DEFLECTION AND STRESS ANALYSIS OF A BEAM ON DIFFERENT ELEMENTS USING ANSYS APDL

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ABSTRACT

This paper studies the maximum deflection and Von-Misses stress analysis of:- a) Simply Supported Beam and b) Cantilever Beam under two different types of loading. The theoretical calculations are done based on the general Euler-Bernoulli’s Beam Equation. The Computational Analysis is done on ANSYS 14.0 software. Comparing the Numerical Results with that of the ANSYS 14.0, excellent accuracy of the present method has been extracted and demonstrated. In ANSYS 14.0 accuracies of different elements are measured and it has been visualized and concluded that Beam 189 element is most suitable element for Beam Analysis as compared to the Beam 188 element and other Solid elements.

Keywords: ANSYS, Beam, Beam Analysis, Euler-Bernoulli’s Beam Equation, 188 Element, 189 Element, Solid Elements.

I. INTRODUCTION

Beams belong to the basic structural members used in the modeling abstraction of mechanical systems.¹ In this paper behavior of beam and solid elements are discussed on the basis of Von-Misses stress and Deflection occurred on beam due to various types of load i.e point load and uniformly distributed load applied on rectangular section beam. A member subjected to bending moment and shear force undergoes certain deformations. The material of the member will offer resistance or stresses against these deformations.² It is possible to estimate these stresses with certain assumptions. The beam cannot have any translational displacements at its support points, but no restriction is placed on rotations at the supports. The deflected distance of a member under a load is directly related to the slope of the deflected shape of the member under that load. While the beam gets deflected under the loads, bending moment occurs in the same plane due to which stresses are developed. Here the deflection of the beam element is calculated by using the Euler-Bernoulli’s beam equation³ and the bending stresses using the general standard bending equation.
analytically.\(^2\) where on other hand Sparse solver is used to solve the Finite Element Model through Ansys 14.0 APDL.\(^4\)

The effect of elements structure on Maximum Von-Misses stress and Deflection are analyzed in this paper. Those elements are Beam 188, Beam 189, Solid 185 and solid 285. And it has been noticed that the most accurate result was measured by Beam 189 followed by Beam 188.

II. THEORETICAL CALCULATIONS

The calculations are done considering a uniform rectangular cross-sectional beam of linear elastic isotropic homogeneous materials. The beam is assumed to be massless, inextensible having developed no strains \(^5\).

Using the bending moment curvature relationship the following equation is obtained:

\[
EI \frac{d^2y}{dx^2} = M \quad \text{(1)}
\]

Using the equation: 

\[
\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{\gamma} \quad \text{(2)}
\]

Stress is calculated.

Where \(E\) is modulus of elasticity, which is constant. \(I\) is moment of inertia=\(bh^3/12\), \(b=\)width of beam, \(h=\)height of beam, \(M=\)moment developed.

Four cases are considered in this paper a) Simply Supported Beam with Uniformly Loading b) Simply Supported Beam with Single Point Load at centre c) Cantilevered Beam with Uniformly Loading d) Cantilevered Beam with Single Point Load at the end.

CASE 1. Simply Supported Beam with Uniformly Distributed Load \(W\) per unit length
Assuming \(L=100\text{m}, b=10\text{m}, h=10\text{m}, \nu=0.3, E=2\times10^7\text{N/m}^2, F=500\text{N}\). The maximum deflection of beam at a distance \(x=\frac{L}{2}\) from one of the fixed end is \(5WL^4/384EI\) and it is calculated as \(0.0003906\text{m}\). With the required boundary conditions the maximum bending moment is obtained as \(WL^2/8\). Using the equation: 

\[
\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{\gamma},
\]

Stress developed \(\sigma_0\) is \(37.502\text{N/m}^2\).

CASE 2: Simply Supported Beam with Single Point Load at Centre
Assuming \(L=100\text{m}, b=10\text{m}, h=10\text{m}, \nu=0.3, E=2\times10^7\text{N/m}^2, F=500\text{N}\). The maximum deflection of beam at a distance \(x=\frac{L}{2}\) from one of the fixed end is \(WL^3/48EI\) and it is calculated as \(0.00063\text{m}\). With the required boundary conditions the maximum bending moment is obtained as \(WL/4\). Using the equation: 

\[
\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{\gamma},
\]

Stress developed \(\sigma_0\) is \(75.003\text{N/m}^2\).
CASE 3: Cantilevered Beam with Uniformly Distributed Load \( W \) per unit run over the whole length
Assuming \( L=100\text{m} \), \( b=10\text{m} \), \( h=10\text{m} \), \( \nu =0.3 \), \( E=2\times10^7\text{N/m}^2 \), \( F=500\text{N} \). The maximum deflection of beam at a distance \( x=L \) from the fixed end is \( WL^4/8EI \) and it is calculated as 0.00375m. With the required boundary conditions the maximum bending moment is obtained as \( WL^2/2 \). Using the equation: \( (M/I)=(E/R)=(\sigma/Y) \), Stress developed \( \sigma_b \) is 150.0060\( \text{N/m}^2 \).

