GL MODEL ON PROPAGATION OF SURFACE WAVES IN MAGNETO-THERMOELASTIC MATERIALS WITH VOIDS AND INITIAL STRESS

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ABSTRACT

In this paper, the surface waves propagation in a generalized magneto-thermoelastic materials taking (Green Lindsay) model with voids and initial stress is investigated. The basic governing equations have been formulated in xz-plane and the magnetic field is considered in y-axis that acts perpendicular to the wave propagation. Lame's potential method is applied to solve the problem. The boundary conditions that the continuity of forces stresses and Maxwell's stresses components, displacement components, heat flux, temperature and volume fraction field are estimated at the interfaces between two dissimilar half-space to obtain the frequency equation of the surface waves in the medium. Some special cases with neglecting: (i) the magnetic field and initial stress, (ii) the magnetic field, initial stress and voids parameters, (iii) the magnetic field, initial stress and thermal parameters, and (iv) the magnetic field, initial stress, thermal field and voids parameters are deduced as special cases from this study.

Keywords: Rotation, thermoelasticity, magnetic field, surface waves, initial stress, voids.

1. INTRODUCTION

In recent years, more attentions have been given to the initial stress on waves with thermal field, magnetic field and voids under relaxation times because of its utilitarian aspects of Seismic waves, Earthquakes, Volcanoes and Acoustics. In the classical theory of thermoelasticity, when an elastic solid is subjected to a thermal disturbance, the effect is felt in a location far from the source, instantaneously. This implies that the thermal wave propagates with infinite speed, a physically impossible result. In contrast to conventional thermoelasticity, non-classical theories came into existence during the last part of the 20th century. For example, Lord and Shulman [1], by incorporating a flux-rate term into Fourier's law of heat conduction, formulated a generalized theory which involves a hyperbolic heat transport equation admitting finite speed for thermal signals. Green and Lindsay [2], by including temperature rate among the constitutive variables, developed a temperature-rate dependent thermoelasticity that does not violate the classical Fourier's law of heat conduction, when body under consideration has a center of symmetry and this theory also
predicts a finite speed for heat propagation. Chandrasekharaih [3] referred to this wavelike thermal disturbance as second sound. The Lord and Shulman theory of generalized thermoelasticity was further extended by Dhaliwal and Sherief [4] to include the anisotropic case. A survey article on representative theories in the range of generalized thermoelasticity is due to Hetnarski and Ignaczak [5]. The reflection of thermoelastic waves from the free surface of a solid half-space and at the interface between two semi-infinite media in welded contact, in the context of generalized thermoelasticity is investigated by Sinha and Sinha [6], Sinha and Elsibai ([7], [8]). Abd-Alla and Al-Dawy [9] studied the reflection phenomena of SV waves in a generalized thermoelastic medium. Sharma et al. [10] investigated the problem of thermoelastic wave reflection from the insulated and isothermal stress-free as well as rigid fixed boundaries of a solid half-space in the context of different theories of generalized thermoelasticity.

Theory of linear elastic materials with voids is an important generalization of the classical theory of elasticity. The theory is used for investigating various types of geological and biological materials for which classical theory of elasticity is not adequate. The theory of linear elastic materials with voids deals the materials with a distribution of small pores or voids, where the volume of void is included among the kinematics variables. The theory reduces to the classical theory in the limiting case of volume of void tending to zero. Non-linear theory of elastic materials with voids was developed by Nunziato and Cowin [11]. Cowin and Nunziato [12] developed a theory of linear elastic materials with voids to study mathematically the mechanical behavior of porous solids. Puri and Cowin [13] studied the behavior of plane waves in a linear elastic material with voids. Iesan [14] developed the linear theory of thermoelastic materials with voids.


Recently, Abo-Dahab [33] investigated the propagation of P waves from stress-free surface elastic half-space with voids under thermal relaxation and magnetic field. Singh and Pal [34] discussed surface waves propagation in a generalized thermoelastic material with voids. This paper is motivated by the linear theory of thermoelasticity with voids developed by Iesan [14]. Recently, Singh and Pal [34] studied the surface wave propagation in a generalized thermoelastic material with voids which is the special case from this paper without effect all of the magnetic field and rotation.

