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HYPERSONIC SIMILITUDE FOR PLANAR WEDGES

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ABSTRACT

A similitude has been obtained for a planar wedge with attached bow shock at high incidence in hypersonic flow. A strip theory in which flow at a span wise location is two dimensional developed by Ghosh is been used. This combines with the similitude to lead to a piston theory which gives closed form of solutions for unsteady derivatives in pitch. Substantially the same results as the theory of Liu and Hui are obtained with remarkable computational ease for some special cases. Effects of wave reflection and viscosity have not been taken into account.

Keywords: Hypersonic Flow, Planar Wedge, Angle of Incidence, Mach number, Piston Theory.

INTRODUCTION

Sychev's [1] high incidence hypersonic similitude is applicable to a wing provided it has an extremely small span in addition to small thickness. The unsteady infinite span case has been analyzed, but mostly for small flow deflections. The piston theory of Lighthill [2] neglects the effects of secondary wave reflection. Appleton [3] and McInthosh [4] have included these effects. Hui's [5] theory is valid for wedges of arbitrary thickness oscillating with small amplitude provided the bow shock remains attached. Erricsson's [6] theory covers viscous and elastic effects for airfoils with large flow deflection. Orlik-Ruckemann [7] has included viscous effect and Mandl[8] has addressed small surface curvature effect for oscillating thin wedges. Ghosh's [9] similitude and piston theory for the infinite span case with large flow deflection is valid for airfoils with planar surfaces. In the present work results have been obtained for hypersonic flow of perfect gas over a wide range of Mach numbers and angle of incidence

ANALYSIS

Figure A. shows an airfoil with attached bow shock, oscillating with small amplitude and frequency, having its windward nonplanar surface at an arbitrary incidence.

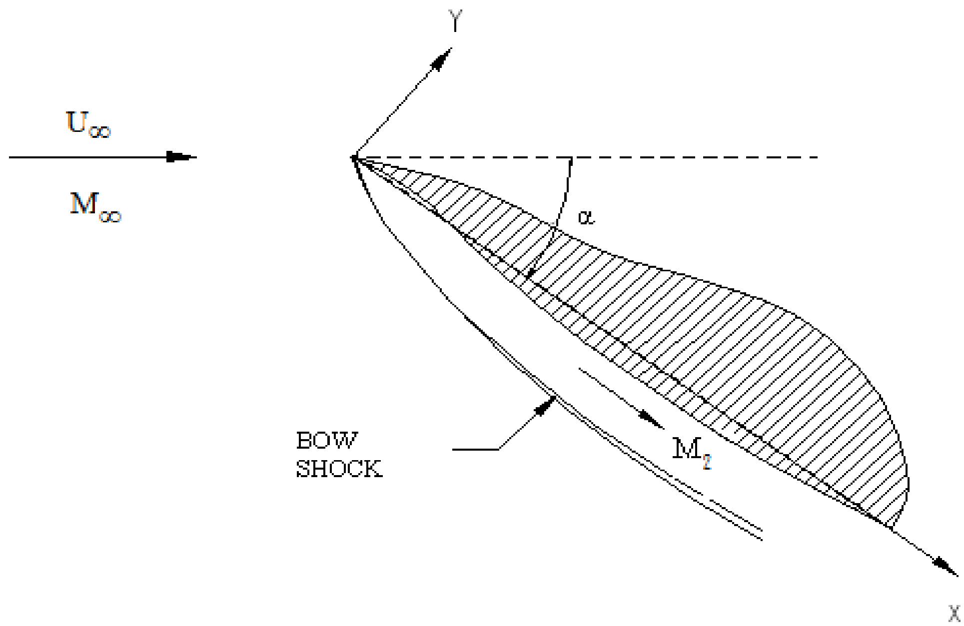

Fig. A Coordinate System

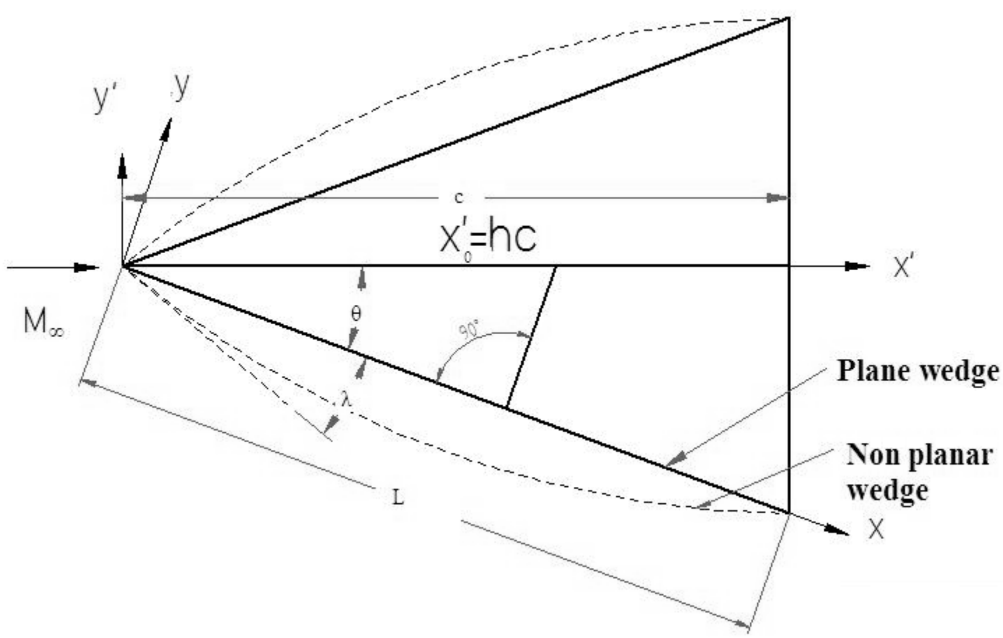

Fig. B. Plane and non-planar wedge; transfer of pivot position from

The surface departs from y = 0 plane by a small amount. The *x*-axis is coincident with the chord of the windward surface in its mean position, and the lee surface pressure is considered negligible. The angle between the *x*-axis and the bow shock at leading edge is ¢, which is a small quantity for hypersonic flow for incidences away from shock detachment. For the purpose of order of magnitude analysis, the windward surface is assumed planar so that
ϕ =β- α, where β is the shock wave angle. From oblique shock relations, for large M_∞ and γ (sp.heat ratio) = 1.4,

$M_2^2 sin^2\phi = 0.143$ or
$sin^{-1}(1/M_2) = sin^{-1}2.64¢ = 2.64¢$ .... (1)

Therefore, δ, inclination of the characteristics behind the shock, is 0(¢). The perturbation velocities in *y* and *x* directions are $\boldsymbol{q_y}$= 0 ($\boldsymbol{U}_\infty$sin α) and $\boldsymbol{q_x}$ = 0 (¢$\boldsymbol{U}_\infty$sin α). This suggests the transformations: $\boldsymbol{q_x}$= ¢ $\boldsymbol{q_x'}$ and *x* = $\boldsymbol{\phi^{-1}x'}$. However, for large ¢, δ=0(¢) is no longer valid, and the second transformation is not appropriate. For the validity of this theory, ¢ ≤ 0.175 radian = 10 deg. This implies, from the Eq. (1) that the lower limit for $\boldsymbol{M_2}$ is around 2.5. An analysis follows resembling the small disturbance theory and, therefore is omitted here. A similitude where flow equations reduce to 1D unsteady form is obtained, and hence, the Piston analogy. The error in this theory is of 0($\boldsymbol{\phi^2}$). The non-planar windward surface may depart from *y* = 0 as much as in case of corresponding compression surface in hypersonic small disturbance theory in which the condition $\boldsymbol{M_2 \geq 2.5}$ is also implicit since the characteristics are required to be at small inclinations. Therefore, the present similitude includes 2 Dimensional small disturbance similitudes for the compression side. The similarity parameters can be shown to be $\boldsymbol{M}_\infty$sinα and $\boldsymbol{\phi M}_\infty$cosα. However, for the flat plate case, the latter is not an independent parameter (since ¢ is wholly determined by $\boldsymbol{M}_\infty$, $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$), but it is automatically satisfied if the former is satisfied, as shown below. From oblique shock relations, for ¢ ≪ 1, α ≫ ¢,

