APPLICATION OF MEDIAN MODEL AND GOAL PROGRAMMING MODELS FOR SELECTION OF TRANSSHIPMENT LOCATION OF A POWER UTILITY COMPANY HAVING TWO POWER PLANTS AND TWO CAPTIVE COAL MINES TO SUPPLY COAL

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ABSTRACT

A coal fired power generating company was seeking an intermediate location. The decision of intermediate location was solved by using median model and goal programming approaches. Decisions/results of both the methods are discussed in this research paper with a case example.

1.0 INTRODUCTION

Indian Power Company generates electricity from coal. It has two power generating plants in different geological location in the eastern part of India. Initially Indian power plant used to get coal for its power plant from nationalized coal company. But after the reform of Indian Government, Indian power Supply Company takes its coal from its own coal mines. It has selected two coal blocks for open cast coal mines to supply the coal of its two power generating companies. This vertical integration has obviously will help the company to reduce the power generating cost per unit.

The company wants to reduce its power generating cost. The management is thinking of to keep its coal in an intermediate location. Coal required to supply the two power plants should be supplied from this intermediate coal stocking yard. Management of Indian power generating company is thinking that this transshipment location of stocking coal may reduce the transportation cost of coal. It will also increase the reliability to supply of coal to the power plants.

2.0 PROBLEM

Indian Power Company has to locate a new intermediate coal stocking yard. This location will receive shipments of coal from two sources: Mine M1 and mine M2. This intermediate coal
stocking yard will supply coal to the power generating plants P1 and P2. The coal production capacities of the two mines are 455 million ton/yr and 550 million ton/yr respectively. The requirement of coal per year for P1 and P2 are 450 million ton/yr and 500 million ton/yr respectively. The locations of mines and power plants with respect to head office at Kolkata (considering it origin) are shown in the Cartesian plane in the following table.

<table>
<thead>
<tr>
<th>Location</th>
<th>Annual Loads between the Intermediate location and power plants and coal mines (in million ton per year)</th>
<th>Cost C_i to move one million ton to one K.M. (1 lakh per million ton)</th>
<th>Coordinate Location (x_i,y_i) of existing mines and power plants w.r.t. the Head office (in Kilometers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>455</td>
<td>1</td>
<td>20, 30</td>
</tr>
<tr>
<td>P2</td>
<td>550</td>
<td>1</td>
<td>10, 40</td>
</tr>
<tr>
<td>P2</td>
<td>450</td>
<td>1</td>
<td>30, 50</td>
</tr>
<tr>
<td>P2</td>
<td>500</td>
<td>1</td>
<td>40, 60</td>
</tr>
</tbody>
</table>

The location of Intermediate/transshipment coal stocking yard (x_o,y_o) is to be determined in the Cartesian plane.
3.0 SOLUTION

This can be solved using a simple median model.

This model considers the volume/quantity of loads transported on rectangular paths [4] (R.C. Vergin and J.D. Rogers “An Algorithm and computational Procedure for Locating Economic facilities”, Management Science 13, no. 6, Feb1967, pp 242-54.). All movements are made in east-west or north-south directions; diagonal moves are not considered. The simple median model provides an optimal solution.

Since all loads must be on rectangular paths, distance between each existing facility and the new plant (intermediate coal stocking place) will be measured by the difference in the x-coordinates and the difference in the y-coordinates. If we let \((x_o, y_o)\) be the coordinates of a proposed intermediate/transshipment location, then

\[ D_i = |x_o - x_i| + |y_o - y_i| \]

Notice we calculate the absolute value of the differences, because distance is always positive. Notice too we could have written

\[ D_i = |x_i - x_o| + |y_i - y_o| \]

Our goal is to find values for \(x_o\) and \(y_o\) (location of intermediate coal stocking yard/transshipment location) that result in minimum transportation costs. We follow three steps:

1. Identify the median value of loads \(L_i\) moved.
2. Find the x-coordinate of the existing facility that sends (or receives) the median load.
3. Find the y-coordinate of the existing facility that sends (or receives) the median load.

The x and y-coordinates found in steps 2 and 3 define the new plant/intermediate coal stocking yard’s best location.

Application of the Model: Let us apply the three steps to the data in Table 1 above.

