(c) Type A evaluation of uncertainty contributions

This is done by statistical analysis of a series of observations, normally by calculation of the arithmetic mean value and its experimental standard deviation.

### 3.2.3 Relationship between legally prescribed error limits and uncertainty

If a measuring instrument is tested for conformity with a given specification or with a requirement with regard to the error limits, this test consists of comparisons of measurements with those resulting from use of a physical standard or calibrated standard instrument. The uncertainty of measurement inherent in the measurement process then inevitably leads to an uncertainty of decision of conformity. Fig. 2 (taken from the standard ISO 14253-1) [21] makes this problem quite clear: between the conformance zones and the upper and lower non-conformance zones there is in each case an uncertainty zone whose width corresponds approximately to twice the expanded uncertainty of measurement at the 95% probability level [W. Shulz, et.al., 1999]. The uncertainty comprises contributions of the standard(s) used and the instrument under test as well as contributions that are related to the measuring procedure and to the incomplete knowledge about the existing environmental conditions. Because of the uncertainty of measurement, measurement results affected by measurement deviations lying within the range of the uncertainty zones cannot definitely be regarded as being, or not being, in conformity with the given tolerance requirement.
4. RESULTS AND DISCUSSIONS

4.1 Effect of ageing on measurement error

4.1.1 Non-Automatic Weighing Instrument(s)

The verification and test data of 20 various centers from the central laboratory for legal metrology has been taken. Five years data of each centre were collected and analyzed. In the laboratory, three different tests such as weighing test, eccentricity test and repeatability test were done as part of the verification and test procedure of non-automatic weighing instruments [NAWIs].

4.1.1.a Weighing test

In this test, standard load from 5 gm to 6200gm is applied and the error is found out (up error). Then the load is taken out from the receptor one by one to get the down error. The various weights that has been used for calibration are 5gm, 500gm, 2000gm, 5000gm and 6200gm. It is observed from the analysis that, the error exceeds maximum permissible error (MPE), error increases with ageing and varies from instrument to instrument. The mpe on service (MPES) is 0.1gm. It can be observed from Table II that the error is maximum with weighing test followed by eccentricity test and repeatability test. In the weighing test the error down is 3% more than error up. The weighing tests error is 98% more compared to the eccentricity test and 99% more compared to repeatability test. The instrument to instrument error difference is evident from Graph 1.

It can also be observed as is given in Table IX that, there is annual average growth (AAG) of error of 91% and 87.831% for weighing test-up and weighing test-down respectively. It can be observed from Table II that there is a significant error growth with respect to year. One-way Anova, Two way Anova and multiple comparison test also has been conducted to prove the significance of the findings. Two-way Anova results are given in Table III.
### Table II Comparison of different tests in NAWI

<table>
<thead>
<tr>
<th>Year</th>
<th>Weighing Test (mpe = 0.1gm)</th>
<th>Eccentricity Test (mpe=0.2mg)</th>
<th>Repeatability Test (mpe=0.3mg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Error up</td>
<td>Error down</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Mean 0.1051</td>
<td>0.1177</td>
<td>0.0053</td>
</tr>
<tr>
<td></td>
<td>SD 0.0597</td>
<td>0.0673</td>
<td>0.00389</td>
</tr>
<tr>
<td></td>
<td>CV 56.8316</td>
<td>57.1453</td>
<td>73.3962</td>
</tr>
<tr>
<td>2</td>
<td>Mean 0.1536</td>
<td>0.1581</td>
<td>0.0066</td>
</tr>
<tr>
<td></td>
<td>SD 0.0849</td>
<td>0.0894</td>
<td>0.0038</td>
</tr>
<tr>
<td></td>
<td>CV 55.2539</td>
<td>56.5465</td>
<td>57.5758</td>
</tr>
<tr>
<td>3</td>
<td>Mean 0.5812</td>
<td>0.5857</td>
<td>0.0078</td>
</tr>
<tr>
<td></td>
<td>SD 1.1149</td>
<td>1.0883</td>
<td>0.00463</td>
</tr>
<tr>
<td></td>
<td>CV 191.8324</td>
<td>185.8101</td>
<td>59.3590</td>
</tr>
<tr>
<td>4</td>
<td>Mean 0.6833</td>
<td>0.7077</td>
<td>0.0097</td>
</tr>
<tr>
<td></td>
<td>SD 1.1090</td>
<td>1.1414</td>
<td>0.00493</td>
</tr>
<tr>
<td></td>
<td>CV 162.3065</td>
<td>161.2873</td>
<td>50.8247</td>
</tr>
<tr>
<td>5</td>
<td>Mean 0.8521</td>
<td>0.8828</td>
<td>0.0116</td>
</tr>
<tr>
<td></td>
<td>SD 1.2789</td>
<td>1.2921</td>
<td>0.0044</td>
</tr>
<tr>
<td></td>
<td>CV 150.0833</td>
<td>146.3672</td>
<td>37.9310</td>
</tr>
</tbody>
</table>