In the present paper, the surface waves propagation in a generalized magneto-thermoelastic materials taking GL model with voids and initial stress is investigated. In Section 2, the governing equations are generalized with the help of Green and Lindsay theory [2]. These equations are solved for general solutions. In Sections 4 and 5, the particular solutions are obtained and applied at required boundary conditions to obtain the frequency equation of surface waves in thermoelastic material with voids. In Section 6, some limiting cases of the problem are discussed and the results obtained are calculated numerically and presented graphically. In the last section, some concluding remarks are given.

NOMENCLATURE

\( \alpha, b, m, \xi \) are the void material parameters,
\( \alpha_0 \) is the coefficient of linear thermal expansion,
\( \beta = \alpha_0 (3\lambda + 2\mu) \),
\( \delta_{ij} \) is the Kronecker delta,
\( \eta \) is the entropy per unit mass,
\( \Theta = T - T_0 \), \( \left| \Theta / T_0 \right| << 1 \),
\( \lambda \) and \( \mu \) are Lamé's parameter,
\( \mu_t \) is the magnetic permeability,
\( \rho \) is the density,
\( \sigma_{ij} \) are the components of stress vector,
\( \tau_{ij} \) are the components of Maxwell's stresses vector,
\( \tau_0 \) and \( \tau_1 \) are the thermal relaxations parameters,
\( \Phi \) is the change in the volume fraction field,
\( \chi \) is the equilibrated inertia,
\( \omega \) is the frequency,
\( \vec{B} \) is the magnetic induction vector,
\( C_e \) is the specific heat per unit mass,
\( e_{ij} \) are the components of strain tensor,
\( \vec{E} \) is the electric intensity vector,
\( \vec{F} \) is Lorentz's body force
\( g \) is the intrinsic equilibrated body forces,
\( \vec{H} \) is the magnetic field vector,
\( \vec{h} \) is the perturbed magnetic field vector,
\( \vec{J} \) is the electric intensity vector,
\( k \) is the wave number,
\( K \) is the thermal conductivity,
m is the thermo-void coefficient,
P is the initial stress,
$q_i$ are the components of the heat flux vector,
$S_i$ are the components of the equilibrated stress vector,
t is the time,
$T_0$ is the natural temperature of the medium,
$T$ is the absolute temperature,
$u_i$ are the components of the displacement vector,
c is the phase speed.

2- GOVERNING EQUATIONS

The governing equations for an isotropic, homogeneous elastic solid with generalized thermoelasticity with voids and incremental heat flux at reference temperature $T_0$ taking into our account GL model and the fields (thermal, voids and elastic) are given as follows

\[
\sigma_{ij} = \left( \lambda e_{kk} - \beta \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) \right) \Theta + b \Phi - P \delta_{ij} + 2 \mu e_{ij} - P w_{ij},
\]

\[
q_i + \tau_0 q_{,i} = K \Theta ,_{,i},
\]

\[
S_i = c \Phi ,_{,i},
\]

\[
\rho \eta = \beta e_{kk} + \alpha \Theta + m \Phi ,
\]

\[
g = -b e_{kk} - \xi \Phi + m \Theta ,
\]

\[
\rho T_0 \eta = q_{,i,i},
\]

\[
e_{ij} = \frac{1}{2} \left( u_{i,j} + u_{j,i} \right), \quad w_{ij} = \frac{1}{2} \left( u_{j,i} - u_{i,j} \right).
\]

The Maxwell’s electro-magnetic stress tensor $\tau_{ij}$ is given by

\[
\tau_{ij} = \mu_e \left( H_i h_j + H_j h_i - (H_k h_k) \delta_{ij} \right).
\]

The equation of motion

\[
\sigma_{je,j} + F_i = \rho \ddot{u}_i
\]

which tends to

\[
\left( \mu - \frac{P}{2} \right) u_{i,jj} + \left( \lambda + \mu - \frac{P}{2} \right) u_{j,i} - \beta \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) \Theta ,_{,i} + b \Phi ,_{,i} + F_i = \rho \ddot{u}_i.
\]

The equation of heat conduction under GL model

\[
\rho C_e (\Theta + \tau_0 \ddot{\Theta}) + \beta T_0 \dot{u}_k u_k + m T_0 (\Phi + \tau_0 \ddot{\Phi}) = K \Theta ,_{,ii},
\]

\[
\alpha \Phi ,_{,ii} - b u_{k,k} - \xi \Phi + m \Theta = \rho \chi \Phi.
\]
where
\[ \vec{F} = \vec{J} \times \vec{B}. \] (13)

