OPTIMAL DESIGN OF MACHINERY SHALLOW FOUNDATIONS WITH CLAY SOILS

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ABSTRACT

Optimal design of shallow foundations used in supporting various machinery is of vital importance. Badly designed foundations result in serious problems such as soil failure, large soil settlement, low vibration efficiency and resonance of structures near by the foundation.

An optimal design approach based on using MATLAB optimization tool box is used to provide the optimal design of a machinery foundation. To minimize the foundation cost the foundation mass is used as an objective function. To control the high technical quality of the foundation from soil mechanics and vibrations point of view, 10 functional constraints are used.

The soil isolation efficiency of the soil is responsible about the vibration transmitted to the surrounding. Therefore, the effect of the minimum isolation efficiency in the range of 80 – 96 % on the foundation optimal design is investigated in details. The performance of the foundation against the minimum isolation efficiency is outlined so that the civil engineer can compromise between cost and performance.

Keywords: Shallow Foundations – Optimal Design – Foundation _ Soil Interaction – Clay Soils.

1. INTRODUCTION

Machinery foundations have to be precisely designed to avoid the side effects of the dynamic loads encountered with all the machinery. The difficulty in this respect is due to the interaction between the machine, foundation and soil. The reason for this is the great variability in soil type , properties and water content.

Design techniques range from following local national codes to optimal approaches. There is a little work done in behalf of optimal design of machinery foundations and this research work comes to fill this gap and introduces an approach for the optimal design of machinery foundations
taking into consideration the cost (minimum mass objective function) and performance during operation (through using nine functional constraints). Most of research activities are focused on soil properties required for the foundation design and the foundation-soil interface models.

Gazetas (1983) reviewed the state of art of analyzing the dynamic response of foundations subjected to machine-type loading. He presented his results in the form of formulae and dimensionless graphs [1]. Pecker (1996) reviewed the evaluation of seismic bearing capacity of shallow foundations resting on cohesive and dry cohesionless soils [2]. Adel et al. (2000) applied the expert system approach to foundation design providing the user with means to assess the effect of various foundation alternatives and ground conditions [3]. Pender (2000) explained a model for the cyclic stress-strain behavior of cohesive soil and the application of the model in estimating the dynamic compliance of footing under vertical cyclic loading [4]. Tripathy et al. (2002) studied the void ratio and water content of specimens at several intermediate stages during swelling. They studied the void ratio and water content characteristics of the soil during swelling and shrinkage [5].

Anteneh (2003) presented the foundation performance requirements and the basic steps employed in the design of a machine foundation. He reviewed all the soil parameters required for the foundation design and the basic concepts in foundation vibrations [6]. Ostadan et al (2004) discussed a series of parametric studies for structure-soil interaction damping and presented an effective approach to estimate the system damping for such systems [7]. Park and Hashash (2004) proposed formulations used in nonlinear site response analysis which showed that the equivalent linear frequency domain solution used to approximate nonlinear site response underestimated the surface ground motion within a period range relevant to engineering applications [8].