### Graph 1. Instrument to Instrument Error Differences
### Table III: Two way ANOVA

**w-error (UP)**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>year</td>
<td>4</td>
<td>4.371</td>
<td>1.093</td>
<td>2.524</td>
<td>0.058</td>
</tr>
<tr>
<td>ws</td>
<td>19</td>
<td>21.486</td>
<td>2.387</td>
<td>5.514</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Residual</td>
<td>76</td>
<td>15.587</td>
<td>0.433</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>99</td>
<td>41.444</td>
<td>0.846</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**w-error (DOWN)**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>year</td>
<td>4</td>
<td>4.596</td>
<td>1.149</td>
<td>2.54</td>
<td>0.056</td>
</tr>
<tr>
<td>NAWI</td>
<td>19</td>
<td>21.242</td>
<td>2.36</td>
<td>5.218</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Residual</td>
<td>76</td>
<td>16.282</td>
<td>0.452</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>99</td>
<td>42.12</td>
<td>0.86</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 4.1.1.b. Eccentricity test

In the eccentricity test, for calibration, large weights are usually used in preference to several small weights. Same load [in this case 2000gm] are placed on different locations of the receptor of the electronic balance as shown below.

The maximum permissible error on service [MPES] is fixed as 0.2gm. The mean eccentricity test error exceeds MPES by 4%. The eccentricity error varies from instrument to instrument [typical mean error variation of 0.0034 to 0.0158 was observed]. The annual average growth [AAG] is 21.9% for eccentricity test.

![Receptor](image)

The eccentricity test error growth with ageing is evident from Table II and Two-way Anova test given in Table IV also gives the significance of error growth. Multiple comparison test also validate the difference between various instruments. Eccentricity test error is 55% more compared to the repeatability test error.

### Table IV: Two-way Anova - Eccentricity test error

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>year</td>
<td>4</td>
<td>0.00025</td>
<td>6.26E-05</td>
<td>23.786</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>NAWI</td>
<td>19</td>
<td>0.000757</td>
<td>8.41E-05</td>
<td>31.957</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Residual</td>
<td>76</td>
<td>9.47E-05</td>
<td>2.63E-06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>99</td>
<td>0.0011</td>
<td>2.25E-05</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 4.1.1.c. Repeatability test [r-test]

In repeatability test two series of weighing shall be performed, one with a load of about 50% and one with a load close to 100% of maximum. Readings shall be taken when the instrument is loaded, and when unloaded. Here 3000gm and 6000gm has been used. Each weight is loaded and unloaded 5times.
The maximum permissible error on service [MPES] of repeatability test is 0.3. The error grows with ageing. There is an instrument to instrument variation. The MPES prescribed by the department is maximum with repeatability test and minimum with weighing test. The mean error exceeds MPES is minimum with repeatability test.

The average annual growth [AAG] of repeatability test error is 54.81%. Two-way Anova test confirms the significance of error growth with ageing as given in Table V.

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>year</td>
<td>4</td>
<td>0.000239</td>
<td>5.97E-05</td>
<td>36.505</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>NAWI</td>
<td>19</td>
<td>0.000104</td>
<td>1.15E-05</td>
<td>7.032</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Residual</td>
<td>76</td>
<td>5.89E-05</td>
<td>1.64E-06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>99</td>
<td>0.000401</td>
<td>8.19E-06</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.1.2 Weights

The working standard solid weights starting from 20kg to 1mg having 23 different denominations within the range were yearly subjected to verification and testing and the error have been found.