¢/tan α = [($\boldsymbol{\gamma}$ -1) $\boldsymbol{M_\infty^2 sin^2}$α + 2] / [($\boldsymbol{\gamma}$ +1) $\boldsymbol{M_\infty^2 sin^2}$α]

Or

¢$\boldsymbol{M}_\infty$cosα = $\boldsymbol{M}_\infty$sinα [($\boldsymbol{\gamma}$ - 1) $\boldsymbol{M_\infty^2 sin^2}$α + 2] / [($\boldsymbol{\gamma}$ + 1) $\boldsymbol{M_\infty^2 sin^2}$α]

But, for a non-planar surface, for example, the biconvex airfoil ¢ is not determined by$\boldsymbol{M}_\infty$, $\boldsymbol{\gamma}$ and α. Then ¢$\boldsymbol{M}_\infty$cosα is an independent similarity parameter

PISTON THEORY

The Airfoil geometry and motion gives piston velocity u_p, which is related to pressure p. since the piston Mach number M_p= u_p /a_∞ ≫1, instead of the approximate expression of Lighthill, the exact expression is used which van be written in quadratic form in pressure ratio, yielding

$$\frac{p}{p_\infty} = 1 + AM_\infty{}^2 sin^2\alpha + AM_\infty sin\alpha\sqrt{B + M_\infty{}^2 sin^2\alpha}$$

$$A = \frac{\gamma(\gamma+1)}{4}$$

$$\boldsymbol{B} = \left(\frac{4}{\gamma+1}\right)^2 \qquad \ldots\ldots\ldots\ldots (2)$$

Pressures on a steady flat plate and biconvex air foil (semi-nose angle 11.3 deg) have been calculated. This theory has been applied for an oscillation plane wedge. The two surfaces of the wedge can be treated separately as flat plates (Fig. B).

Consider the lower one oscillating about $x = x_0$. The nose down moment

$$-m = \int_0^L (x - x_\circ) p dx \qquad \ldots\ldots\ldots\ldots (3)$$

The stiffness and damping derivatives are, respectively,

$$-C_{m_{\alpha_\circ}} = \frac{1}{\frac{1}{2}\rho_\infty U_\infty{}^2 L^2}\left(-\frac{\partial m}{\partial \alpha_\circ}\right)$$

and

$$-C_{m_q} = \frac{1}{\frac{1}{2}\rho_\infty U_\infty L^3}\left(-\frac{\partial m}{\partial q}\right)$$

Evaluated at $\propto = \theta$ and q = 0, Piston Mach number

$$M_p = \frac{1}{a_\infty}[U_\infty Sin^2\alpha + (x - x_o)\, q] \qquad \ldots\ldots\ldots\ldots (4)$$

By combining equations (2 – 4), differentiation within the integral sign and integration are performed. Then shifting axis of oscillation from $\boldsymbol{x_o}$ to $\boldsymbol{x_o'}$ (Fig B) and defining h as $\boldsymbol{x_o}/\boldsymbol{c}$, $\boldsymbol{x_o}$ = hL$\boldsymbol{cos^2}$θ. Multiplying by 2 for effects of two sides and replacing L by C in non-dimensionalizing, the derivatives of a plane wedge are

$$-\boldsymbol{C_{m_\alpha}} = (\boldsymbol{\gamma} + 1)(\tan\theta)(2 + D + 1/D)\left(\tfrac{1}{2} - h\boldsymbol{cos^2}\theta\right) \qquad \ldots\ldots\ldots\ldots (5)$$

$$-\boldsymbol{C_{m_q}} = (\boldsymbol{\gamma} + 1)(\tan\theta/\boldsymbol{cos^2\theta})(2 + D + 1/D)\left(\tfrac{1}{3} - h\boldsymbol{cos^2}\theta + \boldsymbol{h^2 cos^4\theta}\right) \qquad \ldots\ldots\ldots\ldots (6)$$

Where,

$$\boldsymbol{D} = \frac{\sqrt{\left(\frac{4}{\gamma+1}\right)^2 + M_\infty{}^2 sin^2\alpha_\circ}}{M_\infty sin\alpha_\circ}$$

Results of Stiffness and damping derivatives for various mach Numbers and angle of incidences have been studied.

RESULTS AND DISCUSSIONS

Ghosh's large-deflection hypersonic similitude and consequent plane and conico-annular piston theories have been applied to obtain pressure and the pitching moment derivatives for oscillating non-slender wedges. The plane piston theory for a wedge is extended from quasi-steady analysis, which gives the moment derivatives due to pitch rate C_{mq}, to an unsteady analysis; the two analysis combine to give the moment derivative due to incidence rate, which is shown here to be the same for wedges and quasi-wedge of arbitrary shape;

In the present work an attempt is made estimate the stability derivates for planar wedges for a wide range of Mach number and angle of attack for attached shock case. Results for planar wedges for Mach 5 are shown in Figs. 1 and Fig.2. It is seen that the stiffness derivative in pitch increases linearly with increase in the semi-vertex θ from 5 degrees to 25 degrees (Fig. 1). It is interesting to see that the stiffness derivative increases linearly with increase in the semi vertex angle of the wedge. Further, it is seen that the center of pressure lies at a distance of 50 % to 60 % from the nose. There is linear shift of center of pressure with increase in the semi vertex angle of the wedge. This was expected that with the increase in the semi vertex angle of the wedge . Fig.2 shows the variation of damping derivatives with the pivot position for Mach number 5 for various semi vertex angles. For all the values of semi vertex angles namely from 5 degrees to 10 degrees initially the damping derivative in pitch decreases, then reaches to a minimum value and then increases. Further, it is seen that for semi vertex angles 5 & 10 degrees the curve is more or less flat and the minima takes place at 40 % from the nose. The reasons for this behavior may be that for small semi vertex angles there is no much variation in surface pressure of the wedge. The trend in the damping derivatives for semi vertex angles 15, 20, and 30 degrees is different and is on the expected lines, initially the damping derivative decreases, reaches to a minima and then increases. However, the magnitude of this increase is different for different values of semi vertex angles for the pivot position of h = 0.0, also there is a tendency of the shift of the minima towards the rear of the wedge and for h = 1.0 increase in the value of the damping is pitch is almost uniform.

