HYDROMAGNETIC MIXED CONVECTION MICRO POLAR FLOW
DRIVEN BY A POROUS STRETCHING SHEET – A FINITE ELEMENT STUDY

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ABSTRACT

We analyze a finite element solution for the magneto hydrodynamics mixed convection micro polar flow through a porous medium driven by a porous stretching sheet with uniform suction. The governing partial differential equations are solved numerically by using finite element technique. The effect of Hartmann number, Darcy parameter and surface condition on velocity, micro rotation and temperature functions has been study.

KEYWORDS: Porous medium, Micropolar Fluids, Convection, Stretching Sheet.

1. INTRODUCTION

The theory of micro fluids, as developed by Eringen [6] has been a field of active research for the last few decades as this class of fluids represents mathematically many industrially important fluids like paints, blood, body fluids, polymers, colloidal fluids and suspension fluids. In this material points in a volume element can undergo motions about centers of mass along with deformation. The problem of simple micro fluid contains a system of 19 equations with 19 unknowns so that it becomes difficult to find solution. A subclass of these fluids introduced by Eringen [7], is the micropolar fluids, which exhibit the micro rotational effects and micro rotational inertia under these assumptions deformation of the fluid microelements is ignored: nevertheless micro rotational effects are still present and surface and body couples are permitted. Here in the skew symmetric property of the gyration tensor is imposed mathematically in addition to a condition of micro isotropy, so that the system of 19 equations reduces to seven equations in seven unknowns.

This theory has been extended by Eringen [8] incorporating the thermal effects, i.e., heat dissipation, heat conduction to the so called as thermo micropolar fluids. Among the interesting
results Eringen cited were the occurrence of a thermodynamic pressure tensor. The coupling of the temperature gradient with the constitutive equations and the occurrence of the microrotation vector in the heat conduction equation. None of these effects were present in the classical field theories of fluids.

The free convective flow of the fluids with microstructure is an interesting area of research including liquid crystals, dilute solutions of polymers fluids and many types of suspensions, since in many configurations in technology and nature, one continually encounters masses of fluid rising freely in extensive effects. Willaim et. al., [11] and Bhargava et. al.,[4] investigated natural convection case the natural convection effects are also present because of the presence of gravitational body forces. A situation where both the natural and forced convection effects are of comparable order is called mixed or combined convection. Mixed convection flows in channels and ducts and applications in nuclear reactors, heat exchangers etc., and have been studied by various authors namely Yucel [18], Gorla et. al.,[10]. Perhaps the most important question is the effect of buoyancy on forced convection transport rates. The buoyancy forces may aid or oppose forced flow causing an increase or decrease in the heat transfer rate. The problem of the stretching sheet has been of great use in engineering studies. Agarwal et. al.,[3] studied the flow and heat transfer over a stretching sheet while Danborg and Fansler [9] have investigated a problem in which the free stream velocity is constant and the wall is being stretched with a variable velocity.

Recently Kelsan et. al.,[14] studied the effect of surface conditions on the micropolar flow driven by a porous stretching sheet. The purpose of the present paper is to analyze the effect of surface conditions on mixed convection flow of a micropolar fluid driven by a porous stretching sheet, by assuming the most general type of boundary condition on the wall. Such a type of study may be applicable to polymer technology involving the stretching of the plastic sheet. In many metallurgical processes the cooling of continuous strips or filaments is involved by drawing them through a quiescent fluid. During the process of drawing, the strips are some times stretched. In such situations the rate of cooling has a great effect on the properties of the final product. By drawing them n a micropolar fluid the rate of cooling may be controlled, thereby giving the desired characteristics to the final product.

Many stages in nuclear reactors and MHD generators working under the influence of external magnetic fields could be examined and controlled using the present model. Na and Pop [17] investigated the boundary layer flow of a micropolar fluid past a stretching wall. Desscaun and Kelson [5] studied the flow of a micropolar fluid bounded by a stretching sheet. Hady [12] studied the solution of heat transfer to micropolar fluid from a non-isothermal stretching sheet with injection. In all the above studies, the authors took the stretching sheet to be oriented in horizontal direction. Abo-Eldahad and Ghnaim [2] investigated convective heat transfer in an electrically conducting micropolar fluid at a stretching surface with uniform free stream.