If the error exceeds maximum permissible error on service [MPES] corrective measures have been taken to keep it within MPES. Five years verification and test data of 20 different working standards, each having 23 denominations has been analyzed. It has been found from the studies that with ageing the mean error increases as it is evident from Graph 3. The mean error variations from working standard to working standard ranges from 2.1194 to 11.9431. It is also found, there is an average annual average growth of 30.31. in the case of weight measures during verification and testing, correction also were done, if the observed error exceeds MPES.
4.1.3 Volumetric Measures [v-error]

In this study five years calibration data of 20 conical measures has been taken and analyzed. The conical measures shall be fabricated from galvanized steel sheets, Aluminium alloy sheets, copper sheets, brass sheets, stainless steel sheets or tin plate. The measures are so designed that, when they are tilted 120 degree from the vertical, they become empty.

Twenty working standards of liquid measures have been taken for the study. Each working standard consists of 10 different liquid measures which is capable to measure from 10000ml to 10ml. Table VI shows the MPES and error of a typical working standard.

Table VI Typical Error with respect to MPE for Volumetric measures

<table>
<thead>
<tr>
<th>Volume (ml)</th>
<th>Error (ml)</th>
<th>MPE(s) (ml)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10000</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>5000</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>2000</td>
<td>3.2</td>
<td>2</td>
</tr>
<tr>
<td>1000</td>
<td>2.7</td>
<td>1.5</td>
</tr>
<tr>
<td>500</td>
<td>1.9</td>
<td>1</td>
</tr>
<tr>
<td>200</td>
<td>1.85</td>
<td>0.8</td>
</tr>
<tr>
<td>100</td>
<td>1.36</td>
<td>0.6</td>
</tr>
<tr>
<td>50</td>
<td>1.1</td>
<td>0.5</td>
</tr>
<tr>
<td>20</td>
<td>1.15</td>
<td>0.4</td>
</tr>
<tr>
<td>10</td>
<td>0.41</td>
<td>0.2</td>
</tr>
</tbody>
</table>

The growth of error above MPES with ageing is shown in graph 4. Annual average growth of error is 23.82 and it is minimum compared to weight measures and non-automatic weighing instrument. One-way Anova given in Table VII proves the significance of error increase with year.

There is a significant error difference from working standard to working standard as is evident from Table VIII. The mean error above MPES ranges from 0.2323 to 1.2893.
4.1.3 Regression Analysis

Regression is the determination of a statistical relationship between two or more variables. In simple regression, there are only two variables, one variable [defined as independent] is the cause of the behavior of another one [defined as dependent variable]. Regression can only interpret what exists physically i.e. there must be a physical way in which independent variable x can affect dependent variable y.

In this analysis year [x] has been taken as independent variable and error [y] has been taken as the dependent variable.

\[ \text{Graph 4} \quad \text{Error growth with ageing Volumetric Measures} \]

**Table VII** Oneway ANOVA - v-error

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Groups</td>
<td>1.173</td>
<td>4</td>
<td>.293</td>
<td>2.914</td>
<td>.032</td>
</tr>
<tr>
<td>Within Groups</td>
<td>4.526</td>
<td>45</td>
<td>.101</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table VIII** - Two way ANOVA – v-error

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>year</td>
<td>4</td>
<td>1.173</td>
<td>.293</td>
<td>31.497</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>ws</td>
<td>19</td>
<td>4.191</td>
<td>.466</td>
<td>50.038</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Residual</td>
<td>76</td>
<td>0.335</td>
<td>0.00931</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The regression analysis equations for various working standards are derived as follows.

1. Electronic balance
   a. Weighting test Error-up equation \( Y = 0.2024x - 405.9 \)
   b. Weighing test Error-down equation \( Y = 0.208x - 417.133 \)
   c. Eccentricity Test error \( Y = 0.002x - 3.149 \)
   d. Repeatability Test error \( Y = 0.002x - 3.021 \)

2. Weight measure standards \( Y = 1.011x - 2024.436 \)

3. Volumetric measure standards \( Y = 0.108x - 216.57 \)

The error growth with ageing is given in Table IX for various working standards.