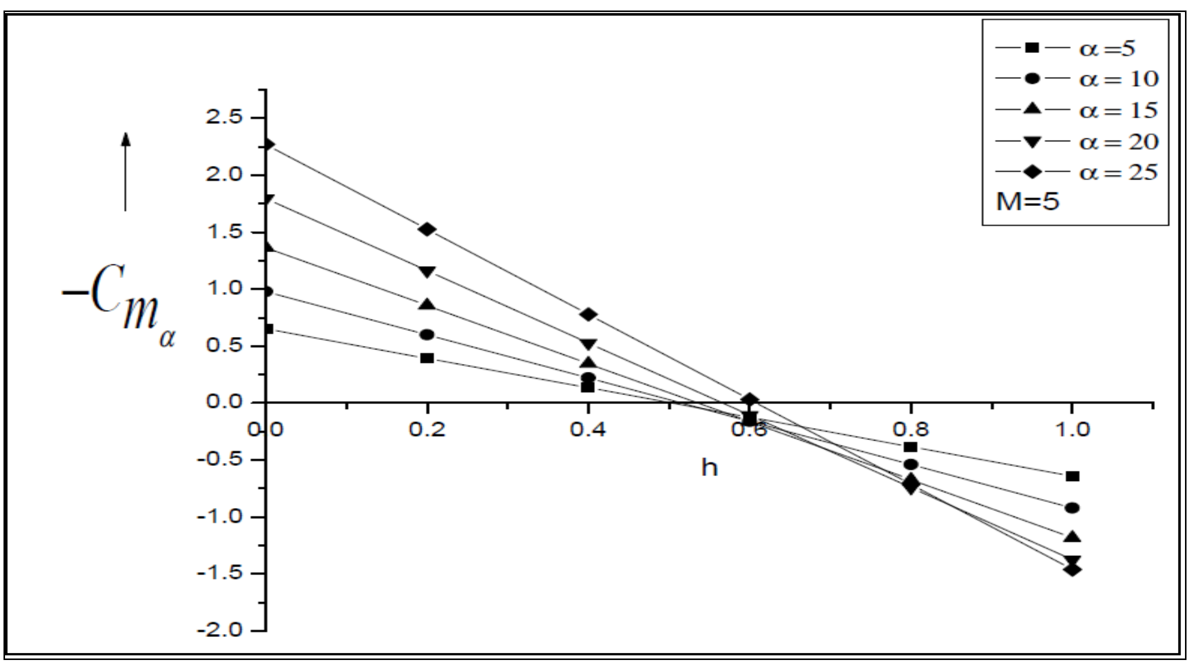

Fig.1: Variation of Stiffness Derivative with Pivot position at Mach Number M = 5

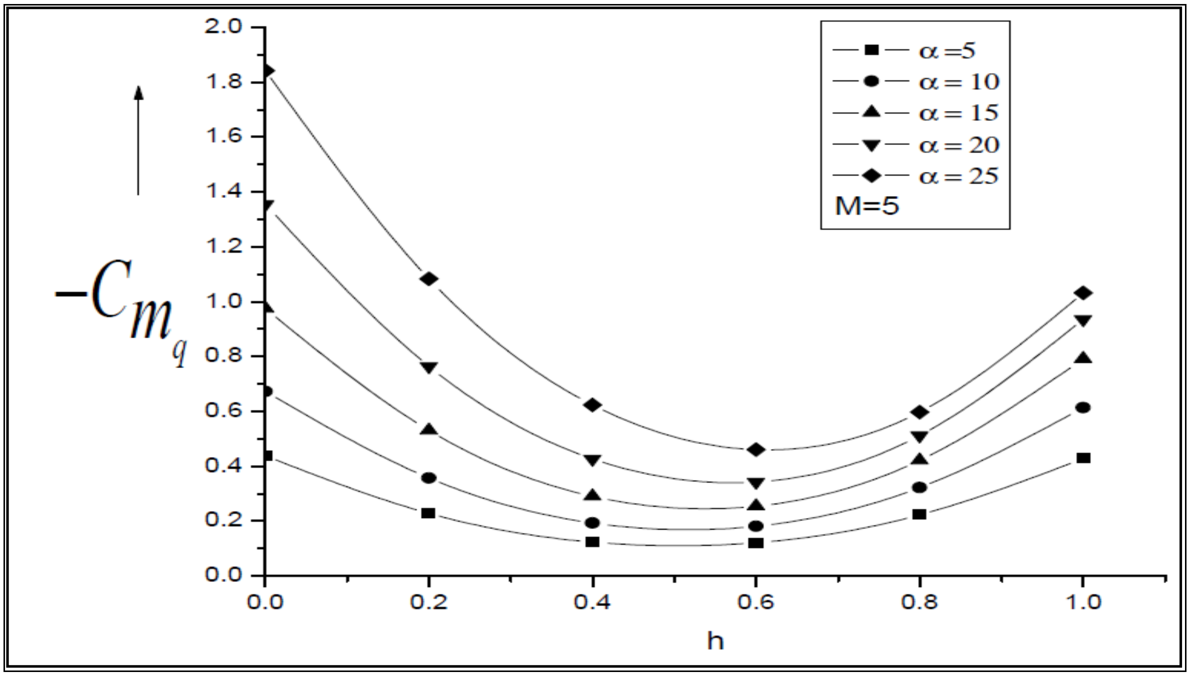

Fig. 2: Variation of Damping Derivative with Pivot position at Mach Number M = 5

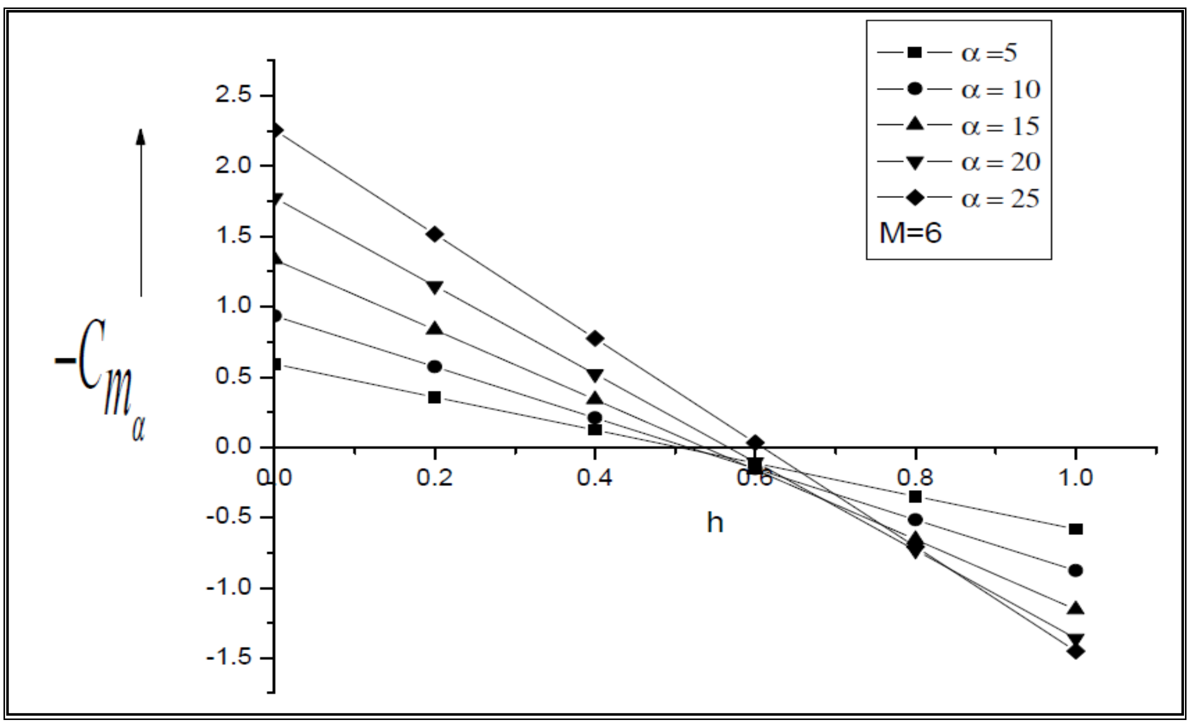

Fig. 3: variation of Stiffness Derivative with pivot position with Mach number M = 6