Consider that the medium is a perfect electric conductor, we take the linearized Maxwell’s equations governing the electromagnetic field, taking into account absence of the displacement current (SI) as the form
\[ \begin{aligned}
&\text{curl} \, \vec{h} = \vec{J}, \\
&\text{curl} \, \vec{E} = -\mu_e \frac{\partial \phi}{\partial t}, \\
&\text{div} \, \vec{h} = 0, \\
&\text{div} \, \vec{E} = 0
\end{aligned} \] (14)

where
\[ \vec{h} = \text{curl} \left( \vec{u} \times \vec{H}_0 \right) \] (15)

where, we have used
\[ \vec{H} = \vec{H}_0 + \vec{h} \left( x, z, t \right), \quad \vec{H}_0 = \left( 0, H_0, 0 \right). \] (16)

For two-dimensional motion in xz-plane, Eqs. (10)-(12) written as
\[ \begin{aligned}
\left( \lambda + 2 \mu + \mu_e H_0^2 - \frac{P}{2} \right) \frac{\partial^2 u_1}{\partial x^2} + \left( \lambda + \mu + \mu_e H_0^2 - \frac{P}{2} \right) \frac{\partial^2 u_3}{\partial x \partial z} \\
+ \left( \mu - \frac{P}{2} \right) \frac{\partial^2 u_1}{\partial z^2} - \beta \tau^1 \frac{\partial \phi}{\partial x} + b \frac{\partial \psi}{\partial x} = \rho \frac{\partial^2 u_1}{\partial t^2}, \\
\left( \lambda + 2 \mu + \mu_e H_0^2 - \frac{P}{2} \right) \frac{\partial^2 u_3}{\partial z^2} + \left( \lambda + \mu + \mu_e H_0^2 - \frac{P}{2} \right) \frac{\partial^2 u_1}{\partial x \partial z} \\
+ \left( \mu - \frac{P}{2} \right) \frac{\partial^2 u_3}{\partial z^2} - \beta \tau^1 \frac{\partial \phi}{\partial z} + b \frac{\partial \psi}{\partial z} = \rho \frac{\partial^2 u_3}{\partial t^2}, \\
\rho C_e \tau^0 \frac{\partial \Theta}{\partial t} + \beta T_0 \left( \frac{\partial^2 u_1}{\partial x \partial t} + \frac{\partial^2 u_3}{\partial z \partial t} \right) \\
+ m \tau^0 \frac{\partial \Phi}{\partial t} = K \left( \frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial z^2} \right), \\
\alpha \left( \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \right) - b \left( \frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial z} \right) - \xi \Phi + m \Theta = \rho \chi \frac{\partial \Phi}{\partial t} \end{aligned} \] (17)

where
\[ \tau^0 = 1 + \tau^0 \frac{\partial}{\partial t}, \quad \tau^1 = 1 + \tau^1 \frac{\partial}{\partial t}. \]

The displacement components \( u_1 \) and \( u_3 \) may be written in terms of the scalar and the vector potential functions, \( \phi \) and \( \psi \), respectively, as the following form
\[ u_1 = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z}, \quad u_3 = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x}. \] (21)
Substituting from Eq. (21) into Eqs. (17)-(20), we get
\[ C^{-2} \nabla^{2} \phi - \beta \tau \Theta + \bar{b} \Phi = \ddot{\phi}, \]  \hspace{1cm} (22)
\[ C^{-2} \nabla^{2} \psi = \ddot{\psi}, \]  \hspace{1cm} (23)
\[ \varepsilon \nabla^{2} \Theta = \left( \Theta + \varepsilon_{1} \nabla^{2} \phi + \varepsilon_{2} \Phi \right), \]  \hspace{1cm} (24)
\[ \alpha \nabla^{2} \Phi - \xi \Phi - b \nabla^{2} \phi + m \Theta = \rho \chi \Phi. \]  \hspace{1cm} (25)

Where
\[ C_{T}^{-2} = \frac{\lambda + 2 \mu + \mu e H^{2}}{\sqrt{p / 2}}, \quad C_{S}^{-2} = \frac{\mu - p / 2}{\rho}, \quad \bar{\beta} = \frac{\beta}{\rho}, \]  \hspace{1cm} (26)
\[ \bar{b} = \frac{b}{\rho}, \quad \varepsilon = \frac{K}{\rho C_{e} \tau^{2}}, \quad \varepsilon_{1} = \frac{\beta T_{0}}{\rho C_{e} \tau^{2}}, \quad \varepsilon_{2} = \frac{m T_{0}}{\rho C_{e}}. \]

