OPTIMAL SHIPMENTS, ORDERING AND PAYMENT POLICIES FOR INTEGRATED SUPPLIER-BUYER DETERIORATING INVENTORY SYSTEM WITH PRICE-SENSITIVE TRAPEZOIDAL DEMAND AND NET CREDIT

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ABSTRACT

In this research, an integrated supplier-buyer inventory system is studied when market demand is price-sensitive trapezoidal and units in inventory are subject to deterioration at a constant rate. The buyer has an option to choose between discount in unit price and delay in settling the account against the purchases made offered by the supplier. This type of trade credit is termed as 'net credit'. In this scenario, if the buyer settles payment within the stipulated time period \( M_1 \), then the buyer receives a cash discount; otherwise the full payment must be paid by the time \( M_2 \); where \( M_2 > M_1 \geq 0 \). The joint profit per unit time of supplier-buyer is maximized with respect to selling price, purchase quantity, number of transfers from the supplier to the buyer and payment time. An algorithm is outlined to obtain optimal solution. The numerical example is given to validate the proposed formulation. The managerial issues are deduced through sensitivity analysis of inventory parameters.

Key Words: Integrated Inventory Model, Deterioration, Price-Sensitive Trapezoidal Demand, Net Credit.
1. INTRODUCTION

The classical EOQ model assumes that the buyer uses cash-on-delivery policy which is no longer a practice followed by the player of the supply chain. Goyal (1985) first proposed the concept of delay payment policy opted by the supplier for the buyer to derive economic order quantity. Thereafter, many researchers analyzed inventory model to study the effect of delay period payment on stimulating the demand. One can refer review by Shah et al. (2010) on trade credit and inventory policy. The most of the articles in this review article established that demand stimulates and lowers on-hand stocking for supplier while buyer can earn interest on the generated revenue. However, the provision for making early payment was not addressed.

Ho et al. (2008) observed that the offer of trade credit delays cash-flow and increases the risk of cash-flow shortage for the supplier. To combat this trade-off, the supplier offers a cash discount in unit purchase price to attract the buyer for early payment. For example, the supplier offers 3% discount on buyer’s unit purchase price if payment is made within 10 days; otherwise the account is to be settled within 30 days for the purchases made. In financial management, this credit term is regarded as ‘3/10 net 30’. Related articles are by Liberia and Orgler (1975), Hill and Rainier (1979), Kim and Chung (1990), Arcelus and Srinivasan (1993), Ouyanget al. (2002), Chang (2002), Huang and Chung (2003). In these articles, the buyer is the sole decision maker.


Shah et al. (2011) analyzed an integrated inventory policy with ‘two-part’ trade credit when demand is quadratic. This type of demand is observed in the fashion market, seasonal products, etc. However, the demand of above mentioned items including branded electronic items decreases drastically after some time. Cheng et al. (2011) discussed trapezoidal
demand in which the demand pattern is linearly increasing with time upto some point of time, becomes constant in some interval of time and thereafter it decreases exponentially. Shah and Shah (2012) developed joint optimal inventory policies for two players of the supply chain when demand is trapezoidal. They (2012) studied effect of deterioration in above problem.

In this paper, the objective is to analyze an integrated inventory system for deteriorating items for price-sensitive trapezoidal demand. The units in inventory of both the players are subject to deterioration at a constant rate. The supplier offers a choice of cash discount in unit purchase price if payment is settled earlier (specified); otherwise, the buyer has to make the full payment by the allowable credit period. The joint total profit per unit time is maximized with respect to payment time, retail price, purchase quantity and number of shipments from the supplier to the buyer. The algorithm is proposed to find best optimal solution. A numerical example is given to validate the developed problem. Sensitivity analysis is carried out and managerial issues are discussed.

2. ASSUMPTIONS AND NOTATIONS

2.1 Assumptions

The model is developed with following assumptions.