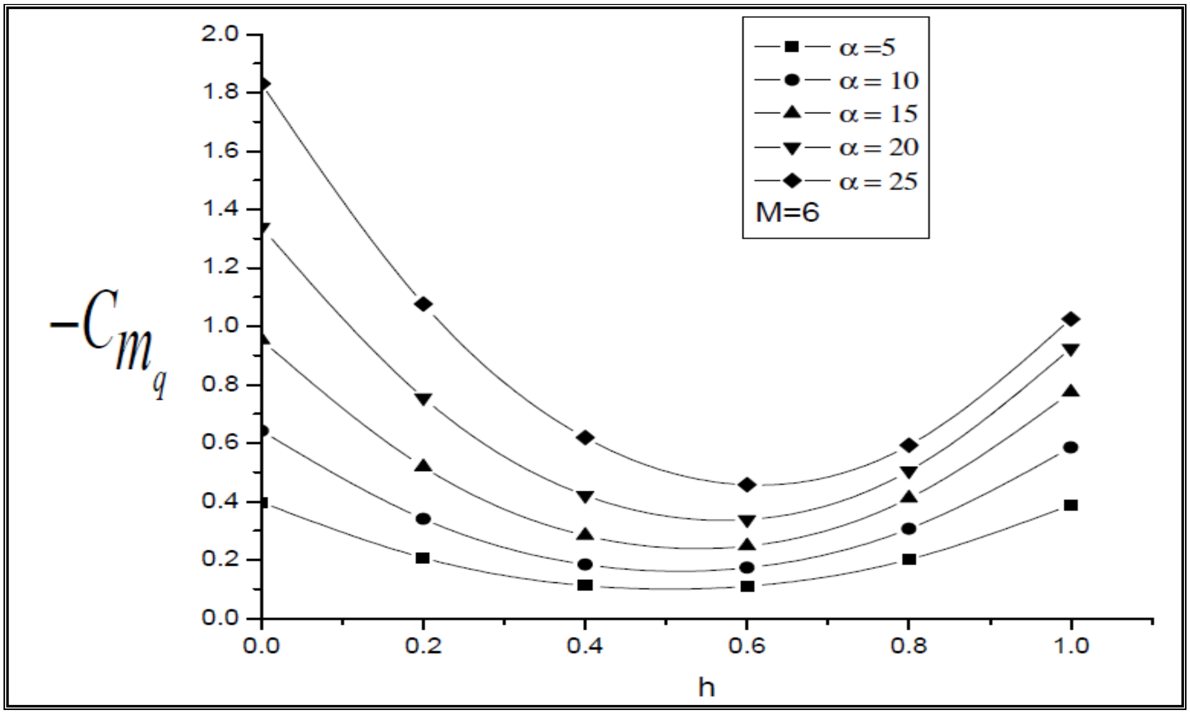

Fig. 4: Variation of Damping derivative with pivot position with Mach number M = 6

Fig. 3 and Fig.4 present the result of stiffness & damping derivative in pitch for Mach 6.0. In this case all the parameters are the same except that the Mach number has become 6. As it evident from the expressions of stiffness & damping derivative that they are directly proportional to the Mach number. The difference in the results for Mach 5 & 6 is that the behavior & trend remains the same but the magnitudes of these derivatives are different.