Mohammadein and Gorla [15] studied the heat transfer characteristics of a laminar boundary layer of a micropolar fluid over a linearly stretching sheet with prescribed uniform surface temperature or prescribed wall heat & flux and viscous dissipation and, internal heat generation. However, of late, the effects of a magnetic field on the micropolar fluid problem are very important. Mohammadein and Gorla [16] presented a numerical study for the boundary layer of a horizontal plate placed in a micropolar fluid. They analyzed the effects of a magnetic field with vectored surface mass transfer and induced buoyancy stream wise pressure gradients on heat transfer. They investigated the impact of the magnetic field, mass transfer, buoyancy, and material parameters on the surface friction and heat transfer rates. Siddheswar and Pranesh [15] investigated magneto-convection in a micropolar fluid.

In this paper a finite element method solution for the fixed convection micropolar fluid flow through a porous medium driven by a porous stretching sheet with uniform suction under the influence of a uniform transverse magnetic filed. The governing partial differential equations are
solved numerically by using Galarekin finite element method analysis with quadratic approximation functions. The effect of porous medium, magnetic field and surface conduction on the velocity microrotation functions has been studied. It is found that the micropolar fluids help in the reduction of stress also acts as a cooling agent.

2. FORMULATION OF THE PROBLEM

Let us consider an isothermal, steady, Laminar, incompressible micropolar fluid through a porous medium flowing past a porous surface coinciding with the plane $y = 0$, the flow being confined in the region $y > 0$. Two equal and opposite forces are introduced along the $x$-axis so that the surface is stretched keeping the origin fixed. A uniform magnetic field of strength $H_0$ is applied normal to the walls. The component of velocity varies linearly along the $x$-axis, i.e., $u(x,0) = Dx$ where $D(>0)$ is an arbitrary constant. A uniform velocity $V_0$ through, and normal, to the stretching surface is also introduced. Let the wall temperature remains steady at $T_w$ while the free stream fluid temperature remains steady at $T_\infty$. Assuming the viscous dissipation effects to be negligible, the governing equations of the flow in two dimensions are as follows:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left(v + \frac{k}{\rho}\right) \frac{\partial^2 u}{\partial y^2} + \frac{k}{\rho_0} \frac{\partial N}{\partial y} + g_n \beta (T - T_\infty) - \left(\sigma \mu e^2 H_0^2 \rho_0\right) u - \left(\frac{v}{k}\right) u
\]

\[
\rho C_p \left[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right] = k \gamma \frac{\partial^2 T}{\partial y^2} + \alpha \left(\frac{\partial T}{\partial x} \frac{\partial N}{\partial y} - \frac{\partial T}{\partial y} \frac{\partial N}{\partial x}\right) + Q (T_\infty - T) - \frac{\partial (u_x)}{\partial y}
\]

where $u, v$ are the velocity components along $x$ and $y$ directions, respectively, $\rho$ is the fluid density, $v$ is the kinematic viscosity, $v_r$ is the kinematic rotational velocity, $g$ is the acceleration of gravity, $\beta_f$ is the coefficient of volumetric thermal expansion of the fluid. $C_p$ is the specific heat at constant pressure, $\sigma$ is fluid electrical conductivity, $B$ is the magnetic induction, $J$ is the micro inertia density, $w$ is the component of the angular velocity vector normal to the $x, y$ – plane, $\gamma$ is the spin gradient viscosity, $T$ is the temperature, $\alpha$ is the effective thermal diffusivity of the fluid and $k$ is the effective thermal conductivity, $q_r$ the radioactive heat flux. The second term on the right hand side of the momentum equation denotes buoyancy effects, and the third is the MHD term. By using the Roseland approximation, the radioactive heat flux in the $y$ direction is given by

\[
q_r = \frac{-4\sigma^*}{3 \beta_x} \frac{\partial T^4}{\partial y}
\]