### Table IX. Table showing regression analysis

<table>
<thead>
<tr>
<th>YEAR</th>
<th>error up</th>
<th>error down</th>
<th>e test error</th>
<th>r test error</th>
<th>weight</th>
<th>volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>.1051</td>
<td>.1177</td>
<td>.005275</td>
<td>.001264</td>
<td>3.2544</td>
<td>.3331</td>
</tr>
<tr>
<td>2007</td>
<td>.1536</td>
<td>.1581</td>
<td>.006638</td>
<td>.001735</td>
<td>4.2288</td>
<td>.4585</td>
</tr>
<tr>
<td>2008</td>
<td>.5812</td>
<td>.5857</td>
<td>.007800</td>
<td>.002957</td>
<td>8.8420</td>
<td>.5514</td>
</tr>
<tr>
<td>2009</td>
<td>.6833</td>
<td>.7077</td>
<td>.009651</td>
<td>.005371</td>
<td>6.6310</td>
<td>.6530</td>
</tr>
<tr>
<td>2010</td>
<td>.8521</td>
<td>.8828</td>
<td>.011631</td>
<td>.006978</td>
<td>7.1092</td>
<td>.7765</td>
</tr>
<tr>
<td>AAG</td>
<td>91.72</td>
<td>87.58</td>
<td>21.90</td>
<td>54.81</td>
<td>30.31</td>
<td>23.82</td>
</tr>
<tr>
<td>Constant</td>
<td>-405.88</td>
<td>-417.13</td>
<td>-3.14</td>
<td>-3.02</td>
<td>-2024.43</td>
<td>-216.57</td>
</tr>
<tr>
<td>Year</td>
<td>0.202</td>
<td>0.208</td>
<td>0.002</td>
<td>0.002</td>
<td>1.011</td>
<td>0.108</td>
</tr>
</tbody>
</table>

### 4.2 Limits of maximum permissible errors on verification and uncertainty

#### 4.2.1. Relationship upon verification

In practice, measuring instruments are considered to comply with the legal requirements for error limits if:

- the absolute value of the measurement deviations is smaller than or equal to the absolute value of the legally prescribed MPES on verification when the test is performed under prescribed test conditions; and
- the expanded uncertainty of measurement of the previous quantitative metrological test, for a coverage probability of 95\%, is small compared with the legally prescribed error limits [Rogosa D. R., et. al., 1983].

The expanded measurement uncertainty at the 95\% probability level, \( U_{0.95} \), is usually considered to be small enough if the following relationship is fulfilled:

\[
U_{0.95} \leq \frac{1}{3} \cdot MPEV \tag{4}
\]

where \( MPEV \) is the absolute value of the MPE on verification.

\( U_{\text{max}} \) [\( U_{0.95} \)] is, therefore, the maximum acceptable value of the expanded measurement uncertainty of the quantitative test. Based on the initial verification data of typical six non-automatic weighing instruments, the criteria for the assessment of compliance are illustrated in Fig. 3, cases a, b, c and f comply with the requirements of the verification regulations, whereas cases d and e will be
rejected. Values in all cases, including their uncertainty of measurement, lie within the tolerances fixed by the MPES in service. It was observed that, the MPE on verification of a newly verified measuring instrument will in the worst case be exceeded by 33%. However, as the legally prescribed MPES in service are valid for the instrument users [S. Bell, 1999], there is, therefore, negligible risk in the sense that no measured value under verification - even if the measurement uncertainty is taken into account - will be outside this tolerance band.

![Diagram](image)

**Fig. 3** Illustration of the criteria for the assessment of compliance in legal metrology and of the bandwidth of measurement deviations $I(E)$ of verified instruments that could be expected when taking the uncertainty $U_{\text{max}}$ into account

- $MPEV$: absolute value of the maximum permissible error on verification
- $MPEV-$: lower maximum permissible error on verification
- $MPEV+$: upper maximum permissible error on verification
- $MPES-$: lower maximum permissible error in service
- $MPES+$: upper maximum permissible error in service
- $U_{\text{max}}$: upper permissible limit of the expanded uncertainty of measurement according to equation
- $y$: best estimate of measurement deviation, $E$
- $I(y)$: acceptance interval with respect to the measurement deviation, $y$
- $A$: range of possible occurring measurement deviations

Due to unavoidable measurement uncertainties from the quantitative tests Fig.3, the legally prescribed error limits $MPEV-$ and $MPEV+$ can be exceeded by the value of $U_{\text{max}}$ without being recognized. Therefore, an interval $I(E)$ may be defined that characterizes the possible bandwidth of measurement errors when using verified instruments.