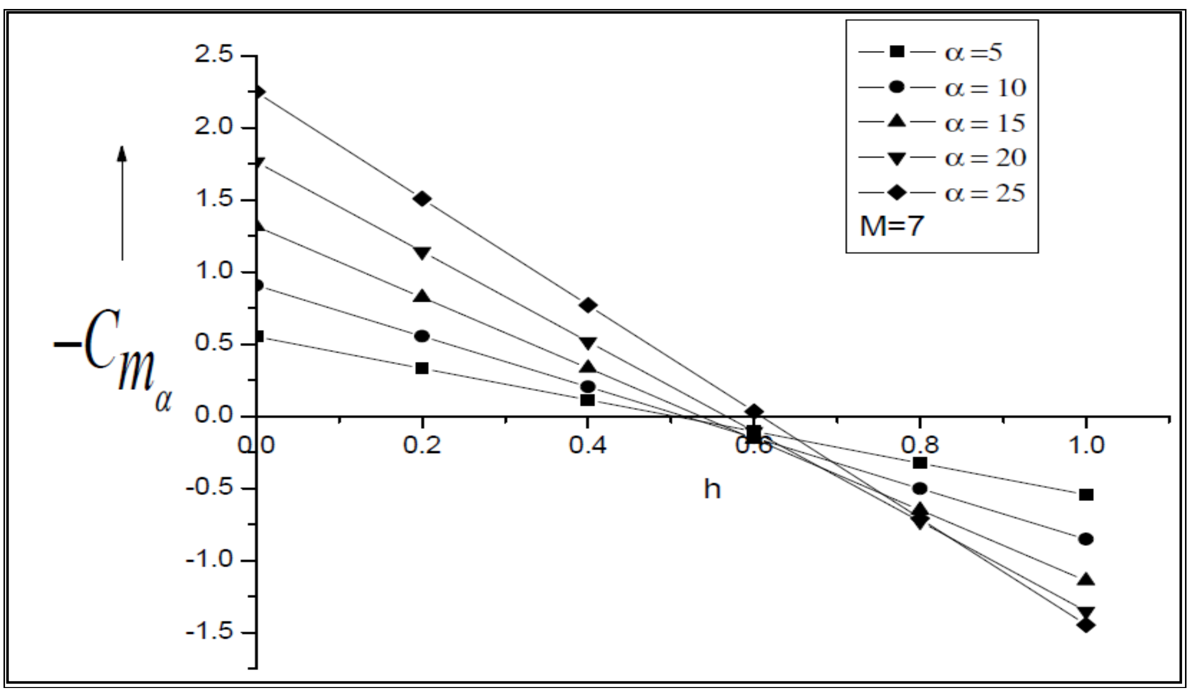

Fig. 5: Variation of stiffness derivative with pivot position at M = 7

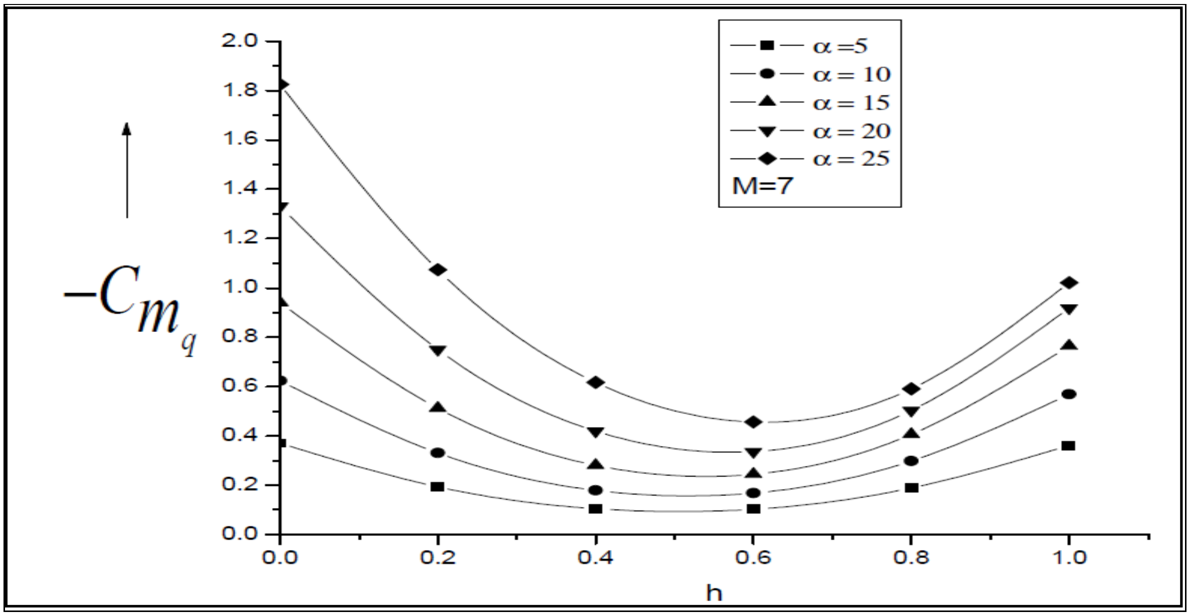

Fig. 6: variation of Damping derivative with pivot position at M = 7

The results for Mach number 7 are presented in Figs. 5 & Fig. 6. Here again due to the increase in the Mach number the magnitude of the stiffness & damping derivative has increased substantially, there is shift of the center of pressure towards the rear and the increase in the values is uniform at h = 1.0. There is peculiar change in the trends of the stiffness & damping derivatives for Mach number 7 that even for semi vertex angle of 15 degrees the variation in the damping derivative is not steep but flat in nature. This trend may be due to the shock wave formation at the nose & its strength leading to the strange behavior in the pressure distribution.

Similar results are seen in Figs. 7 & Fig. 8 for Mach number 8. As explained earlier the trend is the same except the magnitude of the stiffness & damping derivatives.

