AN ANALYSIS OF STABILITY OF TRENDS IN GOLD PRICES USING FRACTAL DIMENSION INDEX (FDI) COMPUTED FROM HURST EXPONENTS

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ABSTRACT

Chaos is a nonlinear, dynamic system that appears to be random but is actually a higher form of order. All chaotic systems have a quantifying measurement known as a fractal dimension. The fractal dimension index (FDI) is a specialized tool that applies the principles of chaos theory and fractals. With FDI one can determine the persistence or anti-persistence of any equity or commodity. In this paper we study the data from gold rates by computing the fractal dimension index. The fractal dimension index is computed from the Hurst exponent and the Hurst exponent is computed from Rescaled Range R/S.

Keywords: chaos, fractals, rescaled range, persistence.

1. INTRODUCTION

In Chaos theory, we define an apparently random event in the marketplace that has some degree of predictability. In order to do this we use fractal as a tool that is a representation of order from chaos. The fractal is commonly defined as an object with self-similar individual parts. In the markets, a fractal can be thought of as an object or “time series” that appears similar across a range of scales. Each frame may zigzag a little differently, but when viewed from afar they have similar attributes on each scale.
All chaotic systems have a quantifying measurement known as a fractal dimension. The fractal dimension is a non-integer dimension that describes how an object takes up space. Fractals will maintain their dimension regardless of the scale used. This is evident in natural phenomena such as mountains, coastlines, clouds, hurricanes and lightning. Similarity across scales is essential in trading because each time frame of a market will have a similar fractal pattern. Such markets can only be forecasted reliably with principles applicable to nonlinear, natural systems using Fractal geometry as tool.

2. FRACTAL DIMENSION INDEX (FDI)

The fractal dimension index (FDI) is a tool that applies the principles of chaos theory and fractals. This specialized indicator identifies the fractal dimension of the market by using re-scaled range analysis and an estimated Hurst exponent. It does so by using all available data on the time/price chart to determine the “volatility” or “trends” of a given market. FDI is the same type of tool used by eminent fractal scholars Benoit Mandelbrot, H.E. Hurst, and Edgar Peters in their examinations of time series analysis. With FDI we can determine the persistence or anti-persistence of any equity or commodity that we display in our graphing program. A persistent time series will result in a chart that is less jagged, subject to fewer reversals, and resembles a straight line. An anti-persistent time series will result in a chart that is more jagged and prone to more reversals. Essentially, FDI will tell us whether a market is a random, independent system or one with bias. The FDI is based on the work of Mandelbrot and Hurst.

3. TRADING WITH THE FDI

The FDI is useful to determine the amount of market volatility. The easiest way to use this indicator is to understand that a value of 1.5 suggests that the market is acting in a completely random fashion. As the market deviates from 1.5, the opportunity for earning profits increases in proportion to the amount of deviation. The entire scale is based on a range of 1-2, suggesting extreme linearity to extreme volatility. If the FDI is closer 2, the probability is higher that the next move will be in the opposite direction of the current move. An FDI closer to 1 signals a trending market in one direction. This knowledge alone gives the trader an incredible advantage, because it can indicate which markets have the most opportunity.
4. COMPUTING THE FDI

For computing the FDI one must first compute the Hurst exponent. For a time series \( x_1, x_2, \ldots, x_n \)

\[
(R/S)_n = c \cdot n^H
\]  

(1)

Hurst computed the Rescaled Range \((R/S)_n\) and found that

\[
\log ((R/S)_n) = \log (c) + H \cdot \log (n)
\]  

(2)

Where \( c \) and \( H \) are constants and \( H \) is the **Hurst coefficient**.

Thus \( H \) is obtained as the slope of the Log \((R/S)_n\) versus Log \(n\) plot. \( H = 0.50 \) implies that the time series is a random time series and implies the absence of long term statistical dependence. The case \( 0.50 < H \leq 1.00 \) implies a persistent time series, a time series characterized by long memory effects. This means that if the trend has been positive in the last observed period, the chances are that it will continue to be positive in the next period. Conversely if it has been negative in the last period it is more likely that it will continue to be negative in the next period. The level of persistence is judged by how far \( H \) is above 0.5. The case \( 0 \leq H < 0.50 \) implies anti-persistent time series. An antipersistent system reverses itself more frequently than a random one.

After the computation of the Hurst exponent, we derive the fractal dimension of the time series which easily accomplished using the formula \( D = 2 - H \). The value of “\( D \)” is the fraction of a dimension between 1 and 2 that the price data represents. Logarithmic returns are used to compute FDI. Because logarithmic returns sum to cumulative returns, most analyst agree that this is most appropriate for financial analysis. When experimenting with the FDI, a large data set is required. The results may appear distorted if there is not enough data.

5. CORRELATION BETWEEN PERIODS

The correlation \( C_N \) between periods is calculated as follows:

\[
C_N = 2^{(2H-1)} - 1
\]

A random time series \( (H = 0.5) \) has zero correlation. A persistent time series (with \( H \) greater than 0.5) results in positive correlation where as an anti-persistent time series
results in negative correlation. \( C_N \) is a measure of the long-term memory present in the time series. \( C_N = 0.25 \) implies that 25% of the data is influenced by the past.

6. **THE V STATISTIC:**

\[
V_n = \frac{(R/S)_n}{\sqrt{n}}
\]

The V statistic was originally used by Hurst for testing stability. If the process is an independent random process, then the plot of \( V \) versus \( \log n \) will be flat. If the process is persistent (\( H > 0.5 \)) then the graph will be upwardly sloping and if the process is antipersistent (\( H < 0.5 \)) the graph will be downward sloping.

7. **DATA ANALYSIS:**

The monthly gold rates (in rupees) required for study was collected from Jan 1971 to Apr 2010 spanning 40 years obtained from World Gold Council.

![Figure 1 Graph of log n versus log R/S](image1)

![Figure 2 Graph of log n versus Vn plot](image2)

8. **CONCLUSION**

From the study carried out, the Hurst Constant \( H = 1.047 \).

- The fractal dimension index \( D = 2 - 1.047 = 0.953 \) which indicates that gold rates are highly persistent resulting in a chart that is less jagged, resembling a straight line.

- The plot of \( V_n \) versus \( \log n \) has upward slope confirming its persistence.

- The correlation between periods \( C_n = 2^{2H-1} - 1 = 1.1347 \) which indicates the presence of long term memory.
9. BIBLIOGRAPHY

7. Mandelbrot, B. (1972), “Statistical Methodology for non-periodic cycles from the co-

variances of R/S Analysis “, Annals of Economics and Social measurement .

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