Where $\sigma^*$ Stefan-Boltzman constant and $\beta$ the mean absorption coefficient. It should be noted that by Roseland approximation we limit our analysis to optical thick fluids. If the temperature differences with in the flow are sufficiently small then equation can be linearized by expanding $T^4$ into the Taylor series about $T_\infty$ and neglecting higher terms takes the form
The heat due to viscous dissipation is neglected for small velocities in energy equation. It is assumed that the porous plate moves with constant velocity in the longitudinal direction, and the plate temperature $T$ varies exponentially with time. Under these assumptions, the appropriate boundary conditions velocity, microrotation, and temperature fields are

$$u = u_p, \quad T = T_w \quad \text{and} \quad w = -\eta \frac{\partial u}{\partial y} \quad \text{at} \quad y = 0, \quad u \to 0, \quad T - T_\infty, \quad w \to 0 \quad \text{as} \quad y \to \infty.$$

A linear relationship between the micro-rotation function $N$ and the surface shear stress $\left( \frac{\partial u}{\partial y} \right)$ is chosen for investigating the effect to different surface conditions for the microrotation. Here $s$ is the boundary parameter and varies from 0 to 1. The first boundary condition ($s = 0$) is a generalization of the no slip condition, which requires that the fluid particles closest to a solid boundary stick to it—neither translating nor rotating. The second boundary condition i.e., micro rotation is equal to the fluid vortices at the boundary ($s \neq 0$) means that in the neighborhood of the boundary. The only rotation is due to fluid shear and therefore, the gyration vector must be equal to fluid vortices.

If $\psi(x, y)$ represents the stream function,

Then

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad 2.5$$

Introducing the dimensionless functions $f(\eta)$ and $g(\eta)$ such that the continuity equation is automatically satisfied and by assuming

$$\eta = \left( \frac{D}{v (1 + p_1)} \right)^{\frac{1}{2}} y, \quad \phi = (D v (1 + p_1))^{1/2} x \ f(\eta) \quad 2.6$$

$$N = \left( \frac{D^3}{v (1 + p_1)} \right)^{\frac{1}{2}} x \ g(\eta), \quad \theta(\eta) = \left( \frac{T - T_\infty}{T_w - T_\infty} \right) \quad 2.7$$

The governing equations (2.2) – (2.4) reduce to the following set of ordinary differential equations.

$$f'^* - f \ f'' = f^* + \frac{p_1}{(1 + p_1)} \ g' + G \theta (M^2 + D^{-1}) \ f' \quad 2.8$$

$$f' g - f' g'' = \frac{-p_1}{p_3} (2g + f') + \frac{p_2}{p_3 (1 + p_1)} \ g'' \quad 2.9$$

$$\theta' + p_3 (f \theta' - \alpha g \theta') - \alpha \theta + \frac{4}{3N_1} \ \theta'' \quad 2.10$$
Where \( p_1 = \frac{k}{\rho v} \), \( p_2 = \frac{D v}{\rho v^2} \) and \( p_3 = \frac{D}{v} \) are the physical micro polar parameters.

\[
G = \left( \frac{g \beta (T_w - T_0)}{D^2 x} \right) \quad \text{(Grashof Number)}, \quad M^2 = \frac{\sigma \mu e^2 H_0^2 L^2}{v^2} \quad \text{(Hartmann Number)}
\]

\[
D^{-1} = \frac{L^2}{k} \quad \text{(Darcy Parameter)}, \quad \alpha_t = \frac{\theta L^2}{k_f} \quad \text{(Heat source parameter)}
\]

\[
N_1 = \frac{3 \beta \cdot k_f}{4 \sigma^* T e^3} \quad \text{(Radiation parameter)}, \quad P_r = \left( \frac{\nu \rho C_p (1 + p_1)}{k_f} \right) \quad \text{(Prandtl number)}
\]

\[
\alpha = \left( \frac{\alpha' D}{\gamma \rho C_p (1 + p_1)} \right) \quad \text{(Material parameter)}
\]

Using a rescaling of parameters as follows:

\[
C_1 = \frac{p_1}{1 + p_1}, \quad C_2 = \frac{p_2}{p_1 (1 + p_1)} \quad \text{and} \quad C_3 = \frac{p_3}{(1 + p_1)}
\]