1. Identify the median load: The total number of tons moved to and from the intermediate location will be 1955. If we think of each number of load/ton individually and number them from 1 to 1955, then the median load/ton number is the “middle” number --- that is the number for which the same number of loads/tons fall above and below. For 1955 loads, the median load number is 978, since 977 numbers of loads/tons falls above and below load number 978. If the total number of loads/tons were even we would consider both middle numbers.

2. Find the x-coordinate of the median load. First we consider movement of loads in the x-direction. Beginning at the origin of figure 1 and moving to the right along the x-axis, observe the number of loads moved to or from existing facilities. Load/ton 1 – 550 are shipped by \(M_2\) from location \(x=10\). Loads/tons 551 - 1005 are shipped by \(M_1\) from location \(x=20\).Since the median load falls in the intervals 551 – 1005, \(x = 20\) is the desired x-coordinate location of the intermediate/transshipment location.

3. Find y coordinate of the median load. Now consider the y-direction of load movements. Begin at the origin of figure 1 and move upward along the y-axis. Movements in the y direction begin with loads/tons 1 – 455 being shipped by \(M_1\) from location \(y = 30\). Loads 456 – 1005, are shipped by \(P_1\) from location \(y = 40\). Since the median load falls, in the interval 551 – 1005, \(y= 40\) is the desired y-coordinate for the new plant.
The optimal intermediate coal stocking yard/plant location, \(x = 20\) and \(y = 40\), results in minimizing annual transportation costs for this network of facilities. To calculate the resulting cost, we use the following equation to calculate \(D_i\) and use these values in the equation.

\[
\text{Total transportation cost} = \sum_{i=1}^{n} C_i L_i (|x - x_i| + |y - y_i|)
\]

The total transportation cost is shown in the Table.

### TABLE: 2

<table>
<thead>
<tr>
<th>Existing Facility (F_i)</th>
<th>x-coordinate of (F_i)</th>
<th>y-coordinate of (F_i)</th>
<th>Distance between (F_i) and Intermediate location</th>
<th>Annual Load (L_i) between (F_i) and intermediate location</th>
<th>Cost to move 1 ton 1 unit distance from (F_i) and intermediate location (in lakh Rs)</th>
<th>Annual transportation cost for (F_i): (C_i D_i L_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>20</td>
<td>30</td>
<td>(</td>
<td>20 - x_i</td>
<td>+</td>
<td>40 - y_i</td>
</tr>
<tr>
<td>M2</td>
<td>10</td>
<td>40</td>
<td>(</td>
<td>20 - x_i</td>
<td>+</td>
<td>40 - y_i</td>
</tr>
<tr>
<td>P1</td>
<td>30</td>
<td>50</td>
<td>(</td>
<td>20 - x_i</td>
<td>+</td>
<td>40 - y_i</td>
</tr>
<tr>
<td>P2</td>
<td>40</td>
<td>60</td>
<td>(</td>
<td>20 - x_i</td>
<td>+</td>
<td>40 - y_i</td>
</tr>
</tbody>
</table>

Total transportation cost == \(\sum_{i=1}^{n} C_i L_i (|x - x_i| + |y - y_i|)\) = Rs 39050

### 3.1 ALTERNATIVE METHOD OF SOLUTION:

In this paper another attempt has been made to solve the transshipment location of the Indian Power generating company with the use of Goal programming [1]. The coordinate location of the mines are M1 (20, 30), M2 (10, 40), and power generating plants P1 (30, 50), and P2 (40, 60). The location of the intermediate location/plant will be such that it will minimize total distance from the mines and power plants. Here the distance is to be measured rectangular. For example, if the location of the intermediate place is \((x_1=30, y_1=50)\), it is considered to be a distance of \(|30 - 20| + |50 - 30| = 10 + 20 = 30\) from mine M1.