3. SOLUTION OF THE PROBLEM

For the analytical solution of Eqs. (22), (24) and (25) in the form of the harmonic traveling wave, we suppose that the solution takes the form
\[ [\phi, \Theta, \Phi](x, z, t) = [\phi_{1}(z), \Theta_{1}(z), \Phi_{1}(z)] \exp [i k (x - ct)]. \]  \hspace{1cm} (27)

Substituting from Eq. (27) into Eqs. (22), (24) and (25), we get
\[ \left( C_{T}^{-2} (D^{2} - k^{2}) + \omega^{2} \right) \phi_{1}(z) - \tau \bar{\beta} \Theta_{1}(z) + \bar{b} \Phi_{1}(z) = 0, \]  \hspace{1cm} (28)
\[ i \omega \varepsilon_{1} (D^{2} - k^{2}) \phi_{1}(z) + (\varepsilon (D^{2} - k^{2}) + i \omega) \Theta_{1}(z) + i \omega \varepsilon_{2} \Phi_{1}(z) = 0, \]  \hspace{1cm} (29)
\[ -b (D^{2} - k^{2}) \phi_{1}(z) + m \Theta_{1}(z) + (\alpha (D^{2} - k^{2}) - \xi + \rho \chi \omega^{2}) \Phi_{1}(z) = 0 \]  \hspace{1cm} (30)

where
\[ D^{2} = \frac{d^{2}}{dz^{2}}, \quad \omega = kc. \]  \hspace{1cm} (31)

Eliminating the constants \( \phi_{1}, \Theta_{1} \) and \( \Phi_{1} \) from Eqs. (28)-(30), we obtain
\[ L \left( D^{2} \right) + M \left( D^{2} \right) + N \left( D^{2} \right) + Q = 0 \]  \hspace{1cm} (32)

where
\[ \begin{align*}
L &= \alpha \varepsilon C_{1,2}^2, \\
M &= C_{1,2}^2 \left[ \varepsilon \left( \rho \chi \omega^2 - \alpha k^2 - \xi \right) + \alpha \left( i \omega - \varepsilon k^2 \right) \right] \\
&\quad + \alpha \varepsilon \left( \omega^2 - C_{1,2}^2 k^2 \right) + i \omega \alpha \varepsilon_1 \tau^1 \overline{\beta} + \varepsilon e \overline{b}, \\
N &= \left[ C_{1,2}^2 \left( i \omega - \varepsilon k^2 \right) + i \omega \varepsilon_1 \tau^1 \overline{\beta} \right] \left( \rho \chi \omega^2 - \alpha k^2 - \xi \right) \\
&\quad + \left( \omega^2 - C_{1,2}^2 k^2 \right) \left[ \varepsilon \left( \rho \chi \omega^2 - \alpha k^2 - \xi \right) + \alpha \left( i \omega - \varepsilon k^2 \right) \right] \\
&\quad + i \omega \left[ \varepsilon_1 \left( \overline{b} m - \alpha \tau^1 \overline{\beta} k^2 \right) + \varepsilon_2 \left( b \tau^1 \overline{\beta} - C_{1,2}^2 m \right) \right], \\
Q &= \left( \omega^2 - C_{1,2}^2 k^2 \right) \left( \varepsilon \left( \rho \chi \omega^2 - \alpha k^2 - \xi \right) - i \omega \varepsilon_z m \right) \\
&\quad - i \omega k^2 \left[ \varepsilon_1 \tau^1 \overline{\beta} \left( \rho \chi \omega^2 - \alpha k^2 - \xi \right) + \varepsilon_2 b \tau^1 \overline{\beta} + \varepsilon_1 \overline{b} m \right] \\
&\quad - b \overline{b} k^2 \left( i \omega - \varepsilon k^2 \right)
\end{align*} \]

we suppose that \( m_1, m_2 \) and \( m_3 \) are the roots of the equation (32), then we can write the general solutions \( \phi, \Theta \) and \( \Phi \) in the forms