And introducing these parameter into equations (2.8) – (3.10) we get

\[
f'' - ff'' = f''' + C_1 g' + G \theta + M_1^2 f'
\]

\[
C_2 g'' = 2 \theta + f'' - C_1 \left( f g' - f' g \right)
\]

\[
\theta' + P_r (f \theta' - \alpha g \theta) - \alpha_2 \theta = 0
\]

\[
M_1^2 = M^2 + D^{-1}
\]

The corresponding boundary conditions then reduce to

\[
f(0) = -\left( \frac{1 - C D}{D v} \right), \quad v_0 = -\lambda, \quad f'(0) = 1,
\]

\[
g(0) = -s f''(0), \quad \theta(0) = 1 \quad \text{and} \quad f' (\infty) = 0,
\]

\[
g(\infty) = 0, \quad \text{and} \quad \theta(\infty) \rightarrow(0) = 0
\]

The local wall shear stress and wall couple stress may be written as

\[
T_w = - \left( \mu + k \right) \frac{\partial u}{\partial y} \bigg|_{y=0} = -x f''(0) \sqrt{(\mu + k) D^3 \rho}
\]

\[
\text{and}
\]

\[
m_w = v \frac{\partial N'}{\partial y} \bigg|_{y=0} = \left( \frac{D^2 v}{v (1 + p_1)} \right) x g'(0)
\]
The rate of heat transfer from the wall is given by

\[ q_w = -k_f \frac{\partial T}{\partial y} \bigg|_{y=0} = -k_f (T_w - T_\infty) \left( \frac{D}{v(1 + p_1)} \right)^{\frac{1}{2}} \theta'(0) \]  

2.18

3. METHOD OF SOLUTION

Finite element method
To solve the differential equations (2.12 – 2.14) with the boundary conditions (2.15), we assume

\[ f' = h \]  

3.1

The system of equation (2.12 – 2.14) then reduces to

\[ h'' + fh' - h^2 + c_1g' + G\theta + (M^2 + D^{-1})h = 0 \]  

3.2

\[ C_2 g h - h' - 2g + \frac{C_3}{C_1} (fg' - gh) = 0 \]  

3.3

\[ \theta'' + \frac{P_f}{C_r} (f \theta' - \alpha g \theta') - \alpha \theta = 0 \]  

3.4

The corresponding boundary conditions now become

\[ f(\theta) = -\lambda, \quad h(0) = 1, \quad g(0) = -s h'(0), \quad \theta(0) = 1, \quad h(\infty) = 0, \quad g(\infty) = 0, \quad \theta(\infty) = 0 \]  

3.5

For computational purposes and without any loss of generality, \( \infty \) has been fixed as 8. The whole domain it divided into a set of 80 line elements of width 0.1. Each element being three nodded.

In order to predict the heat and mass transfer behavior in the porous medium equations (3.1) – (3.4) are solved by using finite element method. A simple 3-noded triangular element is considered. \( \psi, \theta \) and \( C \) vary inside the element and can be expressed as

\[ \psi = N_1 \psi_1 + N_2 \psi_2 + N_3 \psi_3 \]
\[ \theta = N_1 \theta_1 + N_2 \theta_2 + N_3 \theta_3 \]
\[ \phi = N_1 \phi_1 + N_2 \phi_2 + N_3 \phi_3 \]

Gelarkin’s method is used to convert the partial differential equation (2.8) – (2.10) into matrix form of equations. Details of FEM formulations and good understanding of the subject is given in the books [3, 4]. The matrix equations are, assembled to get global matrix equations for he whole domain, which is then solved iteratively, to obtain \( \theta, \psi \) and \( \phi \) in porous medium. In order to get accurate results, tolerance level of solution for \( \theta, \psi \) and \( \phi \) are set at \( 10^{-5} \) and \( 10^{-9} \) respectively. Element size in domain varies. Large number of elements are located near the walls where large variations in \( \theta, \psi \) and \( \phi \) are expected. The mesh is symmetrical about central horizontal and vertical lines of the cavity. Sufficiently dense mesh is chosen to make the solution mesh invariant. The mesh size of 3200 elements has good accuracy in predicting the heat transfer behavior of the porous medium. The computations are carried out on high-end computer.
4. RESULTS AND DISCUSSION