1. The supply chain comprises of single-supplier single-buyer and for single item.
2. Shortages are not allowed. Lead-time is zero.
3. The demand rate is price-sensitive trapezoidal. (Appendix A)
4. The supplier offers a discount \( \beta (0 < \beta < 1) \) in the purchase price if the buyer pays by time \( M_1 \); otherwise full account is to be settled within allowable credit period \( M_2 \), where \( M_2 > M_1 \geq 0 \). The offer of discount in unit purchase price from the supplier will increase cash inflow, thereby reducing the risk of cash flow shortage.
5. By offering a trade credit to the buyer, the supplier receives cash at a later date and hence incurs an opportunity cost during the delivery and payment of the product. On the buyer’s end, the buyer can generate revenue by selling the items and earning interest by depositing it in an interest bearing account during this permissible delay period. At the end of this period, the supplier charges to the buyer on the unsold stock.
6. During the time \([M_1, M_2] \), a cash flexibility rate \( f_{sc} \) is used to quantize the favor of early cash income for the supplier.
7. The units in the inventory system of both the player deteriorate at a constant rate \( \theta (0 < \theta < 1) \). The deteriorated units can neither be repaired nor replaced during the period under review.

2.2 Notations

The mathematical concept is developed using following notations.

\( A_b \)  \quad \text{Buyer’s ordering cost per order ($/order)}
\( A_s \)  \quad \text{Supplier’s set-up cost ($/setup)}
To increase cash inflow and reduce the risk of cash flow shortage, the supplier offers a discount \( \beta (0 < \beta < 1) \) off the purchase price, if buyer settles the account within time \( M_1 \), otherwise, full account is to be settled within an allowable credit period \( M_2 \); where \( M_2 > M_1 \geq 0 \).

\( C_s \) Supplier’s unit manufacturing cost ($/unit)
\( v \) Supplier’s unit sale price ($/unit)
\( P \) Buyer’s unit sale price ($/unit) (a decision variable)

Note: \( P > v > (1 - \beta)v > C_s \),

\( I_b \) Buyer’s carrying charge fraction per unit per year excluding interest charges
\( I_s \) Supplier’s carrying charge fraction per unit per year excluding interest charges
\( I_{sp} \) Supplier’s capital opportunity cost rate per unit/year
\( f_{sc} \) Supplier’s cash flexibility rate per unit/year
\( I_{be} \) Interest earned by the buyer during offered credit period \( M_2 \) per unit per year
\( I_{bc} \) Buyer’s interest paid per unit per year

\( R \) \((= R(P,t))\) Market demand rate (Appendix A), where \( a > 0 \) is scale demand, \( 0 < b_1, b_2 < 1 \) are the rates of change of demand, \( \eta > 1 \) is price-elasticity mark-up and \( u_1 \) and \( u_2 \) are time points at which demand pattern changes. (Fig. 1)

\( \gamma \) The capacity utilization factor which is the ratio of the market demand rate to production rate. \( \gamma < 1 \) is deterministic and constant.

\( \theta \) Constant deterioration rate \((0 < \theta < 1)\) of units
\( T \) Buyer’s cycle time (a decision variable)
\( n \) Number of transfers from a supplier to buyer, \( n \) is a positive integer (a decision variable)
\( Q \) Buyer’s procurement quantity during each transfer (a decision variable)

\( TBP \) Buyer’s total profit per unit time
\( TSP \) Supplier’s total profit per unit time
\( \pi \) \((= TSP + TBP)\) Joint total profit of the integrated system per unit time

3. MATHEMATICAL MODEL

The buyer purchases \( Q \) units in each transfer. So the supplier produces in the batches of size \( nQ \) and hoards set-up cost. The supplier transships \( Q \) units manufactured initially and thereafter, \( Q \) units are transported at \( T \) time units until the supplier’s inventory depletes to zero.