Dewookar and Huzjak (2005) studied the effect of the effective normal stress, liquid limit and plasticity index on the soil friction angle. They viewed the available models relating soil friction angle and plasticity index [9]. Prakash and Puri (2006) discussed the analysis methods used in determining the foundation response when subjected to vibrating loads. They considered the machine foundation as a mass-spring-damper model with one or two degree of freedom [10]. Ahmed et. al. (2006) studied the nature, structural features, engineering behavior and properties of soil samples extracted from Penana Island. The studied the effect of the soil depth on particle size, moisture content, liquid limit, plastic limit, specific gravity, cohesion and friction angle [11]. Anggraini (2006) evaluated the increase of shear strength of fibrous peat due to the application of consolidation pressure between 50 and 200 kPa [12]. Steinhausen (2007) tried to solve the problem of installing large and heavy reciprocating machines implying high dynamic loads at the substructure of offshore systems [13]. Chandrakaran et al. (2007) presented a simplified design procedure for the design of machine foundations subjected to vertical dynamic loading using the half-space theory. They presented characteristic curves that can be used by the practicing engineers to reduce the number of trials in the design process and to arrive at the optimal design [14]. Simoes et al (2007) showed how to predict in a simple way the effect of vibration due to machine operation on its surroundings considering the foundation as a vibration source on the surface of an elastic medium [15]. Chakravarti (2008) studied the design of rectangular block foundation for vibrating machines considering both the effect of damping and coupled modes using time-history analysis [16]. Bhatia (2008) highlighted the need for a better interaction between foundation designers and machine manufacturers to ensure improved machine performance. He discussed the design methodology for foundation design and the vibration isolation system for heavy-duty machines [17]. Gunduz (2008) studied the time dependant settlements calculated with numerical methods and subsequent comparisons with field measurements [18]. Michaels (2008) proposed an alternative to the traditional representation of damping. He considered that the imaginary part of the shear modulus will vary directly with frequency which is useful in cases where the water table may change affecting the foundation design [19]. Karray and Lefebvre (2008) studied the effect of Poisson’s ratio on Rayleigh wave phase velocities leading to more accurate soil characterization [20].
Jain et al. (2010) used an artificial neural network technique to predict the shear strength parameters of medium compressibility soil. They studied the variation of soil cohesion and internal friction angle with soil dry density and degree of saturation [21]. Kenarangi and Rofooein (2010) investigated the performance of tuned mass dampers in reducing the nonlinear response of irregular buildings under bi-directional horizontal ground motions considering soil-structure interaction[22]. Puri and Rao (2010) evaluated the modulus of elasticity and the resilient modulus of subgrade clayey sand soils by laboratory testing. They used the California Bearing Ratio Test and the Unconfined Cyclic Triaxial Test which are useful in studying soil settlement and the mechanistic pavement design [23]. Davidovic et al. (2010) compared the settlements calculated using different settlements prediction techniques and those of experiments in order to test techniques accuracy and reliability [24]. Olsson (2010) focused on how to calculate long-term settlement in soft clays and discussed ways of evaluating some of the important parameters used [25]. Romerco et al. (2010) presented motion formation to study the coupled response of rigid foundations modeled by springs and dampers. Their use of matrix manipulation has led to an expression providing closer approximation to the real phenomenon [26].

Aziz and Ma (2011) focused on the design and analysis of bridge foundation using four codes. They found which code is better for design and control of the high settlement problem due to loading [27]. Moghaddasi et al. (2011) investigated the effects of soil – foundation – structure interaction on seismic response of structures through a robust Monte Carlo simulation using a wide range of soil – foundation systems and earthquake input motions in time history analysis [28]. Clayton (2011) studied the factors affecting the stiffness of soils and weak rocks. He described the impact of a number of stiffness parameters on the displacements around a retaining structure [29]. Kim et al. (2011) used an electrical resistivity cone probe to determine the electrical resistivity and void ratio of seashore soft soils. They concluded that their technique may be an effective one for the estimation of the offshore soil void ratio [30].

Kaptan (2012) proposed an empirical formulation for the rapid determination of the allowable bearing pressure of shallow foundations in soils and rocks. The proposed expression was consistent with the results of the classical theory and proved to be rapid and reliable [31]. Miyata and Bathurst (2012) used the results of load measurements and that reported in the literature to examine the prediction accuracy of the Coherent Gravity Method [32]. Palacios et al. (2012) studied the geotechnical characteristics of foundation design under seismic conditions in towns in the Granada province of Spain. They recommended using deep foundations [33]. Tafreshi and Dawson (2012) presented the results of laboratory-model tests on strip footings supported on unreinforced and geocell-reinforced sand beds under combination of static and dynamic loads. They studied the influence of various parameters including the amplitude of the dynamic load [34]. Zamani and El-Shamy (2012) presented a 3-dimensional micro-scale framework utilizing the discrete element method to analyze the seismic response of soil-foundation-structure systems. They examined the impact of structure and damping, foundation embedment, exciting amplitude and frequency on the system response [35]. Hassaan, Lashin and Al-Gamil (2012) collected a large number of soil and foundation parameters available in the literature and molded them in the form of mathematical models suitable for the computer-aided analysis and design of foundations. The models used had high multiple correlation coefficient for accurate representation of the data [36].

