RESISTANCE DISTANCE METHOD FOR DETERMINATION OF ISOMORPHISM AMONG PLANAR KINEMATIC CHAINS AND THEIR DERIVED MECHANISMS

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ABSTRACT

The detection of isomorphism among kinematic chains and their derived mechanism has been a hot area of research for last several years. In the present communication a resistance distance based computerized method is proposed and it is tested for planar kinematic chains up to ten links. In this method the kinematic chains are represented in the form of resistance graph and from that graph a laplacian matrix is generated which is further transformed into a resistance distance matrix. The Square sums of resistances are used as an index for isomorphism identification among kinematic chains. This method is successfully examined for one degree of freedom, 6, 8, 10, links planar kinematic chains, 9 links two degree of freedom and 10 links three degree of freedom planar kinematic chains.

Keywords: Distinct Mechanism [DM]; Kinematic Chain [KC]; laplacian matrix [L], Resistance Distance matrix [RD].

1. INTRODUCTION

Structural synthesis of the kinematic chain (KC) and mechanism has been the subject of a number of studies in recent years. One important area of structural synthesis is to develop all possible mechanisms derived from a given KC so that the designer has the liberty to select the best or optimum mechanism depending upon the application. In the course of development of mechanisms, duplication or isomorphism may be possible. So for the identification of duplication or isomorphism, the researchers have proposed several methods in recent past. The methods proposed so far are based on adjacency matrix method [1], distance matrix [2] to determine the structurally distinct mechanisms of a kinematic chain. The flow matrix method [3] and the extended adjacency matrix
methods [4, 5] are also proposed. Minimum code [6], characteristic polynomial of matrix [7], link path code [8], the representation set of links by Vijayananda [9], interactive weighted distance approach [10] is used to characterize the kinematic chains. Recently Lu et al [15, 16] derived valid contracted graphs with the help of characteristic strings and identified the isomorphism. Most of these methods either have lack of uniqueness or are very time consuming. The flow matrix method [3] is a lengthy process to identify the distinct mechanism (DM), as ‘n’ flow matrices are required to be developed. The row-sum of extended adjacency matrix method [4] distinguishes only 69 distinct mechanisms derived from the family of 8-link, 1-DOF kinematic chains instead of the 71 reported by other researchers. The characteristic polynomial of matrix [7] approach is a very lengthy process in which the invariants of each matrix are tabulated and compared for the identification of distinct mechanism. Hence, there is always a need to develop a computationally optimized method to detect isomorphism in kinematic chains and their derived mechanisms.

In the present work, a new method is proposed in which interaction effect of connecting links has been taken into consideration along with a squared laplacian matrix [L]. A new matrix named as resistance distance matrix [RD] is formed using [L]. This matrix is further transformed in the form of a string where each string represents a link of KC. The string directly gives the number of DM derived from a KC. It is also used as an index for determination of isomorphism among KCs. The resistance distance method is widely used to solve the electrical network and to analyze the mathematical molecular chemistry problems. The proposed method is efficient and accurate and only one matrix for a given kinematic chain is developed for the determination of DM i.e. [RD].

2. DEFINITIONS OF TERMINOLOGY

2.1 Resistance equivalent graph

In the proposed method a link of KC is represented in the form of a graph and then a resistance value is assigned between the two nodes of a graph and the whole KC is represented in the form of a resistance network. To illustrate this a four bar KC as shown in Fig-1 is represented in the form of a resistance equivalent graph as shown in Fig-2. The resistance value assigned between the nodes is taken as one unit.

![Fig.1: Four bar chain](image1)

![Fig.2: Equivalent resistance graph of a four bar chain](image2)

2.2 Laplacian matrix

The laplacian matrix is widely used for electrical networks problems. A laplacian matrix can be derived from a simple graph for no loops and no parallel edges. Let G be a simple graph for n vertices 1,2,----n. Then an Adjacency matrix A of G is
\[ a_{ij} = \begin{cases} 1 & \text{if } i \text{ is adjacent to } j \\ 0 & \text{otherwise} \end{cases} \]

Laplacian Matrix \( L \) of \( G \): \( L = D - A \)

Where \( D \) is the diagonal degree matrix

For Fig.1 the adjacency and Laplacian matrix are

\[
A = \begin{bmatrix}
0 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 \\
\end{bmatrix}
\]

\[
L = \begin{bmatrix}
2 & -1 & 0 & -1 \\
-1 & 2 & -1 & 0 \\
0 & -1 & 2 & -1 \\
-1 & 0 & -1 & 2 \\
\end{bmatrix}
\]

2.3 Resistance distance matrix [RD]

The equivalent resistances between all the nodes are calculated. For example, the equivalent resistance \( R_{1432} \) between nodes 1 and 2 is calculated taking the resistance \( R_{14}, R_{43}, R_{32} \) in series as

\[
R_{1432} = R_{14} + R_{43} + R_{32} = 1 + 1 + 1 = 3
\]