The supplier offers the buyer a two-part trade credit period to encourage early payment reducing risk of cash inflows. During the available credit period buyer earns interest on the generated revenue. The aim is to maximize the joint profit per unit time of the integrated system with respect to buyer’s selling price, payment time, procurement quantity and the number of transfers from the supplier to the buyer.
3.1 Supplier’s total profit per unit time

The supplier manufactures $nQ$ units in batches where $Q$ is defined in Appendix B and incurs a batch set-up cost $A_s$. The supplier’s set-up cost per unit time is $A_s/(nT)$. Following Joglekar (1988), the supplier’s inventory holding cost per unit time is

$$\int_0^T I(t) dt.$$ (See Appendix C for computation of $\int_0^T I(t) dt$).

The purchase cost of an item for the buyer is $(1 - K_j \beta) v$, when account is settled at time $M_j$; where $j = 1, 2; K_1 = 1, K_2 = 0$. Hence, for the permissible delay period, the opportunity cost per unit time is $1/(1 - K_j \beta) v I_{jsp} M_j Q$; where $j = 1, 2; K_1 = 1, K_2 = 0$. When the buyer pays at time $M_1$, the supplier can use the revenue $(1 - \beta) v$ to shrink a cash flow crisis during time $M_2 - M_1$. This timely payment acquires gain at the cash flexibility rate per unit time and is given by $1/(1 - \beta) v f_{sc} (M_2 - M_1) Q$. Hence, the supplier’s total profit per unit time is, sales revenue plus the interest earned on the timely payment, minus total cost which is sum of the manufacturing cost, set-up cost, inventory holding cost and opportunity cost, is given by

$$TSP_j(n) = \frac{(1 - K_j \beta) v Q}{T} - \frac{C_s Q}{T} - \frac{A_s}{nT} \left[ (1 - \gamma) + \gamma \right] \int_0^T I(t) dt$$

$$\left[ \frac{C_s (I_s + I_{sp})}{T} \left( (n - 1)(1 - \gamma) + \gamma \right) \right] \int_0^T I(t) dt$$

$$\frac{(1 - K_j \beta) v I_{jsp} M_j Q}{T} + \frac{(1 - \beta) v f_{sc} (M_2 - M_1) Q}{T}$$

$$j = 1, 2; K_1 = 1, K_2 = 0$$

(1)

3.2 Buyer’s total profit per unit time

The ordering cost per unit time is $A_p / T$ for each transfer of $Q$ units. The buyer’s purchase cost per unit time is $(1 - K_j \beta) v Q / T$ and inventory holding cost per unit time is

$$\int_0^T I(t) dt$$

$$\frac{1}{T}$$

where $j = 1, 2; K_1 = 1, K_2 = 0$.

On the basis of choice of payment time of the buyer two cases may arise.

1. $T < M_j$
2. $T \geq M_j; j = 1, 2$.

**Case 1**: $T < M_j$ (Fig. 2)
In this case, the buyer’s stock level depletes to zero before the permissible delay period. So, the opportunity cost for the buyer is zero. The interest earned on the generated revenue per unit time is given by

\[
PI_{be} \left[ \int_{0}^{T} t \cdot R(P,t) \, dt + Q(M_j - T) \right] / T ; j = 1,2.
\]

(See Appendix D for \( \int_{0}^{T} t \cdot R(P,t) \, dt \)). Hence, buyer’s total profit per unit time is

\[
TBP_{j1}(P,T) = \frac{PQ}{T} - \frac{(1-K_j \beta)vQ}{T} - \frac{A_b}{T} - \frac{(1-K_j \beta)vI_b \int_{0}^{T} I(t) \, dt}{T}
\]

\[
+ \frac{PI_{be} \left[ \int_{0}^{T} t \cdot R(P,t) \, dt + Q(M_j - T) \right]}{T}
\]

\( j = 1,2; \ K_1 = 1, \ K_2 = 0 \) \hspace{1cm} (2)