\[ \phi(x, z, t) = \left[ A_1 e^{m_1 z} + A_2 e^{m_2 z} + A_3 e^{m_3 z} + A_4 e^{m_4 z} + A_5 e^{m_5 z} \right] e^{i k (x - ct)}, \]
\[ \Phi(x, z, t) = \left[ \eta_1 A_1 e^{m_1 z} + \eta_2 A_2 e^{m_2 z} + \eta_3 A_3 e^{m_3 z} + \eta_4 A_4 e^{m_4 z} + \eta_5 A_5 e^{m_5 z} \right] e^{i k (x - ct)}, \]
\[ \Theta(x, z, t) = \left[ \xi_1 A_1 e^{m_1 z} + \xi_2 A_2 e^{m_2 z} + \xi_3 A_3 e^{m_3 z} + \xi_4 A_4 e^{m_4 z} + \xi_5 A_5 e^{m_5 z} \right] e^{i k (x - ct)} \]

where

\[ \eta_n = \frac{i \omega \varepsilon_z \left[ C_{1,2}^2 \left( m_n^2 - k^2 \right) + \omega^2 \right] - i \omega e \overline{b} \left( m_n^2 - k^2 \right)}{\overline{b} \left[ \varepsilon \left( m_n^2 - k^2 \right) + i \omega \varepsilon_1 \tau^1 \beta \right] + i \omega \varepsilon_z \tau^1 \beta}, \]
\[ \xi_n = \frac{\tau^1 \beta \eta_n - C_{1,2}^2 \left( m_n^2 - k^2 \right) + \omega^2}{\overline{b}}, \quad n = 1, 2, 3. \]

The general solutions \( \psi \) of the equation (23) is getting as the form

\[ \phi(x, z, t) = \left[ B_1 e^{-m_4 z} + B_2 e^{m_4 z} \right] e^{i k (x - ct)} \]

where

\[ m_4 = k^2 - \frac{\rho}{\mu} \omega^2. \]
4. FORMULATION OF THE PROBLEM

We consider plane waves propagate through the two semi-infinite half-spaces of thermoelastic solid with voids, which we identify as the region \( z > 0 \) the medium \( M_1 \) \( \left[ \lambda, \mu, \alpha, \beta, b, m \right] \) and the region \( z < 0 \) the medium \( M_2 \) \( \left[ \bar{\lambda}, \bar{\mu}, \bar{\alpha}, \bar{\beta}, \bar{b}, \bar{m} \right] \) as shown in Figure 1.

\[
\phi(x, z, t) = \left[ A_1 e^{-m_1 z} + A_2 e^{-m_2 z} + A_3 e^{-m_3 z} \right] e^{ik(x-ct)}, \quad (40)
\]

\[
\Phi(x, z, t) = \left[ \eta_1 A_1 e^{-m_1 z} + \eta_2 A_2 e^{-m_2 z} + \eta_3 A_3 e^{-m_3 z} \right] e^{ik(x-ct)}, \quad (41)
\]

\[
\Theta(x, z, t) = \left[ \xi_1 A_1 e^{-m_1 z} + \xi_2 A_2 e^{-m_2 z} + \xi_3 A_3 e^{-m_3 z} \right] e^{ik(x-ct)}, \quad (42)
\]

\[
\psi(x, z, t) = B_1 e^{-m_4 z} e^{ik(x-ct)}, \quad (43)
\]

For medium \( M_2 \)

\[
\phi'(x, z, t) = \left[ A'_1 e^{m_1 z} + A'_2 e^{m_2 z} + A'_3 e^{m_3 z} \right] e^{ik(x-ct)}, \quad (44)
\]

\[
\Phi'(x, z, t) = \left[ \eta'_1 A'_1 e^{m_1 z} + \eta'_2 A'_2 e^{m_2 z} + \eta'_3 A'_3 e^{m_3 z} \right] e^{ik(x-ct)}, \quad (45)
\]

\[
\Theta'(x, z, t) = \left[ \xi'_1 A'_1 e^{m_1 z} + \xi'_2 A'_2 e^{m_2 z} + \xi'_3 A'_3 e^{m_3 z} \right] e^{ik(x-ct)}, \quad (46)
\]

\[
\psi'(x, z, t) = B'_1 e^{m_4 z} e^{ik(x-ct)}, \quad (47)
\]