Pan, Tsai and Lin (2013) presented the ultimate bearing capacity of shallow foundations. They utilized the weighted genetic programming and soft computing polynomials for the accurate prediction and visible formulas for the ultimate bearing capacity [37]. Karamitros, Bouckovalos and Chaloulous (2013) presented a simplified analytical methodology for the computation of the seismic settlements of stripped rectangle foundations resting on liquefiable soil with clay crust [38]. Nagah and Nwankwoala (2013) recommended appropriate foundation design and construction for large-scale ground and elevated water storage in the Onne area, Rivers State, Nigeria [39].
Shahnazari, Shahin and Tutunchran (2014) utilized the evolutionary polynomial regression, classical genetic programming and gene expression programming for the accurate prediction of shallow foundation settlements on cohesionless soils [40]. Cunha and Albuquerque (2014) summarized the historical development of foundation engineering in Brazil since its early beginning illustrating how foundation engineering technologies were improved and approached [41]. Fellenius (2014) studied in details aspects of bearing capacity of shallow foundations considering inclined and eccentric loading, inclination and shape factors, overturning and sliding [42].

2. **ANALYSIS**

**Objective function:**

The foundation mass, \( M_f \) is used as the objective function of the optimization problem to reduce its cost. It is related to the foundation length \( L \), width \( B \) and height \( H_f \) through:

\[
M_f = \rho LBH_f \tag{1}
\]

Where \( \rho \) is the foundation density.

**Functional constraints:**

(i) **Length / width ratio constraint, \( C_1 \):**

\[
C_1 = B / L \ (\leq 1) \tag{2}
\]

This constraint means that the foundation has a rectangular or square shape.

(ii) **Height / Width ratio constraint, \( C_2 \):**

\[
C_2 = H_f / B \ (\leq 2) \tag{3}
\]

This constraint may be less than 1 for shallow foundations or the limit may go up to 4 [43].

(iii) **Soil working stress constraint, \( C_3 \):**

\[
C_3 = M_t \gamma / (BL) \ (\leq Q_{all}) \tag{4}
\]

where:
- \( M_t \) = total foundation-machine mass = \( M_f + M_m \)
- \( M_m \) = machine mass.
- \( Q_{all} \) = allowable bearing capacity of the soil = \( Q_u / FS \)
- \( Q_u \) = ultimate bearing capacity of the soil.
- \( FS \) = design factor of safety

The soil ultimate bearing capacity \( Q_u \) is related to the soil properties parameters through [44]:

\[
Q_u = \left[ (c/\tan\phi) + 0.5\gamma B\sqrt{K_p} \right] [K_p e^{\pi \tan\phi} - 1] \tag{5}
\]

Where:
- \( c \) = soil cohesion
- \( \phi \) = soil internal friction angle
- \( K_p \) = Rankin's coefficient = \((1 + \sin\phi)/(1-\sin\phi)\)
(iv) **Soil elastic settlement constraint, \( C_4 \):**

The soil settlement is the resultant of 3 types of settlements: elastic, primary consolidation and secondary consolidation. Here, we are going to consider only the elastic type with setting the upper bound the lowest range of the allowable settlement which is 25 mm [45].

The elastic settlement as given by Meyerhof depends on the foundation width \( B \) as follows [45]:

\[
C_4 = \frac{2C_3}{N_{60}} \quad \text{for} \quad B \leq 1.22 \text{ m} 
\]

And

\[
C_4 = \left(\frac{2C_3}{N_{60}}\right) \frac{B}{(B+0.3)} \quad \text{for} \quad B > 1.22 \text{ m} 
\]

Where \( N_{60} \) = standard penetration number.