**Case 2:** \( T \geq M_j; j = 1,2 \) (Fig.3)
Fig. 3 Interest earned and charged when \( T \geq M_j; j = 1, 2 \)

In this case, the buyer’s permissible payment time offered by the supplier or before the cycle time. The interest earned per unit time by the buyer at the rate \( I_{be} \) during \([0, M_j]; j = 1, 2\) is

\[
\frac{1}{T} \pi_{be} \int_0^{M_j} t \cdot R_1(P, t) dt = \frac{1}{T} \pi_{be} \left\{ \begin{array}{l}
\int_0^{u_1} t \cdot R_1(P, t) dt + \int_{u_1}^{M_j} t \cdot R_2(P, t) dt \\
\int_{u_2}^{M_j} t \cdot R_3(P, t) dt
\end{array} \right\}; 0 \leq M_j \leq u_1
\]

\[
\frac{1}{T} \pi_{be} \int_0^{M_j} t \cdot R(P, t) dt = \frac{1}{T} \pi_{be} \left\{ \begin{array}{l}
\int_0^{u_1} t \cdot R_1(P, t) dt + \int_{u_1}^{u_2} t \cdot R_2(P, t) dt + \int_{u_2}^{M_j} t \cdot R_3(P, t) dt \\
\end{array} \right\}; u_1 \leq M_j \leq u_2
\]

\[
\frac{1}{T} \pi_{be} \int_0^{M_j} t \cdot R(P, t) dt = \frac{1}{T} \pi_{be} \left\{ \begin{array}{l}
\int_0^{u_1} t \cdot R_1(P, t) dt + \int_{u_1}^{u_2} t \cdot R_2(P, t) dt + \int_{u_2}^{M_j} t \cdot R_3(P, t) dt \\
\end{array} \right\}; u_2 \leq M_j \leq T
\]

and interest paid per unit time at the rate \( I_{bc} \) during \([M_j, T]; j = 1, 2\) is

\[
\frac{1}{T} \pi_{bc} \int_{M_j}^{T} t \cdot R(P, t) dt = \frac{1}{T} \pi_{bc} \left\{ \begin{array}{l}
\int_{M_j}^{u_1} t \cdot R_1(P, t) dt + \int_{u_1}^{u_2} t \cdot R_2(P, t) dt + \int_{u_2}^{T} t \cdot R_3(P, t) dt \\
\end{array} \right\}; u_2 \leq M_j \leq T
\]
Therefore, total profit of buyer per unit time is

\[
\frac{1}{T} (1 - K_j \beta) v I_{bc} \left\{ \int_{M_j}^{u_1} I_1(t) \, dt + \int_{u_1}^{u_2} I_2(t) \, dt + \int_{u_2}^{T} I_3(t) \, dt \right\}; \quad M_j \leq u_1 \leq T
\]

\[
\frac{1}{T} (1 - K_j \beta) v I_{bc} \int_{M_j}^{T} I(t) \, dt = \frac{1}{T} (1 - K_j \beta) v I_{bc} \left\{ \int_{M_j}^{u_2} I_2(t) \, dt + \int_{u_2}^{T} I_3(t) \, dt \right\}; \quad M_j \leq u_2 \leq T
\]

\[
\frac{1}{T} (1 - K_j \beta) v I_{bc} \int_{M_j}^{T} I_3(t) \, dt; \quad u_2 \leq M_j \leq T
\]

\[j = 1, 2; K_1 = 1, K_2 = 0\]

3.3 Joint total profit per unit time

The joint profit per unit time of integrated system is given by

\[
\pi_j(n, P, T) = \begin{cases} 
\pi_{j1}(n, P, T); & T < M_j \\
\pi_{j2}(n, P, T); & T \geq M_j 
\end{cases} \quad j = 1, 2\]

\[\pi_{j1}(n, P, T) = TSP_j(n) + TBP_{j1}(P, T)\]

\[\pi_{j2}(n, P, T) = TSP_j(n) + TBP_{j2}(P, T); \quad j = 1, 2\]

The objective is to decide optimal values of discrete variable \(n\) and continuous variables \(P\) and \(T\), which maximize \(\pi_j(n, P, T); \quad j = 1, 2\). We use following steps to maximize the joint profit of the supply chain.
4. COMPUTATIONAL PROCEDURE

To maximize joint profit, execute following steps:

Step 1: Assign parametric values in proper units to all model parameters.