So far, the MPES on verification may be seen as supporting the conclusion that an instrument would be in conformity with required MPES in service ($MPES$) taking into consideration the above-mentioned influences. The advantages of this verification system are that it is practical in terms of legal enforcement, and due to the widened tolerance band in service ($MPES-$ to $MPES+$), it is potentially tolerant of external influences and of drifts in indication over the legally fixed validity periods. Verification validity only expires early in cases of unauthorized manipulations and damage that could reduce the accuracy of the instrument [Stanley J. C., 1971].
4.2.2. Relationship upon testing of working standards

In legal metrology, working standards are the standards that are used routinely to verify measuring instruments. In several economies, some of the working standards used in legal metrology must be tested or verified according to special regulations [Klaus-Dieter Sommer, et. al., 1994] The MPES of such working standards depend on their intended use. In general, they should be significantly lower than the expanded uncertainties that are required by equation (4).

![Diagram of relationship between MPES and measurement uncertainty upon conformity evaluation in calibration](ref:17) and testing of working standards

**Fig. 4** Relationship between MPES and measurement uncertainty upon conformity evaluation in calibration (ref. 17) and testing of working standards

- **Ys**: conventional true value
- **Y**: best estimate of the measurement deviation, E
- **MPEws-**: Lower limit of maximum permissible error of working standard
- **MPEws+**: Upper limit of maximum permissible error of working standard
- **I[y]**: Acceptance interval with respect to the measurement deviation y.

Usually, a working standard, e.g. non-automatic weighing instrument is considered to comply with the respective requirements for legal error limits if the difference between its indication, or measured value, and the corresponding value realized by a reference standard is equal to or less than the difference between the prescribed error limits, MPEws, and the expanded uncertainty of measurement, \( U_{0.95} \):

\[
|I_{ws} - y_s| \leq MPE_{ws} - U_{0.95} \quad \text{..................................................(5)}
\]

where:

- **I_{ws}** = the indication of the working standard under test;
- and
- **y_s** = the value provided by a reference standard.
In practice, this means that with respect to measurement deviations, a tolerance band is defined that is significantly reduced when compared with the range between the legally prescribed error limits \([MPE_{ws} - , MPE_{ws} +]\) (see fig. 4). The magnitude of this tolerance band may be described by the interval \([MPE_{ws} - + U \text{ to } MPE_{ws} + - U]\).

This approach is consistent with the prescribed procedures for statements of conformity on calibration certificates.

4.2.3 Uncertainty contribution of verified instruments

In practice, it is often necessary or desirable to determine the uncertainty of measurements that are carried out by means of legally verified measuring instruments. If only the positive statement of conformity with the legal requirements is known, for example in the case of verified instruments without a certificate, the uncertainty of measurements for such instruments can be derived only from the information available about the prescribed error limits (on verification and in service) and about the related uncertainty budgets according to the requirements established in 3.1 and 4.2.1 earlier.

On the assumption that no further information is available, according to the principle of maximum entropy, the following treatment is justified:

- The range of values between the MPES on verification can be assumed to be equally probable [T. Lammi, 2001]
- Due to uncertainty in measurement, the probability that indications of verified instruments are actually beyond the acceptance limits of the respective verification declines in proportion to the increase in distance from these limits [J. Ren-e van Dorp, et. al., 2003]. A trapezoidal probability distribution according to Fig. 5 can, therefore, reflect adequately the probable dispersion of the deviation of verified measuring instruments [J. R. Van Dorp et. al. 2007].
- Immediately after verification, the indications of measuring instruments may exceed the MPES on verification by the maximum value of the expanded uncertainty of measurements at most.
- After prolonged use and under varying environmental conditions, it can be assumed that the expanded measurement uncertainty, compared with its initial value, may have increased significantly.

\[
u = \frac{1}{1.33MPEV \sqrt{(1 + \beta^2) / 6}} \quad ; \quad \beta = \frac{3}{4}
\]

\[
u = \frac{1}{2 \times MPEV \sqrt{(1 + \beta^2) / 6}} \quad ; \quad \beta = \frac{1}{2}
\]

Fig. 5 Suggested probability distributions for evaluating the standard uncertainty contribution of verified measuring instruments
In particular, the following evaluation of the uncertainty contribution of verified instruments seems to be appropriate:

(i) Immediately after verification, the trapezoidal probability distribution of the errors according to plot (a) of Fig. 8 can be taken as a basis for the determination of the uncertainty contribution of the instruments.