Velocity, Micro rotation and temperature functions as obtained by using the finite element method are shown in figs 1-18. The values of material parameter c1, c2 and c3 are taken to be fixed as 0.5, 2 and 0.05 respectively. The Prandtl number \( \text{Pr} \), and material parameter \( \alpha \), each are kept fixed at 1, where as the effect of other important parameters, namely Grashof number G, Hartman number M, suction parameter \( \lambda \), heat source parameter \( \alpha \), and radiation parameter N over these functions has been studied.

Fig.1 demonstrates the variation of velocity distribution with buoyancy parameter G. It is found that the velocity increase continuously with increasing G. This revolves that the connective parameter G increases the boundary layer thickness. Also near the boundary it is positive while away from the boundary it is negative i.e., it retards the flow field away from the boundary for small G. for small values of G \( \leq \frac{5}{0.05} \) the velocity continuously decreases while for large values of G, it increases till it attains maximum value near the boundary and then decreases. Fig.2 represents the micro rotation distribution with G. that is found that for all values of G the micro rotation positive in the vicinity of the boundary and negative for away from the boundary and finally goes to zero. An increasing Grashof number G reduces the micro rotation in the presence of magnetic field. Thus convection effects produce a reverse rotation away from the boundary. Fig.3 represents the temperature distribution. The temperature continuously decreases. It has been found that the temperature depreciation increases in the Grashof number G. i.e., Temperature can be controlled by the convection parameter G. Thus we conclude that a desired temperature can be generated by controlling the buoyancy parameter G.

The variation of u, N and \( \theta \) with Darcy parameter \( D^{-1} \) is shown in fig. 4 to 6. From fig. 4 we note that the velocity is negative in the vicinity of the boundary and it remains positive in the whole domain lesser the permeability of the porous medium. Larger the velocity in the vicinity of the boundary and smaller the velocity in the whole domain. The micro rotation is represented the fig. 5. It is observed that for any \( D^{-1} \) near the boundary micro rotation positive where as away from the boundary. It becomes negative and finally goes to zero. Lesser the permeable porous medium larger the micro rotation the presence of the porous medium produce a reverse rotation for away from the boundary. Fig. 6 illustrates the temperature distribution on with different \( D^{-1} \). It is observed that lesser the permeable porous medium larger the actual temperature near the boundary and smaller the actual temperature for away from the boundary. The variation of magnetic field on u, N, \( \theta \) is shown in fig. 7, 8, 9. It is found that the velocity is –ve in the region 1 to 4 and positive for away from the boundary for M \( \leq \frac{5}{10} \) and for higher M \( \geq \frac{5}{10} \). It is negative in the whole domain. The reverse of the flow which appears in the region 1 \( \leq \eta \leq \frac{4}{4} \) enlarges in its size also \( \eta \), experiences and enhancement in the region 1 \( \leq \eta \leq \frac{4}{4} \) and depreciates the marginally for away from the boundary.