Step 2: Set $n = 1$.

Step 3: Solve $\frac{\partial \pi_j}{\partial P} = 0$ and $\frac{\partial \pi_j}{\partial T} = 0$, $j = 1, 2$ simultaneously for $P$ and $T$.

Step 4: Increment $n$ by 1.

Step 5: Continue steps 3 and 4 until

$$\pi_j(n-1, P(n-1), T(n-1)) \leq \pi_j(n, P, T) \geq \pi_j(n+1, P(n+1), T(n+1)); j = 1, 2$$

is satisfied.

Step 6: Stop.

The optimal value of $(n, P, T)$ determines the optimal purchase quantity $Q$ (Appendix B) per transfer for the buyer.

5. NUMERICAL EXAMPLE

Let us illustrate the developed model with the following numerical values to model parameters.

$a = 1,00,000, b_1 = 7\%, b_2 = 5\%, \eta = 1.5, u_1 = 15 \text{ days}, u_2 = 45 \text{ days}, \gamma = 0.9, C_s = $ 2/unit, $v = $ 4.5/unit, $A_s = $ 1000/set-up, $A_b = $ 300/order, $I_s = 5\% /\text{unit/year}, I_b = 8\% /\text{unit/year}, I_{sp} = 9\% /\$/year, $I_{bc} = 16\% /\$/year, $I_{be} = 12\% /\$/year and $f_{sc} = 17\% /\$/year and $\theta = 0.12$. The supplier offers buyer the credit term ‘3/10 net 30’ means if buyer pays by 10 days then he will be offered 3% discount in the unit purchase price otherwise the buyer has to settle the account due against purchases in 30 days.

From Table 1, we see that for 10-shipments, the buyer’s selling price is $ 6.59/unit and cycle time is 122 days maximizing joint total profit of $ 25319 of the integrated system. The corresponding profit of the supplier is $ 13507 and that of buyer is $ 11812. Each transfer is of 2018 units. Optimal payment time is 10 days in ‘3/10net 30’ credit terms. The concavity of joint total profit with respect to number of transfers, $n$ and retail sale price, $P$ are shown in figures 3 and 4 respectively. 3-D plot given in figure 5 for $n = 10$ establishes the concavity of the total joint profit. The variations in permissible delay periods; $M_1$ and $M_2$ are worked out to study the changes in decision variable and total joint profit in Table 1. The profit gain is compared with benchmark of no credit period.
Fig. 4: Concavity of Joint Profit w.r.t. no. of Shipments (n)

Fig. 5: Concavity of Joint Profit w.r.t. Retail Price (P)

Fig. 6: Concavity of joint profit w.r.t. cycle time and retail price
The last column in table 1 represents percentage of profit gain which is calculated by the formula: \[
\left(\frac{\text{Profit with trade credit}}{\text{Profit without trade credit}} - 1\right) \times 100%.
\]