The following may, therefore, be assumed for this standard uncertainty contribution $U_{INSTR}$ (J. R. Van Dorp et.al. 2007).

$$U_{instr} = \frac{1}{a \sqrt{(1 + \beta^2) / 6} = 0.70MPEV}$$

where $a = 1.33 MPEV$, $\beta = \frac{3}{4}$

$MPEV$ is the absolute value of the MPES on verification.

(ii.) After prolonged use and under varying environmental conditions, it can be assumed that, in the worst case, the measurement error extended by the measurement uncertainty will reach the values of the MPES in service. The resulting trapezoidal distribution could more or less be represented by plot (b.) of Fig. 5. In this case, the following may be assumed for the standard uncertainty contribution [J.G.M. Grinten, et.al. 1994]:

$$U_{instr} = \frac{1}{a \sqrt{(1 + \beta^2) / 6 = 0.90MPEV}$$

where: $a = 2 MPEV$, $\beta = \frac{1}{2}$

CONCLUSION

In this research work five years verification and test data of various working standards such as non-automatic weighing instrument, weights and volumetric measures were analyzed for time variance to check the error growth with ageing and also a study has been conducted to find out the relationship between legally prescribed error limits and uncertainty.

In the verification and test data analysis, it is observed that the error on an average exceeds maximum permissible error [MPE]. The error increases with ageing of the instrument and there was a significant error variation from instrument to instrument. Three tests namely weighing test, eccentricity test and repeatability test were conducted in the verification procedure of non-automatic
The annual average growth [AAG] of error was maximum with weighing test and is 89.8% followed by repeatability test were AAG was 54.81% and AAG was 21.9% in eccentricity test. Verification results of twenty sets of working standards of weight measures, each having 23 different solid weights ranging from 20kg to 1mg have been analyzed. It was observed that there was an AAG of error of 30.31%. The mean error variation from working standard to working standard ranges from 2.1194 to 11.9431.

The mean error above MPES ranges from 0.2323 to 1.2893 for different sets of volumetric measure working standards. The AAG was minimum with volumetric measure compared to weights and non-automatic weighing instrument and was only 23.82. One-way, two-way ANOVA and multiple comparison tests were conducted to check the significance of the findings and found that the analysis was significant. Regression analysis has been conducted and the regression equations were derived.

The maximum permissible error is a concept which does not take into account the uncertainty of measurement. The study has extended to a market surveillance focusing on the performance on non-automatic weighing instrument which are practically in use, not on the needs of users and to introduce the concept of uncertainty in the criteria of conformity in type approval and verification. The following are the conclusion obtained from the investigation.

1. In type approval tests, if the measurement uncertainty is equal to or less than one third of the MPES, the measurement uncertainty should be considered in conformity decision. In this case, if the measurement results including the measurement uncertainty lie within the MPE, it should be decided as conforming, and the rest, non-conforming. Before applying these criteria, the consensus among the manufacturers of each instrument shall be achieved.

2. Technical procedures followed in verification is equivalent to those used in calibration and provides confidence in the correctness of indication of verified instruments although no expert knowledge by the instrument’s user is required. Verification therefore, may be considered a strong tool in both legal metrology and quality assurance, when large numbers of measuring instruments are involved. In particular, it excels as a single means by which enforcement can be realized, and because the user is only affected by the MPES in service, it provides a high degree of confidence over a long time period.

3. One disadvantage in verification is that the influence of uncertainty on a decision of conformity of a measuring instrument to specific requirements is not completely clear.

In comparison, traditional calibration is considered an important basic procedure for legal metrology activities and also for fundamental measurement application in scientific and industrial metrology. It is practically not limited as far as the measurement task is concerned, but does require sound expert knowledge on the part of the instrument’s user in carrying out and evaluating measurements.

The legal metrology department is handling all type of working standards, which includes the working standard for weighing precious metals and upto weigh bridge. They follow uniform procedures for the conformity study of various types of working standards. The findings of this study of uncertainty can be incorporated for the unsurpassed conformity, at-least, of the instruments for sophisticated measurements. The regression analysis curve can be used as a bench-mark for the error growth with ageing of the instrument. Verifying authorities may fit their observed values in the regression curve and if these values are well above the regression curve, the instruments can be rejected.
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BIOGRAPHIES

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