\( N_{60} \) is function of the soil internal friction angle \( \phi \). Carter and Bentley gave this relation in a graphical form [46]. The following polynomial model if fitted by the authors to the graphical data with 0.9998 correlation coefficient:

\[
N_{60} = 42.422122955322 - 4.69926404953\phi + 0.120638884604\phi^2
\]

Where \( \phi \) is in degrees.

(v) **Maximum vibration amplitude in lateral direction, \( C_5 \):**

According to the work of Prakash and Puri, the foundation vibration in the horizontal and vertical directions can be considered as uncoupled and defined by a SDOF dynamic model [10]. The maximum vibration amplitude of a SDOF system excited by a rotating unbalance is function of the specific unbalance \( me/Mt \) and the system damping ratio. That is:

\[
C_5 = \frac{(me/M_t)}{[2\zeta_x \sqrt{(1-\zeta_x^2)}]} 
\]

Where \( me \) is the machine unbalance, \( \zeta_x \) is the damping ratio of the soil in the lateral direction.

According to Gazetas, the damping ratio in the lateral direction \( \zeta_x \) is given by [1]:

\[
\zeta_x = 0.29/\sqrt{B_x} 
\]

where \( B_x \) is the mass ratio in the lateral direction:

\[
B_x = \frac{M(2-\nu)}{(8\rho R^3)} 
\]

\( \nu \) = soil Poisson’s ratio

\( M \) = foundation mass

\( R \) = equivalent radius of the equivalent circular foundation

\( = \sqrt{(BL/\pi)} \)

(vi) **Maximum vibration amplitude in vertical direction, \( C_6 \):**

The maximum vibration amplitude of a SDOF system excited by a rotating unbalance is function of the specific unbalance \( me/M_t \) and the system damping ratio in the z-direction \( \zeta_z \). That is:

\[
C_6 = \frac{(me/M_t)}{[2\zeta_z \sqrt{(1-\zeta_z^2)}]} 
\]
Where according to Gazetas [1]:

\[ \zeta = 0.425/\sqrt{B_z} \]  
(11)

\( B_z \) is the mass ratio in the vertical direction:

\[ B_z = M(1 - \nu)/(4\rho R^3) \]  
(vii)

**Soil isolation efficiency in the lateral direction, \( C_7 \):**

The isolation efficiency in the lateral direction is defined as:

\[ C_7 = 100(1-TR_x) \]  
(12)

where \( TR_x \) is the vibration transmissibility in the lateral direction given by:

\[ TR_x = \left\{ \frac{1 + (2\zeta r_x)^2}{(1 - r_x^2)^2 + (2\zeta r_x)^2} \right\}^{0.5} \]  
(13)

\( r_x \) is the frequency ratio in the lateral direction:

\[ r_x = \omega/\omega_{nx} \]  
(14)

where \( \omega \) is the angular exciting frequency of the forced vibrations of the foundation, and \( \omega_{nx} \) is the lateral natural frequency of the foundation-soil dynamic system given by [10]:

\[ \omega_{nx} = \sqrt{k_x/M_t} \text{ rad/s} \]  
(15)

\( k_x \) is the soil stiffness in the lateral direction given according to Gazetas by [1]:

\[ k_x = 8GRC_x(L/B) / (2-\nu) \]  
(16)

where:

\( G \) = soil modulus of rigidity

\( C_x \) is a correction factor in the lateral direction depending only on the foundation ratio \( L/B \) as indicated by Barken [47]. The authors fitted the following third order polynomial to Barken’s data with an 0.99977 correlation coefficient:

\[ C_x = 1.026129841805 - 0.04249420017(L/B) + 0.011028882116(L/B)^2 - 0.000512405066(L/B)^3 \]  
(17)

**Soil isolation efficiency in the vertical direction, \( C_8 \):**

The isolation efficiency in the vertical direction is defined as:

\[ C_8 = 100(1-TR_z) \]  
(18)

where \( TR_z \) is the vibration transmissibility in the vertical direction given by:
\[
TR_x = \left\{ \frac{1 + (2\zeta_x r_x)^2}{0.5} \right\} \left\{ \frac{1}{\left(1 - r_x^2\right)^2 + (2\zeta_x r_x)^2} \right\}
\]