Then the final equivalent resistance \( R_{eq} \) is calculated taking \( R_{1432} \) and \( R_{12} \) in parallel.

\[
R_{eq} = \frac{1}{R_{12}} + \frac{1}{R_{1432}} = \frac{3}{4}
\]

As the graph becomes complex, the no of links increases in the KC and the computation of equivalent resistance becomes tedious. I. Gutman, R. B. Bapat et al [11] D. J. Klein, M. Randić [12] proposed a method RD matrix to calculate the values of equivalent resistance between two nodes. For a four bar kinematic chain RD is represented as

\[
RD = \begin{bmatrix}
0 & 3/4 & 1 & 3/4 \\
3/4 & 0 & 3/4 & 1 \\
1 & 3/4 & 0 & 3/4 \\
3/4 & 1 & 3/4 & 0 \\
\end{bmatrix}
\]

3. STRUCTURAL INVARIANT [RD^2Σ]

The structural invariant is a column matrix whose elements are obtained from the [RD] matrix. The sum of squares of all the row elements for every link is tabulated as square resistance distance [RD^2Σ] matrix. Therefore the [RD^2Σ] are used as the identification number of a KC as well as distinct mechanism derived from a KC. The square of [RD] element is used for better comparison for each row, as a very little change in any matrix value can cause a much larger difference in its
square to observe so the matrix values square is done. If the values of \([RD^2\Sigma]\) for any two links are same then the two inversions are identical otherwise different. The \([RD^2\Sigma]\) values are sorted in ascending order for ease of computation.

4. ALGORITHM TO FIND THE DM FROM A GIVEN KC

Step 1: Define the number of links & their names arbitrarily.
Step 2: Convert the KC into a graph and then a resistance circuit graph is drawn.
Step 3: Define the connectivity of each node, e.g. node ‘1’ is connected to node ‘2’, and ‘4’ as shown in Fig-1.
Step 4: Find out the degree of nodes on the basis of their connectivity to other nodes.
Step 5: Find out the \([A]\), \([L]\), \([RD]\) and \([RD^2\Sigma]\) as explained earlier.
Step 6: Sort the values of string \([RD^2\Sigma]\)

5. PROCEDURE TO TEST THE ISOMORPHISM AMONG KCs

Step1: Steps 1 to 4 are same as above.
Step2: Determine the sum of string \([RD^2\Sigma]\)
Step3: If the sums of \([RD^2\Sigma]\) value for both the KCs are equal then the KCs are isomorphic, otherwise non-isomorphic.

Illustrative Example 1

Fig.3. consists of a revolute joint, 1-DOF KC having 10 links and 13 joints. The task is to determine the DM derived from the given KC.

Fig.3: 10-link, 1-DOF, Planar kinematic chain
The adjacency matrix \( A \) and the laplacian matrix \( L \) can be written as

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
L = \begin{bmatrix}
3 & -1 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 \\
-1 & 3 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & -1 & 3 & -1 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & -1 & 3 & -1 & 0 & 0 & 0 & 0 & -1 \\
-1 & 0 & 0 & -1 & 3 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 3 & -1 & -1 & -1 & 0 \\
-1 & 0 & 0 & 0 & 0 & -1 & 3 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & -1 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 2 & 0 \\
0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 2 \\
\end{bmatrix}
\]

The equivalent resistance distance matrix is then calculated as explained earlier. A MATLAB program is made to calculate the resistance distance matrix from laplacian matrix. The \([RD]\) matrix for Fig.3 is

\[
RD = \begin{bmatrix}
0.00 & 0.67 & 0.91 & 0.88 & 0.63 & 0.91 & 0.73 & 1.16 & 1.04 & 1.10 \\
0.67 & 0.00 & 0.62 & 0.74 & 0.88 & 1.12 & 1.17 & 1.12 & 1.28 & 0.68 \\
0.91 & 0.62 & 0.00 & 0.62 & 0.91 & 1.00 & 1.23 & 0.75 & 1.23 & 0.94 \\
0.88 & 0.74 & 0.62 & 0.00 & 0.67 & 1.12 & 1.28 & 1.12 & 1.17 & 0.68 \\
0.63 & 0.88 & 0.91 & 0.67 & 0.00 & 0.91 & 1.04 & 1.16 & 0.73 & 1.10 \\
0.91 & 1.12 & 1.00 & 1.12 & 0.91 & 0.00 & 0.73 & 0.75 & 0.73 & 1.14 \\
0.73 & 1.17 & 1.23 & 1.28 & 1.04 & 0.73 & 0.00 & 1.23 & 1.16 & 1.54 \\
1.16 & 1.12 & 0.75 & 1.12 & 1.16 & 0.75 & 1.23 & 0.00 & 1.23 & 1.44 \\
1.04 & 1.28 & 1.23 & 1.17 & 0.73 & 0.73 & 1.16 & 1.23 & 0.00 & 1.54 \\
1.10 & 0.68 & 0.94 & 0.68 & 1.10 & 1.44 & 1.54 & 1.44 & 1.54 & 0.00 \\
\end{bmatrix}
\]

The \([RD2]\) is

\[
\]

Arranging in ascending order gives

\[
\]

After inspecting the values obtained by \([RD2]\) it is clear that link ‘1’ & ‘5’, link ‘2’ & ‘4’, link ‘7’ & ‘9’ are equivalent link as their \(RD2\) values are same. So the distinct mechanisms that can be derived from the KC as shown in Fig.3 are 10-3=7.