Figure 3: Cantilevered Beam with Uniformly Distributed Load \( W \) per unit run over the whole length

CASE 4: Cantilevered Beam with Single Point Load at the end
Assuming \( L=100\text{m} \), \( b=10\text{m} \), \( h=10\text{m} \), \( \nu =0.3 \), \( E=2\times10^7\text{N/m}^2 \), \( F=500\text{N} \). The maximum deflection of beam at a distance \( x=L \) from the fixed end is \( WL^3/3EI \) and it is calculated as 0.01m. With the required boundary conditions the maximum bending moment is obtained as \( WL \). Using the equation: \( (M/I)=(E/R)=(\sigma/Y) \), Stress developed \( \sigma_b \) is 300.0120\( \text{N/m}^2 \).

Figure 4: Cantilevered Beam with Single Point Load at the end

III. COMPUTATIONAL RESULT

CASE 1: Simply Supported Beam with Uniformly Distributed Load \( W \) per unit length
A. 188 element- (Fig.5 & Fig.6) Maximum Deflection obtained=\( 0.4e^{-3} \text{m} \) and Maximum Von-Mises Stress obtained= 37.4925 \( \text{N/m}^2 \).
B. 189 element- (Fig.7 & Fig.8) Maximum Deflection obtained=\( 0.4e^{-3} \text{m} \) and Maximum Von-Mises Stress obtained= 37.5025 \( \text{N/m}^2 \).
C. 185 element- (Fig.9 & Fig.10) Maximum Deflection obtained=\( 0.401e^{-3} \text{m} \) and Maximum Von-Mises Stress obtained= 37.0599 \( \text{N/m}^2 \).
D. 285 element- (Fig.11 & Fig.12) Maximum Deflection obtained=\( 0.394e^{-3} \text{m} \) and Maximum Von-Mises Stress obtained= 37.6991 \( \text{N/m}^2 \).
CASE 2: Simply Supported Beam with Single Point Load at Centre

A. 188 element- (Fig.13 & Fig.14) Maximum Deflection obtained=0.644e-3m and Maximum Von-Mises Stress obtained= 74.2500 N/m².
B. 189 element- (Fig.15 & Fig.16) Maximum Deflection obtained=$0.644e^{-3}$m and Maximum Von-Mises Stress obtained= $75 \text{ N/m}^2$.

C. 185 element- (Fig.17 & Fig.18) Maximum Deflection obtained=$0.655e^{-3}$m and Maximum Von-Mises Stress obtained= $111.6060 \text{ N/m}^2$.

D. 285 element- (Fig.19 & Fig.20) Maximum Deflection obtained=$0.644e^{-3}$m and Maximum Von-Mises Stress obtained= $74.2500 \text{ N/m}^2$.
CASE 3: Cantilevered Beam with Uniformly Distributed Load W per unit run over the whole length

A. 188 element- (Fig.21 & Fig.22) Maximum Deflection obtained=0.3797e$^{-2}$ m and Maximum Von-Mises Stress obtained= 148.5070 N/m$^2$.
B. 189 element- (Fig.23 & Fig.24) Maximum Deflection obtained=0.3797e$^{-2}$ m and Maximum Von-Mises Stress obtained= 148.9980 N/m$^2$.
C. 185 element- (Fig.25 & Fig.26) Maximum Deflection obtained=0.3761e$^{-2}$ m and Maximum Von-Mises Stress obtained= 161.8340 N/m$^2$.
D. 285 element- (Fig.27 & Fig.28) Maximum Deflection obtained=0.3703e$^{-2}$ m and Maximum Von-Mises Stress obtained 151.3950 N/m$^2$. 

Figure 19: Displacement At Different Nodes In 285 Element
Figure 20: Stress Distribution At Different Nodes In 285 Element
Figure 21: Displacement At Different Nodes In 188 Element
Figure 22: Stress Distribution At Different Nodes In 188 Element
Figure 23: Displacement At Different Nodes In 189 Element
Figure 24: Stress Distribution At Different Nodes In 189 Element
CASE 4: Cantilevered Beam with Single Point Load at the end
A. 188 element- (Fig. 29 & Fig. 30) Maximum Deflection obtained = 0.010105 m and Maximum Von-Mises Stress obtained = 298.5000 N/m².
B. 189 element- (Fig. 31 & Fig. 32) Maximum Deflection obtained = 0.010105 m and Maximum Von-Mises Stress obtained = 300.0000 N/m².
C. 188 element- (Fig. 33 & Fig. 34) Maximum Deflection obtained = 0.010043 m and Maximum Von-Mises Stress obtained = 321.0030 N/m².
D. 188 element- (Fig. 29 & Fig. 30) Maximum Deflection obtained = 0.010105 m and Maximum Von-Mises Stress obtained = 298.5000 N/m².
IV. COMPARISON OF RESULTS