5. BOUNDARY CONDITIONS
The boundary conditions at the surface take the following form

\[
\begin{align*}
\sigma_{zz} + \tau_{zz} &= \sigma'_{zz} + \tau'_{zz}, \\
\sigma_{xz} + \tau_{xz} &= \sigma'_{xz} + \tau'_{xz}, \\
\frac{\partial \Theta}{\partial z} &= \chi_1 \frac{\partial \Theta'}{\partial z}, \\
\frac{\partial \Phi}{\partial z} &= \chi_2 \frac{\partial \Phi'}{\partial z}, \\
u_1 &= u'_1, \quad u_3 = u'_3, \quad \Theta = \Theta', \quad \Phi = \Phi', \quad \text{at} \quad z = 0
\end{align*}
\]

where

\[
\chi_1 = \frac{K'(1 - ikc \tau_0)}{K(1 - ikc \tau_0)}, \quad \chi_2 = \frac{\alpha'}{\alpha}
\]

we can write equations (47) in the forms

\[
\begin{align*}
\left( \lambda + \mu_e H_0^2 - \frac{P}{2} \right) \nabla^2 \phi + 2\mu \left( \frac{\partial^2 \phi}{\partial z^2} + \frac{\partial^2 \psi}{\partial x \partial z} \right) - \beta \tau' \Theta + b \Phi = \\
\left( \lambda' + \mu_e H_0^2 - \frac{P}{2} \right) \nabla^2 \phi' + 2\mu \left( \frac{\partial^2 \phi'}{\partial z^2} + \frac{\partial^2 \psi'}{\partial x \partial z} \right) - \beta \tau' \Theta + b \Phi', \\
2\mu \frac{\partial^2 \phi}{\partial x \partial z} + \mu \left( \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial z^2} \right) = 2\mu \frac{\partial^2 \phi'}{\partial x \partial z} + \mu \left( \frac{\partial^2 \psi'}{\partial x^2} - \frac{\partial^2 \psi'}{\partial z^2} \right), \\
\frac{\partial \Theta}{\partial z} &= \chi_1 \frac{\partial \Theta'}{\partial z}, \\
\frac{\partial \Phi}{\partial z} &= \chi_2 \frac{\partial \Phi'}{\partial z}, \\
\Theta &= \Theta', \quad \Phi = \Phi', \quad \text{at} \quad z = 0.
\end{align*}
\]

Substitution from equations (1), (8), (21) and (27) into the boundary conditions equations (48) we get

\[
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \\
a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} & a_{28} \\
a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & a_{37} & a_{38} \\
a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & a_{47} & a_{48} \\
a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & a_{57} & a_{58} \\
a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & a_{67} & a_{68} \\
a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & a_{77} & a_{78} \\
a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & a_{86} & a_{87} & a_{88}
\end{bmatrix}
= 0
\]

where
If the initial stress and magnetic field are neglected, the results obtained are deduced to the relevant results obtained by Singh and Pal [34].

6. NUMERICAL RESULTS AND DISCUSSION

The following values of elastic constants are considered Singh [24], for mediums $M_1$ and $M_2$, respectively.
Using these values, it was found that
\[ \omega = 0.01, \quad T_0 = 298K . \]
In order to gain physical insight the roots, Stoneley wave velocity \( \text{Re}(\Delta) \) and attenuation coefficient \( I(\Delta) \), have been discussed by assigning numerical values to the parameter encountered in the problem in which the numerical results are displayed with the graphical illustrations. The variations are shown in Figs. (2)-(7), respectively.

Figs. (2) and (3) show the variations of the roots of equation (32) with respect to the initial stress \( P \) with varies values of the magnetic field for the two medium \( M_1 \) and \( M_2 \). From Fig. (2), it is appear that the roots \( m_1, m_2 \) and \( m_3 \) for medium \( M_1 \) have oscillatory behavior in the whole range of the \( P \)-axis for different values of the magneti field \( H \) and decrease with an increasing of the initial stress \( P \). From Fig. (3), it is clear that the roots \( m_1', m_2' \) and \( m_3' \) for medium \( M_2 \) increase with an increasing of \( P \) but decreases with an increasing of the magnetic field. It is seen that for large values of the magnetic field, \( m_1 \) and \( m_2 \) have an oscillatory behavior with the variation of \( P \).