R_x is the frequency ratio in the lateral direction:
\[
R_x = \omega / \omega_{nz}
\]

\(\omega_{nz}\) is the vertical natural frequency of the foundation-soil dynamic system given by [10]:
\[
\omega_{nz} = \sqrt{\frac{k_z}{M_t}} \text{ rad/s}
\]

k_z is the soil stiffness in the vertical direction given according to Gazetas by [1]:
\[
k_z = 4GRC_z(L/B) / (1-\nu)
\]

C_z is a correction factor in the vertical direction. The authors fitted the following fourth order polynomial to Barken’s data with an 0.9998 correlation coefficient:
\[
C_z = 0.968339145184 - 0.044979844242(L/B) + 0.033221840858(L/B)^2 - 0.004676759709(L/B)^3 + 0.000208628044(L/B)^4
\]

(ix) \textit{Vibration velocity in the lateral direction, }C_9:\

It is recommended by the Building Department of the Government of Hong Kong that the vibration velocity in lateral or vertical directions not to exceed 15 mm/s [48]. Therefore, this and the next constraint is set on vibration velocity of the foundation.

The vibration velocity in the lateral direction for the rotating unbalance excited vibrations is given by:
\[
C_9 = (\omega_m r_x^2 / M_t) / \sqrt{\left(1 - r_x^2\right)^2 + (2\zeta_x r_x)^2}
\]

(x) \textit{Vibration velocity in the vertical direction, }C_{10}:\

The vibration velocity in the vertical direction for the rotating unbalance excited vibrations is given by:
\[
C_{10} = (\omega_m r_z^2 / M_t) / \sqrt{\left(1 - r_z^2\right)^2 + (2\zeta_z r_z)^2}
\]

3. \textbf{CONSTRAINTS LIMITS}

The optimization problem in hand is a constrained one on both foundation dimensions and functional constraints. The upper and lower limits used in this optimal design problem are as follows:

(i) Foundation dimensions limits (depend on machine dimensions for L and B):
\[
\begin{array}{ccc}
2 & \leq & L & \leq & 10 & \text{m} \\
1 & \leq & B & \leq & 5 & \text{m} \\
0.2 & \leq & H_t & \leq & 2 & \text{m} \\
\end{array}
\]

\[\text{(27)}\]
(ii) Functional constraints limits:

\[
\begin{align*}
1 \leq C_1 & \leq 10 \\
0 \leq C_2 & \leq 2 \\
0 \leq C_3 & \leq \text{allowable bearing capacity} \\
0 \leq C_4 & \leq 25 \text{ mm} \\
0 \leq C_5 & \leq \text{Allowable vibration amplitude} \text{ mm} \\
0 \leq C_6 & \leq \text{Allowable vibration amplitude} \text{ mm} \\
\text{Minimum isolation efficiency} & \leq C_7 \leq 100 \% \\
\text{Minimum isolation efficiency} & \leq C_8 \leq 100 \% \\
0 \leq C_9 & \leq 15 \text{ mm/s} \\
0 \leq C_{10} & \leq 15 \text{ mm/s}
\end{align*}
\]

4. ALLOWABLE VIBRATION AMPLITUDES

The limit of the peak vibration amplitude for machinery foundations is function of the vibration frequency [44]. The vibration limit – frequency is presented in a graphical form. To suit computer-aided application of the foundation design, this relation is defined by a power model in the form [36]:

\[
A = 112.429489135742 f^{(-1.969963312149)} \text{ mm}
\]

Where f is the vibration frequency in Hz.

5. MINIMUM ISOLATION EFFICIENCY

The isolation efficiency is a key factor in controlling the vibrations transmitted to surrounding during machine operation. This is a famous known problem facing contracting companies. To help in solving this problem, the lower limit of the isolation efficiency constraint is left adjustable to examine the effect of its level of the foundation design. The following limits are used for the minimum isolation efficiency: 80, 82, 84, 86, 88, 90, 92, 94 and 96 %.