### Table 1: Optimal Solution for Various Credit Terms

<table>
<thead>
<tr>
<th>M_1 (days)</th>
<th>M_2 (days)</th>
<th>Optimal Payment Time (days)</th>
<th>n</th>
<th>P ()</th>
<th>T (days)</th>
<th>Q (units)</th>
<th>Buyer</th>
<th>Supplier</th>
<th>Joint</th>
<th>Buyer</th>
<th>Supplier</th>
<th>Joint</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>11</td>
<td>6.53</td>
<td>112</td>
<td>1878</td>
<td>10336</td>
<td>14463</td>
<td>24800</td>
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<td>-</td>
<td>-</td>
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<td>0</td>
<td>30</td>
<td>30</td>
<td>11</td>
<td>6.37</td>
<td>111</td>
<td>1925</td>
<td>10130</td>
<td>15149</td>
<td>25279</td>
<td>-0.023</td>
<td>0.453</td>
<td>0.189</td>
</tr>
<tr>
<td>10</td>
<td>30</td>
<td>10</td>
<td>10</td>
<td>6.59</td>
<td>122</td>
<td>2018</td>
<td>11812</td>
<td>13507</td>
<td>25319</td>
<td>12.50</td>
<td>-0.708</td>
<td>0.025</td>
</tr>
<tr>
<td>20</td>
<td>30</td>
<td>20</td>
<td>10</td>
<td>6.66</td>
<td>123</td>
<td>2005</td>
<td>12021</td>
<td>13176</td>
<td>25197</td>
<td>14.02</td>
<td>-0.977</td>
<td>0.158</td>
</tr>
<tr>
<td>0</td>
<td>60</td>
<td>60</td>
<td>11</td>
<td>6.27</td>
<td>113</td>
<td>2005</td>
<td>10152</td>
<td>15638</td>
<td>25790</td>
<td>-0.181</td>
<td>0.751</td>
<td>0.384</td>
</tr>
<tr>
<td>10</td>
<td>60</td>
<td>60</td>
<td>11</td>
<td>6.41</td>
<td>116</td>
<td>1997</td>
<td>11597</td>
<td>14266</td>
<td>25863</td>
<td>10.87</td>
<td>-0.138</td>
<td>0.041</td>
</tr>
<tr>
<td>20</td>
<td>60</td>
<td>60</td>
<td>11</td>
<td>6.41</td>
<td>116</td>
<td>1995</td>
<td>11595</td>
<td>14140</td>
<td>25735</td>
<td>10.86</td>
<td>-0.228</td>
<td>0.036</td>
</tr>
</tbody>
</table>

The positive profit gain proves that players of the supply chain are advantageous under two-level trade credit policy. It is observed that buyer entices to pay at early date in net credit scenario of ‘3/10net 30’ with maximum profit.

In table 2, independent and joint decisions are compared under different credit terms. It is seen that the offer of trade credit lowers retail price of the buyer and purchase of larger order is encouraged. The retail price of the buyer is almost double in independent decision compared to co-ordinated decision, while procurement quantity is halved. It is observed that the buyer’s profit decreases and that of supplier increases, which forces buyer to be dominant player in terms of making decision. Goyal (1976) favored the reallocation of profit for attracting buyer to opt for joint decision in the supply chain. Reallocate profit of buyer and supplier as follows:

\[
\text{Buyer’s profit} = \pi(n,P,T) \times \left[ \frac{TBP(P,T)}{TBP(P,T)+TSP(n)} \right]
\]
\[
= 25319 \times \frac{17534}{17534 + 4348} = 20288
\]

\[
\text{Supplier’s profit} = \pi(n,P,T) \times \left[ \frac{TSP(n)}{TBP(P,T)+TSP(n)} \right]
\]
\[
= 25319 \times \frac{4348}{17534 + 4348} = 5031
\]

The reallocated profits for buyer and supplier are exhibited in the last row of Table 2.
Table 2: Optimal solutions for different strategies

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Credit Term</th>
<th>Optimal Payment Time (days)</th>
<th>n</th>
<th>P ($)</th>
<th>T (days)</th>
<th>R(P,T) (units)</th>
<th>Q (units)</th>
<th>Profit ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Buyer</td>
</tr>
<tr>
<td>Independent</td>
<td>Cash on delivery</td>
<td>0</td>
<td>13</td>
<td>14.15</td>
<td>168</td>
<td>198</td>
<td>886</td>
<td>16991</td>
</tr>
<tr>
<td></td>
<td>Trade Credit 3/10 net 30</td>
<td>10</td>
<td>13</td>
<td>13.72</td>
<td>167</td>
<td>206</td>
<td>925</td>
<td>17534</td>
</tr>
<tr>
<td>Joint</td>
<td>Cash on delivery</td>
<td>0</td>
<td>11</td>
<td>6.53</td>
<td>112</td>
<td>283</td>
<td>1878</td>
<td>10336</td>
</tr>
<tr>
<td></td>
<td>Trade Credit 3/10 net 30</td>
<td>10</td>
<td>10</td>
<td>6.59</td>
<td>122</td>
<td>330</td>
<td>2018</td>
<td>11812</td>
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<tr>
<td>Adjusted</td>
<td></td>
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<td></td>
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</tr>
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</table>