In table.1 and table.3 the Analytical Results of Maximum Von-Mises stress and Maximum Deflection on different elements after considering the loading conditions as mentioned above are compared with the Computational Results. And table.2 and table.4 demonstrates the percentage of error between the Analytical Results and Computational Results.
Table 1: DEFLECTION

<table>
<thead>
<tr>
<th>CASE</th>
<th>188ele</th>
<th>189ele</th>
<th>185ele</th>
<th>285ele</th>
<th>Analytical Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASE 1</td>
<td>0.400e^{-3}</td>
<td>0.400e^{-3}</td>
<td>0.401e^{-3}</td>
<td>0.394e^{-3}</td>
<td>0.391e^{-3}</td>
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<tr>
<td>CASE 2</td>
<td>0.644e^{-3}</td>
<td>0.644e^{-3}</td>
<td>0.655e^{-3}</td>
<td>0.610e^{-3}</td>
<td>0.630e^{-3}</td>
</tr>
<tr>
<td>CASE 3</td>
<td>0.3797e^{-2}</td>
<td>0.3797e^{-2}</td>
<td>0.3761e^{-2}</td>
<td>0.3703e^{-2}</td>
<td>0.3750e^{-2}</td>
</tr>
<tr>
<td>CASE 4</td>
<td>0.010105</td>
<td>0.010105</td>
<td>0.010043</td>
<td>0.009886</td>
<td>0.01000</td>
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Table 2: DEFLECTION ERROR PERCENTAGE

<table>
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<tr>
<th>CASE</th>
<th>188ele</th>
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<th>285ele</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASE 1</td>
<td>2.30</td>
<td>2.30</td>
<td>2.56</td>
<td>0.77</td>
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<tr>
<td>CASE 2</td>
<td>2.20</td>
<td>2.20</td>
<td>3.97</td>
<td>3.17</td>
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<tr>
<td>CASE 3</td>
<td>1.25</td>
<td>1.25</td>
<td>0.29</td>
<td>1.25</td>
</tr>
<tr>
<td>CASE 4</td>
<td>1.01</td>
<td>1.01</td>
<td>0.43</td>
<td>1.14</td>
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Table 3: VON-MISES STRESS

<table>
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<tr>
<th>CASE</th>
<th>188ele</th>
<th>189ele</th>
<th>185ele</th>
<th>285ele</th>
<th>Analytical Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASE 1</td>
<td>37.4925</td>
<td>37.5025</td>
<td>37.0599</td>
<td>37.6991</td>
<td>37.5020</td>
</tr>
<tr>
<td>CASE 2</td>
<td>74.2500</td>
<td>75.0000</td>
<td>111.6060</td>
<td>73.4405</td>
<td>75.0030</td>
</tr>
<tr>
<td>CASE 3</td>
<td>148.5070</td>
<td>148.9980</td>
<td>161.8340</td>
<td>151.3950</td>
<td>150.0060</td>
</tr>
<tr>
<td>CASE 4</td>
<td>298.5000</td>
<td>300.0000</td>
<td>321.0030</td>
<td>303.4560</td>
<td>300.0120</td>
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Table 4: VON-MISES STRESS ERROR PERCENTAGE

<table>
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<th>285ele</th>
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<tbody>
<tr>
<td>CASE 1</td>
<td>0.025</td>
<td>0.001</td>
<td>1.179</td>
<td>0.526</td>
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<tr>
<td>CASE 2</td>
<td>1.000</td>
<td>0.004</td>
<td>48.802</td>
<td>2.080</td>
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<tr>
<td>CASE 3</td>
<td>0.999</td>
<td>0.672</td>
<td>7.885</td>
<td>0.926</td>
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<tr>
<td>CASE 4</td>
<td>0.504</td>
<td>0.004</td>
<td>6.997</td>
<td>1.148</td>
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</tbody>
</table>

V. CONCLUSION

After going through all the tables’ data, it can be concluded that the ELEMENT 189 is the best element to do BEAM ANALYSIS rather than 188 element and other SOLID ELEMENTS. Rather, if we define the priority of the elements for beam analysis than it would be as follows 189 element, 188 element, Solid 285 element, solid 185 element. Hence, it is very well justified to mention that to solve beam type of problem we always need to rely on beam 189 element rather than any other elements.
REFERENCES