Fig.8 illustrates the micro rotation with Hartman number M. It is found that the micro rotation is positive in the vicinity of the boundary and negative in the whole domain. Thus the Lorentz force reduces a reverse rotation for away from the boundary. Also higher the Lorentz force larger the micro rotation in the region 1 \( \leq N \leq \frac{4}{4} \) and smaller the magnetic field of micro rotation in the remaining region. Fig. 9 illustrates the temperature distribution for a different M. It is found that the temperature positive in the whole domain. Higher the Lorentz force an increasing the Hartman, Number – M enhance the actual temperature in the vicinity of the boundary and depreciates the boundary in the whole domain. Figs. 10-12 illustrate the variation of velocity micro rotation and temperature functions with the surface temperatures. Fig. 10 shows the variation of velocity distribution with S. It is clear that the velocity decreases with increasing parameters near the boundary it remains positive where as away from it becomes negative and thus retards the flow. The velocity values corresponding to no-spin condition are the maximum.
Fig. 11 represents the micro rotation distribution with S. It is found that the micro rotation is negative for $S \leq 0.5$. In the neighborhood of the boundary and negative in the remaining region and increasing the surface parameters enhances the micro rotation near by the boundary and depreciates in the remaining region. The temperature distribution is shown in fig. 12 for a different. It is found that an increasing the surface parameter $S$ results in a depreciation in the actual temperature. Figs. 13-15 represented the variation $u$, $N$, and $\theta$ for different values of the section parameter $\lambda$. It shows that an injection increases a velocity increases while a reversed effect is observed in the case of suction. Large suction represented a reverse flow in the region $1 \leq \eta \leq 3$. Thus the speed of flow can be controlled by suction, which is important for many engineering applications. Fig. 14 depicts the micro rotation distribution, with different $\lambda$. It is found that a micro rotation is positive near by the boundary and negative in the remaining region. An injection increases the micro rotation experiences and enhances in the vicinity of the boundary and depreciates in the remaining region. An injection increases the micro rotation experiences and enhances in the vicinity of the boundary and depreciates in the remaining region, while the reverse is true for suction. It is observed that the micro rotation effects are more dominant in the vicinity of the boundary. The micro rotation remains unaffected for large injection. Because for large injection micro effects distribution, its values continuously increase with increase in suction while a reverse pattern is noticed for suction. Thus a higher temperature can be obtained by using injection. Therefore we can conclude that suction and injection can be used for controlling/increasing the temperature function, which is required in many engineering applications. Like nuclear reactors, generators etc. The variation of $u$, $N$, and $\theta$ with heat source parameter $\alpha$ is shown in fig 16-18 respectively. Fig. 16 represents the velocity distribution with heat source parameter ($\alpha$). It is found that the velocity exhibits a reversal flow in the vicinity of $\eta = 0$ and the region of reversal flow enlarges with increase in $\alpha$. An increase in the strength of the heat generating source results in an enhancement in the velocity field in the whole domain. Fig. 17 illustrates micro rotation with $\alpha$. It is found that micro rotation positive near by the boundary and negative in the remaining region. An increase in the strength of the heat source produces reverse rotation away from the boundary and this reverse rotation enhance with increase in the strength of heat generating source.

The temperature distribution with $\alpha$ is exhibited in fig. 18. It is found that the actual temperature experiences depreciation with increase in $\alpha$. Thus the presence of the heat source in the fluid region reduces actual temperature.
Fig. 3: Variation of $\theta$ with $G$

- I: $5 \times 10^3$
- II: $10^3$
- III: $3 \times 10^3$
- IV: $5 \times 10^3$

Fig. 4: Variation of $u$ with $D^{-1}$

- I: $10^2$
- II: $2 \times 10^2$
- III: $3 \times 10^2$

Fig. 5: Variation of $N$ with $D^{-1}$

- I: $10^2$
- II: $2 \times 10^2$
- III: $3 \times 10^2$

Fig. 6: Variation of $\theta$ with $D^{-1}$

- I: $10^2$
- II: $2 \times 10^2$
- III: $3 \times 10^2$

Fig. 7: Variation of $u$ with $M$

- I: 2
- II: 5
- III: 10

Fig. 8: Variation of $N$ with $M$

- I: 2
- II: 5
- III: 10
Fig. 9: Variation of $\theta$ with $M$

- $M$: 2, 5, 10

Fig. 10: Variation of $u$ with $S$

- $S$: 0.25, 0.5, 0.75, 1

Fig. 11: Variation of $N$ with $S$

- $S$: 0.25, 0.5, 0.75, 1

Fig. 12: Variation of $\theta$ with $S$

- $S$: 0.25, 0.5, 0.75, 1

Fig. 13: Variation of $u$ with $\lambda$

- $\lambda$: -2, -1, 0, 1, 2

Fig. 14: Variation of $N$ with $\lambda$

- $\lambda$: -2, -1, 0, 1, 2
5. REFERENCES