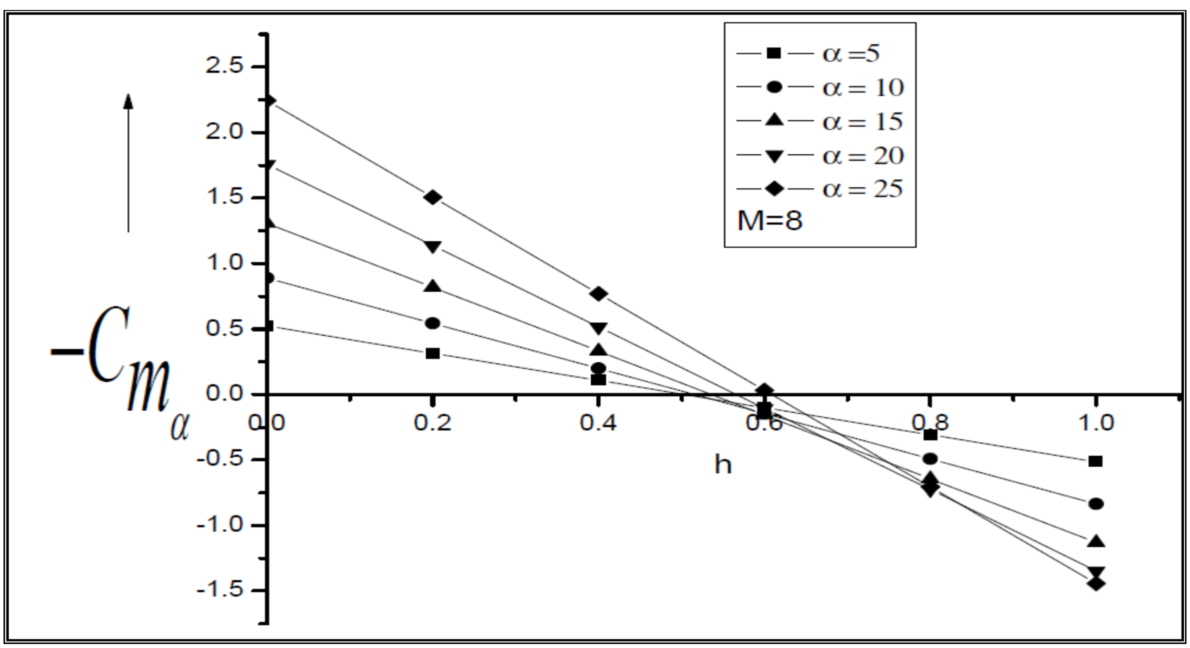

Fig. 7: Variation of Stiffness derivative with pivot position with M = 8

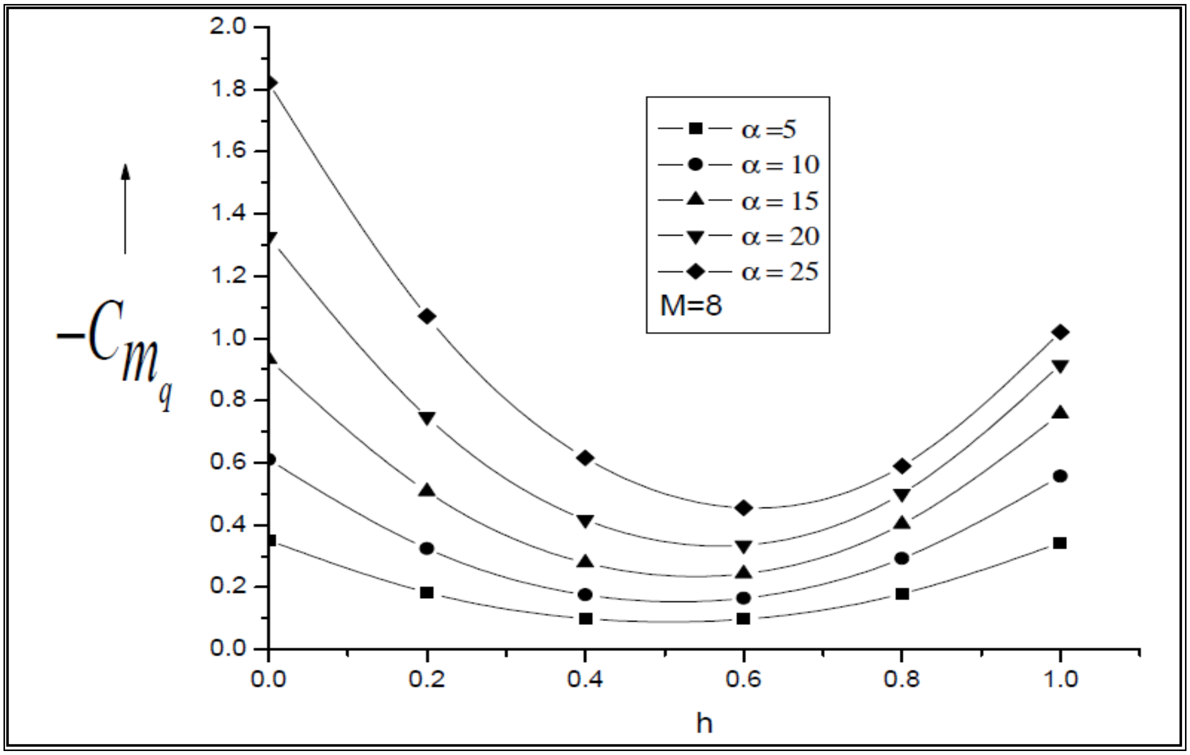

Fig. 8: variation of Damping derivative with pivot position at M = 8

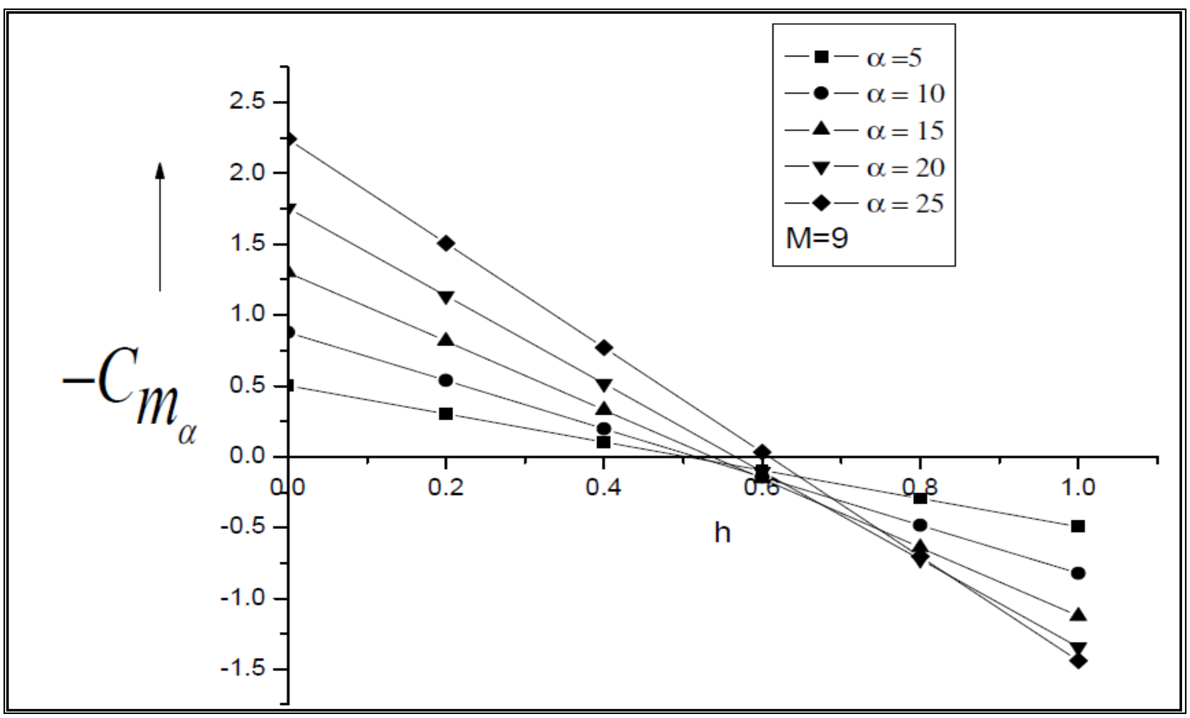

Fig. 9: variation of Stiffness derivative with pivot position at M = 9

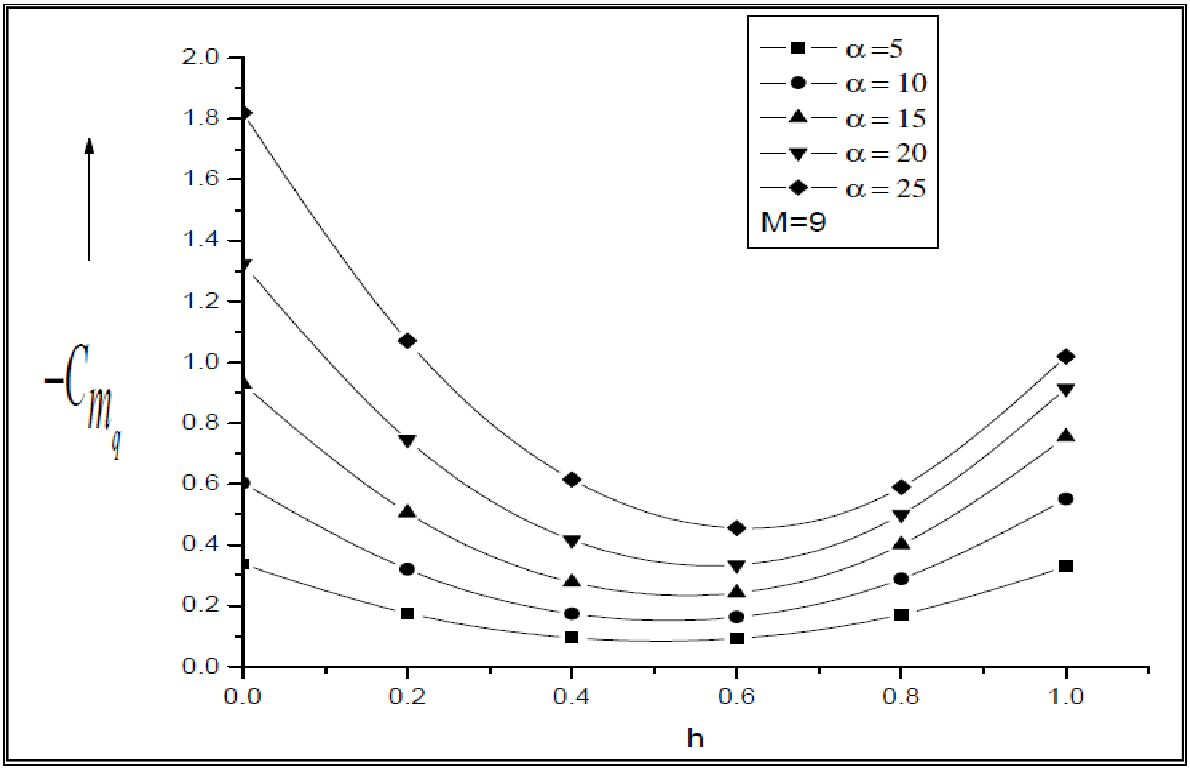

Fig. 10: Variation of damping derivative with pivot position at M = 9