In the goal programming formulation, let

\[
x = \text{first coordinate/ } x - \text{coordinate of the intermediate/transshipment location}
\]
\[
y = \text{second coordinate/ } y - \text{coordinate of the intermediate/transshipment location}
\]

\[
d^-_i = \text{amount by which } x \text{ coordinate of transshipment location exceeds } x \text{ coordinate of mines and power plants } i \ (i = 1, 2, 3, 4). \text{ Here } i=1, 2 \text{ for two mines and } i=3,4 \text{ for two power plants. More elaborately, here, } x1=20, x2=10, x3=30, x4=40. \text{ This is also called positive deviation variable.}
\]

\[
d^+_i = \text{amount by which } x \text{ coordinate of mines and power plants } a \text{ exceeds } x \text{ coordinate of transshipment location. This is } negative \text{ deviation variable.}
\]

\[
e^-_i = \text{amount by which } y \text{ coordinate of transshipment location exceeds } y \text{ coordinate of mines and power plants } i \ . \text{ A positive deviation variable.}
\]

\[
e^+_i = \text{amount by which } y \text{ coordinate of mines and power plants } i \text{ exceeds } y \text{ coordinate of transshipment location. } A \text{ negative deviation variable.}
\]
The goal programming model is given below.

Minimize:

\[ Z = \sum_{i=1}^{n} d_1^+ + d_1^- + e_1^+ + e_1^- + d_2^+ + d_2^- + e_2^+ + e_2^- + d_3^+ + d_3^- + e_3^+ + e_3^- + d_4^+ + d_4^- + e_4^+ + e_4^- \]

Subject to the constraints:

\[ x + d_1^- - d_1^+ = 20 \quad \text{...(eqn. no. 1)} \]
\[ y + e_1^- - e_1^+ = 30 \quad \text{...(eqn. no. 2)} \]
\[ x + d_2^- - d_2^+ = 10 \quad \text{...(eqn. no. 3 )} \]
\[ y + e_2^- - e_2^+ = 40 \quad \text{...(eqn. no. 4 )} \]
\[ x + d_3^- - d_3^+ = 30 \quad \text{...(eqn. no. 5)} \]
\[ y + e_3^- - e_3^+ = 50 \quad \text{...(eqn. no. 6)} \]
\[ x + d_4^- - d_4^+ = 40 \quad \text{...(eqn. no. 7)} \]
\[ y + e_4^- - e_4^+ = 60 \quad \text{...(eqn. no. 8)} \]

The optimal solution is given by

\[ x = 30 \]
\[ y = 40 \]

To solve this problem “TORA” software [3] has been used. The solution is not shown but if reviewer/researcher wants it can be supplied on request to the author.

The total transportation cost (using goal programming method) is shown in the Table below:-.

<table>
<thead>
<tr>
<th>Existing Facility F_i</th>
<th>x-coordinate of F_i</th>
<th>y-coordinate of F_i</th>
<th>Distance between F_i and Intermediate location</th>
<th>Annual Load L_i between F_i and Intermediate location</th>
<th>Cost to move 1 ton 1 unit distance from F_i and Intermediate location (inRs lakh)</th>
<th>Annual transportation cost for F_i: C_iD_iL_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>20</td>
<td>30</td>
<td>x-direction</td>
<td>30 - x_i</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>M2</td>
<td>10</td>
<td>40</td>
<td>x-direction</td>
<td>30 - x_i</td>
<td>10</td>
<td>10</td>
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<tr>
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<td>30</td>
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<td>x-direction</td>
<td>30 - x_i</td>
<td>10</td>
<td>10</td>
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<tr>
<td>P2</td>
<td>40</td>
<td>60</td>
<td>x-direction</td>
<td>30 - x_i</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Total transportation cost \[ = \sum_{i=1}^{n} C_iL_i (|x - x_i| + |y - y_i|) = 39600 \text{ (Rs)} \]

4.0 LIMITATION

First, we have considered the case in which only one new intermediate/ transshipment location is to be added. Second, we should note an important assumption of this model: Any point in
the x – y coordinate system is an eligible point for locating the new intermediate location/transshipment location. The model does not consider road availability, physical terrain, population densities, or any other of the many important location considerations. The task of blending model results with other major considerations to arrive at a reasonable location choice is a major managerial responsibility[2].

5.0 ADVANTAGES

For simplicity we have considered the cost of transport of one ton of coal to one unit distance is 1 lakh rupee. But we can take variable transportation cost also for each pair of locations.

6.0 COMPARISON OF THE TWO MODELS

If we compare the results of these two models, we will observe that the median model has given the best result as the total transportation cost is lower. Although mathematically, the median model is less precision than that of goal programming methods. This has happened perhaps too many compromises in the goal programming methods. Reason of worse result in goal programming is still a question to be investigated in future research works.

7.0 REFERENCES