![Fig. 2: Variation of the the absolute values of the roots \( m_1, m_2 \) and \( m_3 \) with respect to initial stress with varies values of the magnetic field](image-url)
Fig. 3: Variation of the absolute values of the roots $m_1, m_2, m_3$ with respect to initial stress with varies values of the magnetic field

Fig. 4: Variation of the Stoneley waves velocity and attenuation coefficient with respect to phase speed $c$ with varies values of relaxation times with and without magnetic field
Fig. 5: Variation of the Stoneley waves velocity and attenuation coefficient with respect to phase speed $c$ with varies values of relaxation times with and without initial stress.

Fig. 6: Variation of the Stoneley waves velocity and attenuation coefficient with respect to phase speed $c$ with varies values of initial stress with and without magnetic field.
Fig. (4) displays the variation of the Stoneley waves velocity and attenuation coefficient with respect to phase speed c with varies values of relaxation times with and without magnetic field. It is clear that Stoneley waves velocity decreases with the increased values of the phase speed c and with an increasing of the relaxation times, but the attenuation coefficient increases with an increasing of the relaxation times and the small values of phase speed c and after that return decreases with the large values of c, also, it is shown that the Stoneley waves velocity and attenuation coefficients take large values with the presence of the magnetic field comparing with the corresponding values in the absence of H. From Fig. (5), it is clear that the stoneley waves velocity and attenuation coefficient decrease with an increasing of c, also, it is obvious that Stonely waves velocity decreases with an increasing of the relaxation times in the presence of the magnetic field but increases with the relaxation times variation in the absence of H, vice versa for the attenuation coefficients.

Fig. (6) plots the Stoneley waves velocity and attenuation coefficient with respect to phase speed c with varies values of initial stress with and without magnetic field, it is obvious that Stoneley waves velocity decreases with the increasing values of the initial stress P but the attenuation coefficient increases, also, it is seen that they tends to zero as $c=25$. From Fig. (7), we can see that Stoneley waves velocity and attenuation coefficient increase with an increasing of the magnetic field H. Finally, it is clear that Stonely waves velocity takes small values in the presence of the initial stress P comparing with the corresponding values in the absence of P but the attenuation coefficient takes the largest values in the presence of P comparing with it in the absence of P.

7. CONCLUSION

It is clear from the previous results that the effect of initial stress, magnetic field and thermal relaxation time on velocity of Stoneley waves and attenuation coefficient in magneto-thermoelastic materials with voids, we have the values of this waves increased or decreased, with increasing of the value of magnetic field, initial stress and thermal relaxation time.
to the complicated nature of the governing equations for the generalized magneto-thermoelasticity theory, the work done in this field is unfortunately limited in number. The method used in this study provides a quite successful in dealing with such problems. This method gives exact solutions in the elastic medium without any assumed restrictions on the actual physical quantities that appear in the governing equations of the problem considered.

Important phenomena are observed in all these computations:

1 - The influence of the initial stress on the roots via the two media $M$ and $M'$ has an pronounced effects.
2-The Stoneley waves velocity and attenuation coefficients take large values with the presence of the magnetic field comparing with the corresponding values in the absence of $H$.
3-Stoneley waves velocity decreases with an increasing of the relaxation times in the presence of the magnetic field but increases with the relaxation times variation in the absence of $H$, vice versa for the attenuation coefficients.
4- Stoneley waves velocity decreases with the increasing values of the initial stress $P$ but the attenuation coefficient increases, also, it is seen that they tend to zero as $c=25$.
5- Stoneley waves velocity and attenuation coefficient increase with an increasing of the magnetic field $H$.
6- Stonley waves velocity takes small values in the presence of the initial stress $P$ comparing with the corresponding values in the absence of $P$ but the attenuation coefficient takes the largest values in the presence of $P$ comparing with it in the absence of $P$.

The results presented in this paper should prove useful for researchers in material science, designers of new materials, low-temperature physicists, as well as for those working on the development of a theory of hyperbolic propagation of hyperbolic thermoelastic. Relaxation time, voids, and initial stress exchange with the environment arising from and inside nuclear reactors influence their and operations. Study of the phenomenon of relaxation time and initial stress and magnetic field is also used to improve the conditions of oil extractions. Finally, it is concluded that the influence of magnetic field, initial stress, voids parameters, and thermal relaxation times are very pronounced in the surface waves propagation phenomena.

The results obtained are deduced to the results obtained by Singh and Pal [34] in the absence of initial stress and magnetic field.

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8. REFERENCES