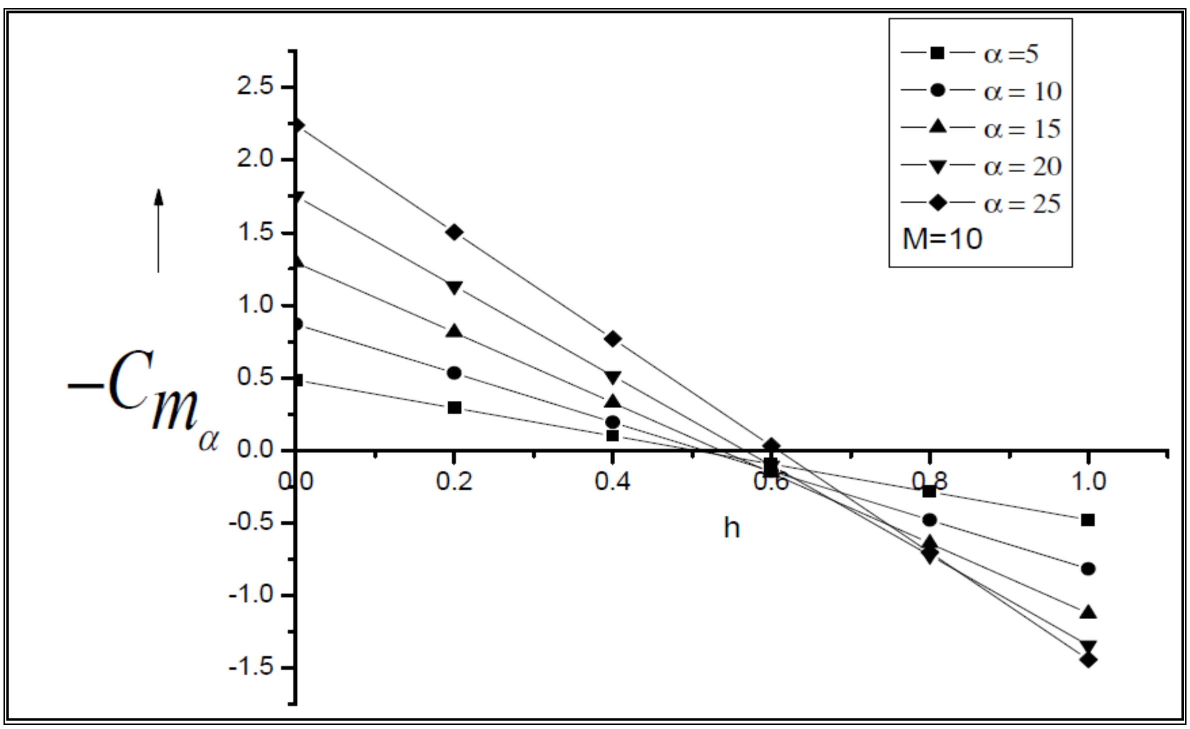

Fig. 11: variation of Stiffness derivative with pivot position with M = 10

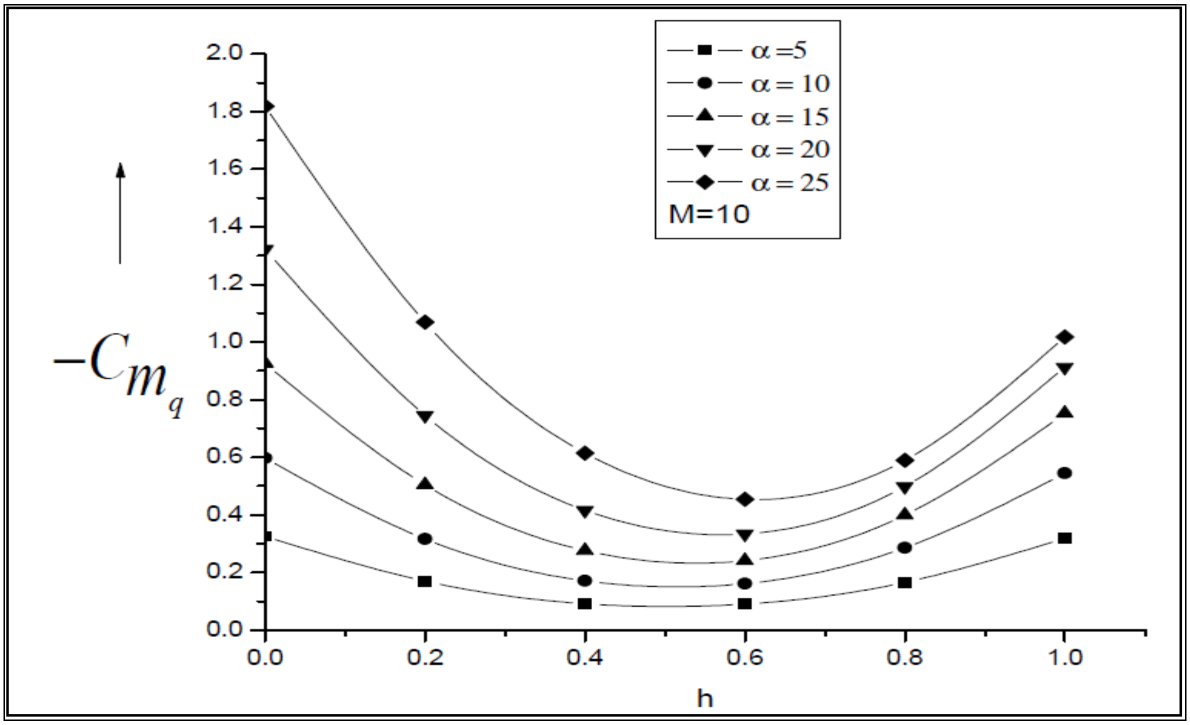

Fig. 12: Variation of damping derivative with pivot position at M = 10

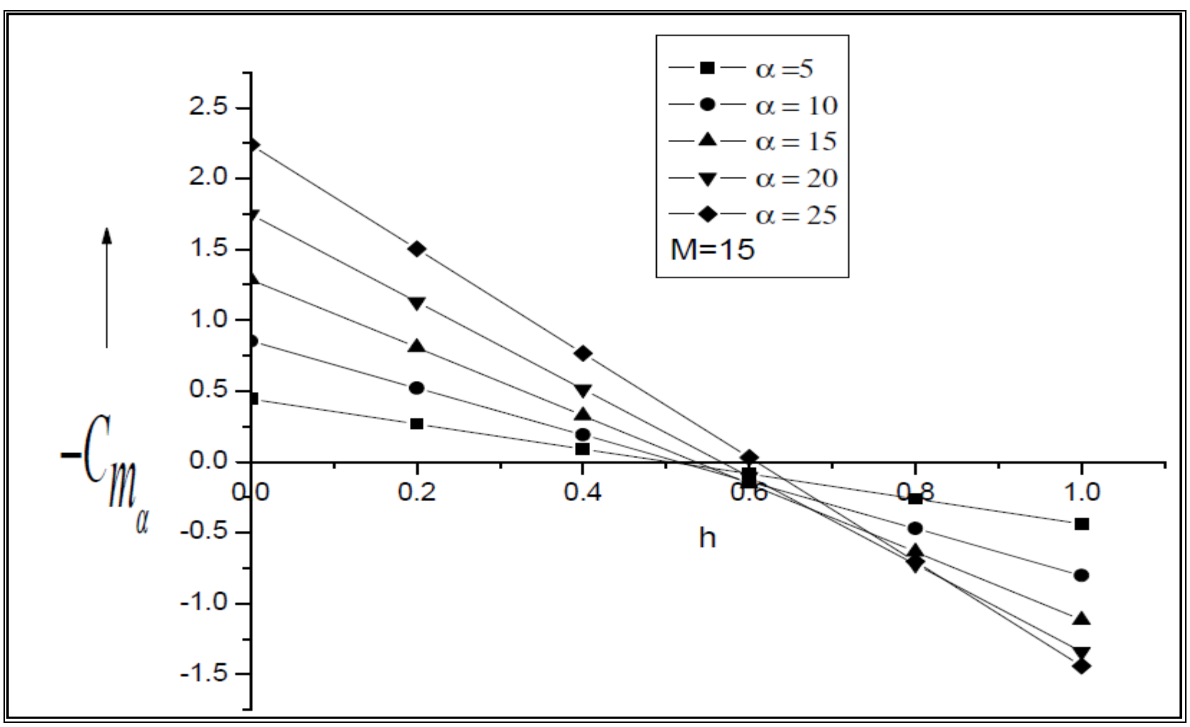

Fig. 13: variation of Stiffness derivative with pivot position at M = 15

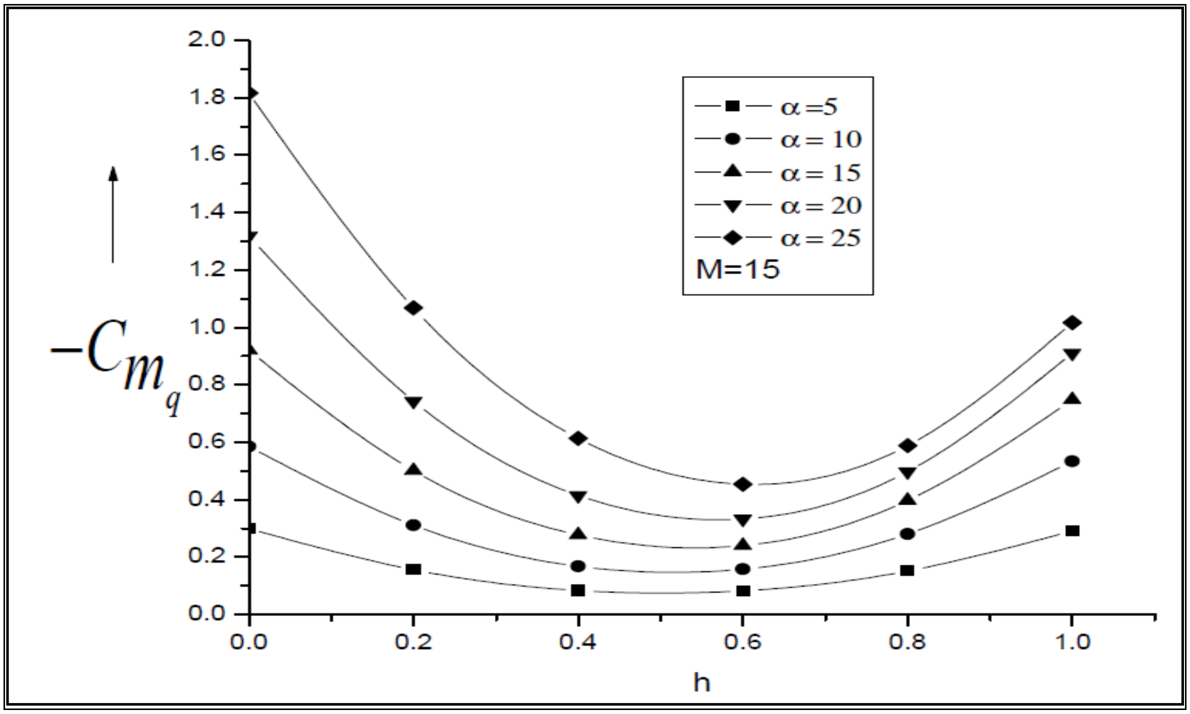