6. OPTIMAL FOUNDATION DESIGN

The objective function given by Eq.1 has to be minimized subject to 10 functional constraints given by Eqs.2, 3, 4, (6 and 7), 8, 10, 12, 18, 25 and 26, and the design variables constraints in Eq.27. The MATLAB optimization toolbox is used to perform this task [49, 50].

7. CASE STUDY

A 150 kW motor-centrifugal pump unit has the parameters:
- Speed: 1800 rev/min.
- Total mass: 500 kg
- Rotor mass: 300 kg
- Overall length: 2 m
- Overall width: 0.60 m
- Residual unbalance: 12 kgmm
The clay soil has the properties:

- Plasticity index: 30%
- Poisson’s ratio: 0.2
- Water content: 20%
- Cohesion: 29.03 kN/m²
- Unit weight: 18 kN/m²
- Friction angle: 29 degrees
- Shear modulus: 0.517 MN/m²
- Shear strength: 23.3 kN/m²
- Foundation density: 2400 kg/m³
- Design factor of safety: 3

Requirements: The foundation dimensions supporting the motor-pump unit for the stated clay soil properties.

Minimum foundation dimensions: The machine dimensions set the minimum length and width of the foundation at 2 and 1 m respectively.

Optimization results: The optimization results is presented graphically against the minimum isolation efficiency:

- Foundation dimensions and mass: Figs.1 and 2.

![Fig.1: Optimal foundation dimensions](image1)

![Fig.2: Optimal foundation mass](image2)

- Soil stress at foundation interface and elastic settlement: Figs.3 and 4.

![Fig.3: Optimal soil stress at foundation Interface](image3)

![Fig.4: Optimal soil elastic settlement](image4)
- Vibration maximum peak amplitudes and isolation efficiencies: Figs. 5 and 6.

![Fig. 5: Optimal vibration peak amplitude of the foundation](image1)

![Fig. 6: Optimal vibration isolation Efficiencies](image2)

- Vibration peak velocities: Fig. 7.

![Fig. 7: Optimal vibration peak velocities](image3)

8. DISCUSSIONS

- Optimization is a powerful technique which leads to successful machinery foundation design fulfilling all the required objectives in case of machinery foundations subjected dynamic loads.
- All the design constraints are simultaneously considered without any trial work.
- Recommendations on soil bearing capacity, foundation relative dimensions, foundation maximum vibrations and isolation efficiency are all considered.
- Isolation efficiency is a very important parameter in foundation design since it controls the vibration and noise induced vibrations of surrounding structures.
- 10 functional constraints are considered increasing the level of the foundation performance and effectiveness associated with a specific dynamic machine.
- The minimum isolation efficiency is used to direct the design giving the structural engineer a chance to compromise between cost and performance.
- The isolation efficiency range considered is from 80% to 96% (lower limit).
- The foundation mass ranges is from 3.12 to 20.96 ton.
- The foundation height/width ratio ranges from 0.65 to 2.
- The vertical soil stress at the foundation interface ranges from 17.76 to 71.21 kN/m².
- The elastic settlement ranges from 4.67 to 19.42 mm.
- The maximum peak vibration amplitude in the lateral direction ranges from 1.46 to 4.99 µm.
- The maximum peak vibration amplitude in the vertical direction ranges from 0.96 to 3.60 µm.
- The isolation efficiency in the lateral direction ranges from 87.56 to 97.52 %.
- The isolation efficiency in the vertical direction ranges from 80 to 96 %.
- The peak vibration velocity in the lateral direction ranges from 0.106 to 0.637 mm/s.
- The peak vibration velocity in the vertical direction ranges from 0.105 to 0.632 mm/s.
- All the functional constraints were within the pre-assigned limits.
- When trying to increase the minimum limit of the isolation efficiency than 0.96, non-consistent values for the foundation dimensions were obtained.

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