The sensitivity analysis for model parameters is carried out by changing parameter as -20%, -10%, 10%, 20%. The figure 6 suggests that joint total profit is very sensitive to utilization factor and scale demand. This insights that the supplier should maintain production and demand ratio nearly 1. The joint profit is very sensitive to buyer’s ordering cost. It directs the buyer to place larger order and do saving in transportation cost. The joint profit decreases with increase in mark-up, supplier’s production cost, interest charged to the buyer, supplier’s opportunity cost and deterioration rate of units in inventory systems of both the players. The mark-up is controllable because it depends on economy of the business. The supplier’s opportunity cost depends on when the buyer is willing to pay. However, supplier can reduce production cost and deterioration rate by using modern machinery and latest storage facilities. The joint profit increases linearly with time suggesting that supplier and buyer are benefited when product enters into the system i.e. demand is in increasing phase.

![Fig. 7 Sensitivity Analysis for Model Parameters on Joint Profit](image-url)
6. CONCLUSIONS

A co-ordinated supplier-buyer inventory policy is addressed when demand is price-sensitive trapezoidal and units in inventory deteriorate at a constant rate. The analysis is focused on two payment scenarios namely ‘net credit’. The total joint profit is maximized with respect to number of transfers from supplier to the buyer, optimal payment time, the retail price and cycle time. To attract the buyer for joint decision, reallocation of the profit scheme is suggested. This result helps the buyer to make a decision between two promotional incentives, viz. price discount and permissible delay payment. In future, one can analyze integrated inventory system for different deterioration rates of units in buyer and supplier’s warehouses. It is worth incorporating imperfect production processes and optimizing production.

REFERENCES


Appendix A : Trapezoidal demand

The demand \( R(P,t) \) is considered to be a trapezoidal type whose functional form is

\[
R(P,t) = \begin{cases} 
  f(t)P^{-\eta}; 0 \leq t \leq u_1 \\
  R_0 P^{-\eta}; u_1 \leq t \leq u_2 \\
  g(t)P^{-\eta}; u_2 \leq t \leq T 
\end{cases}
\]

where \( u_1 \) is time point when the increasing demand function \( f(t) \) changes to constant demand and \( u_2 \) is the time point from where constant demand starts decreasing exponentially. In this study, we take \( f(t) \) to be liner in \( t \), \( R_0 = f(u_1) = g(u_2) \) and \( g(t) \) to be exponentially decreasing in \( t \). So the demand function is

\[
R(P,t) = \begin{cases} 
  R_1(P,t); 0 \leq t \leq u_1 \\
  R_2(P,t); u_1 \leq t \leq u_2 \\
  R_3(P,t); u_2 \leq t \leq T 
\end{cases}
\]

\[
R_1(P,t) = a(1+b_t)tP^{-\eta}
\]

; where \( R_2(P,t) = a(1+b_tu_1)P^{-\eta} \)

\[
R_3(P,t) = a(1+b_tu_1)e^{-b_2(t-u_2)}P^{-\eta}
\]
Appendix B: Computation of inventory at any instant of time $t$ and purchase quantity $Q$

The inventory level in warehouse changes due to price-sensitive trapezoidal demand and deterioration rate of units in the warehouse. The rate of change of inventory at any instant of time $t$ is governed by the differential equation

$$\frac{dI(t)}{dt} = -R(P,t) - \theta I(t); 0 \leq t \leq T$$

with the initial condition $I(T) = 0$.