**Illustrative Example 2**

Fig.4 consists of three 12 link KCs. They have the same characteristic polynomial and eigen values [14]. The characteristic polynomial and eigen values of adjacency matrix failed to detect isomorphism for these chains. These KCs are tested for isomorphism by the proposed method.
The adjacency and laplacian matrix for these three KCs is written by observation and the resistance distance matrix \([RD]\) is obtained by MATLAB program. Then the \([RD^2\Sigma]\) matrix is evaluated for all the three cases and the values are sorted as follows.

\[
[RD^2\Sigma]_a = [8.094\ 8.118\ 8.716\ 9.727\ 9.808\ 9.848\ 10.393\ 11.941\ 13.862\ 14.228\ 14.582,\ (128.58)]
\]

\[
[RD^2\Sigma]_b = [RD^2\Sigma]_c = [9.933\ 9.933\ 9.957\ 9.957\ 10.314\ 10.389\ 11.156\ 11.997\ 15.485\ 15.485\ 15.674,\ (146.76)]
\]

From the above string it is concluded that 12 DM can be derived from KC (a) and 7 DM from KC (b) & (c). Also Fig. 3 (b) & (c) are isomorphic as their sum of string values of \([RD^2\Sigma]\) (146.76) are same. Further (a) & (b) as well as (a) & (c) are non-isomorphic. These results are in accordance with the results of Chang et al [14].

6. RESULTS AND CONCLUSIONS

The proposed method is tested to obtain all DM derived from a family of KC upto 10 links 3-DOF. The results of DM derived from the family of some known cases of planar chains with simple revolute joints are given in table-1. The results are in agreement with those available in the literature given by Vijayananda [9] and Yadav et al [8] and no counter example is found.
Table -1: distinct mechanism of some known cases of planar chains with simple revolute joints

<table>
<thead>
<tr>
<th>Type of chain</th>
<th>Total no of chain</th>
<th>Total no of distinct mechanisms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Previous Methods</td>
<td>Resistance Distance</td>
</tr>
<tr>
<td>1-F,6-link</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>2-F,7-link</td>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>3-F,6-link</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3-F,8-link</td>
<td>7</td>
<td>26</td>
</tr>
<tr>
<td>3-F,10-link</td>
<td>98</td>
<td>684</td>
</tr>
<tr>
<td>1-F,8-link</td>
<td>16</td>
<td>71</td>
</tr>
<tr>
<td>2-F,9-link</td>
<td>40</td>
<td>254</td>
</tr>
<tr>
<td>3-F,10-link</td>
<td>230</td>
<td>1834</td>
</tr>
<tr>
<td>1-F,10-link</td>
<td>230</td>
<td>1834</td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3-F,8-link</td>
<td>7</td>
<td>26</td>
</tr>
<tr>
<td>3-F,10-link</td>
<td>98</td>
<td>684</td>
</tr>
</tbody>
</table>

CONCLUSION

In the present communication a resistance distance approach in relation with the laplacian matrix determines DM from a given KC as well as detects isomorphism among KCs. In this method the usual adjacency matrix is written by observation and it is easily converted into a laplacian matrix. The computer algorithm is also written for this method which can easily be converted into a computer program by using any computer language. All the information is available in just one [RD] matrix and this method is fast and easy for computation as there is no complexity involved to find the structural invariant. Determination of [DM] from n-link kinematic chain using a flow matrix [4] requires the formation of n-flow matrices; however; in the proposed method only a single [RD] matrix is sufficient to identify the DM from a given KC. The [RD2Σ] which compares the total resistance distance information in [RD] is used as an invariant for the identification of DM in KCs. It has been shown with the help of examples that the sum of [RD2Σ] is a unique property of [RD] and that can be used to identify the isomorphism among KCs.

The proposed methods are heuristic and intuitive in nature. This worked well on all the known cases of planar chains with simple joints. Such a new identification system would be extremely selective and would minimize, if not completely eliminate the possibility of duplicate identification for structurally different mechanism. The result of this work contributes to the automation for the process of creative mechanism design. The future scope of this work is to extend by taking the weighted value of joints as we are converting the graph from a given KC and then the resistance value is taken as unity between the nodes.

7. REFERENCES