The solution of the differential equation is

$$I(t) = \begin{cases} 
    I_1(t); & 0 \leq t \leq u_1 \\
    I_2(t); & u_1 \leq t \leq u_2 \quad \text{here} \\
    I_3(t); & u_2 \leq t \leq T 
\end{cases}$$

Using $I(0) = Q$, we get

$$Q = \frac{aP^{-\eta}}{\theta} \left[ -\frac{1 + b_2t}{\theta} + b_1t + \frac{1 + b_1u_1}{\theta} e^{-\theta_1 \theta t} - \frac{b_1}{\theta^2} e^{-\theta_1 \theta t} \right] + \frac{aP^{-\eta}(1 + b_1u_1)}{\theta} \left[ 1 - e^{\theta_1 \theta t} \right] + \frac{aP^{-\eta}(1 + b_1u_1)}{\theta - b_2} \left[ e^{-b_2T + \theta T - \theta t} - e^{-b_2u_2 + \theta u_2 - \theta t} \right]$$
Appendix C: Computation of total inventory during \([0, T]\)

Total inventory during \([0, T]\) is given by
\[
T \int I(t) \, dt = \int_{0}^{u_1} I_1(t) \, dt + \int_{u_1}^{u_2} I_2(t) \, dt + \int_{u_2}^{T} I_3(t) \, dt
\]

Appendix D: Computation of total demand
\[
T \int t \cdot R(P, t) \, dt = \int_{0}^{u_1} t \cdot R(P, t) \, dt + \int_{u_1}^{u_2} t \cdot R(P, t) \, dt + \int_{u_2}^{T} t \cdot R(P, t) \, dt
\]

Appendix E: Buyer’s total profit when \(T \geq M_j; j = 1, 2\)

\[
TBP_{j2}(\{(P, T); 0 \leq M_j \leq u_1\}) = \frac{PQ}{T} - \frac{(1-K_j \beta)vQ}{T} - \frac{A_b}{T} - \frac{1}{T} \left(1-K_j \beta\right) v I_{bc}^{T} \int I(t) \, dt
\]
\[
- \left[ \frac{1}{T} \left(1-K_j \beta\right) v I_{bc}^{u_2} \left\{ \int_{M_j}^{u_2} I_2(t) \, dt + \int_{u_2}^{T} I_3(t) \, dt \right\} \right]
\]
\[
+ \frac{1}{T} \left\{ \int_{0}^{u_1} t \cdot R_1(P, t) \, dt + \int_{u_1}^{M_j} t \cdot R_2(P, t) \, dt \right\}
\]

\[; 0 \leq M_j \leq u_1\]

\[
TBP_{j2}(\{(P, T); u_1 \leq M_j \leq u_2\}) = \frac{PQ}{T} - \frac{(1-K_j \beta)vQ}{T} - \frac{A_b}{T} - \frac{1}{T} \left(1-K_j \beta\right) v I_{bc}^{T} \int I(t) \, dt
\]
\[
- \left[ \frac{1}{T} \left(1-K_j \beta\right) v I_{bc}^{u_2} \left\{ \int_{M_j}^{u_2} I_2(t) \, dt + \int_{u_2}^{T} I_3(t) \, dt \right\} \right]
\]
\[
+ \frac{1}{T} \left\{ \int_{0}^{u_1} t \cdot R_1(P, t) \, dt + \int_{u_1}^{M_j} t \cdot R_2(P, t) \, dt \right\}
\]

\[; u_1 \leq M_j \leq u_2\]

\[
TBP_{j2}(\{(P, T); u_2 \leq M_j \leq T\}) = \frac{PQ}{T} - \frac{(1-K_j \beta)vQ}{T} - \frac{A_b}{T} - \frac{1}{T} \left(1-K_j \beta\right) v I_{bc}^{T} \int I(t) \, dt
\]
\[
- \left[ \frac{1}{T} \left(1-K_j \beta\right) v I_{bc}^{T} I_3(t) \, dt \right]
\]
\[
+ \frac{1}{T} \left\{ \int_{0}^{u_1} t \cdot R_1(P, t) \, dt + \int_{u_1}^{u_2} t \cdot R_2(P, t) \, dt + \int_{u_2}^{M_j} t \cdot R_3(P, t) \, dt \right\}
\]

\[; u_2 \leq M_j \leq T\]