Fig. 14: variation of Damping derivative with pivot position at M = 15

Fig. 9, 10, 11, 12, 13 and Fig .14 presents the results for Mach number 9, and 15. The same trend continues as that of for Mach number 8.

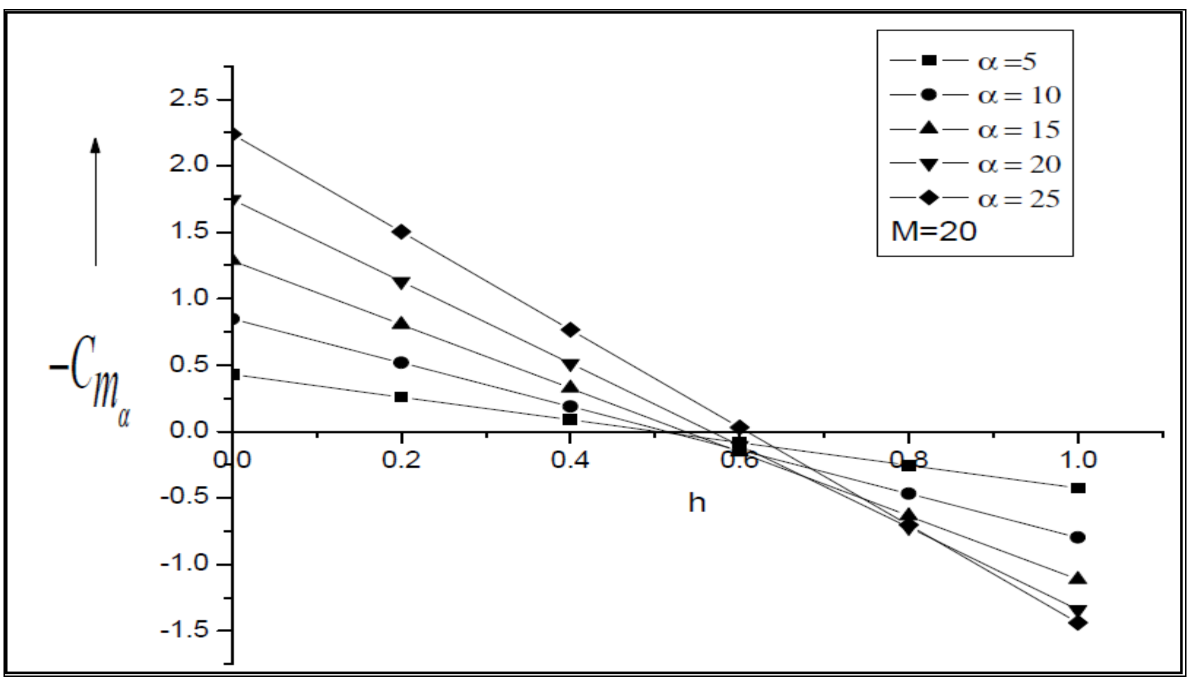

Fig. 15: variation of stiffness derivative with pivot position at M = 20

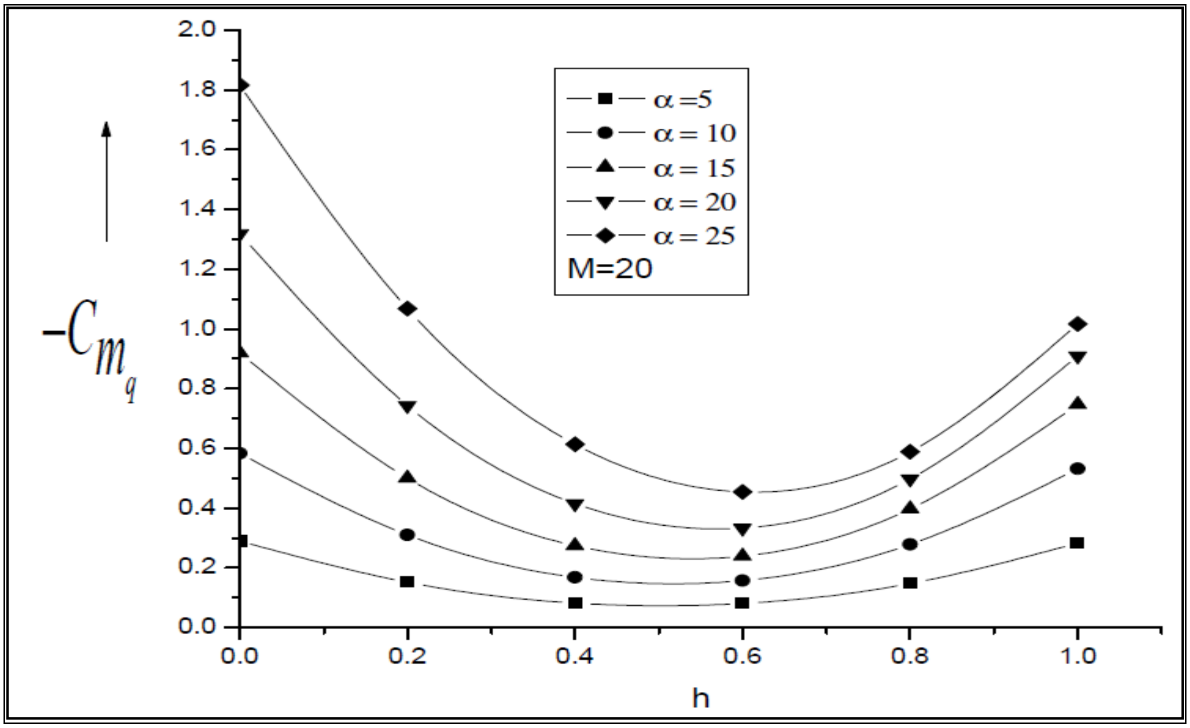

Fig. 16: variation of damping derivative with pivot position M = 20

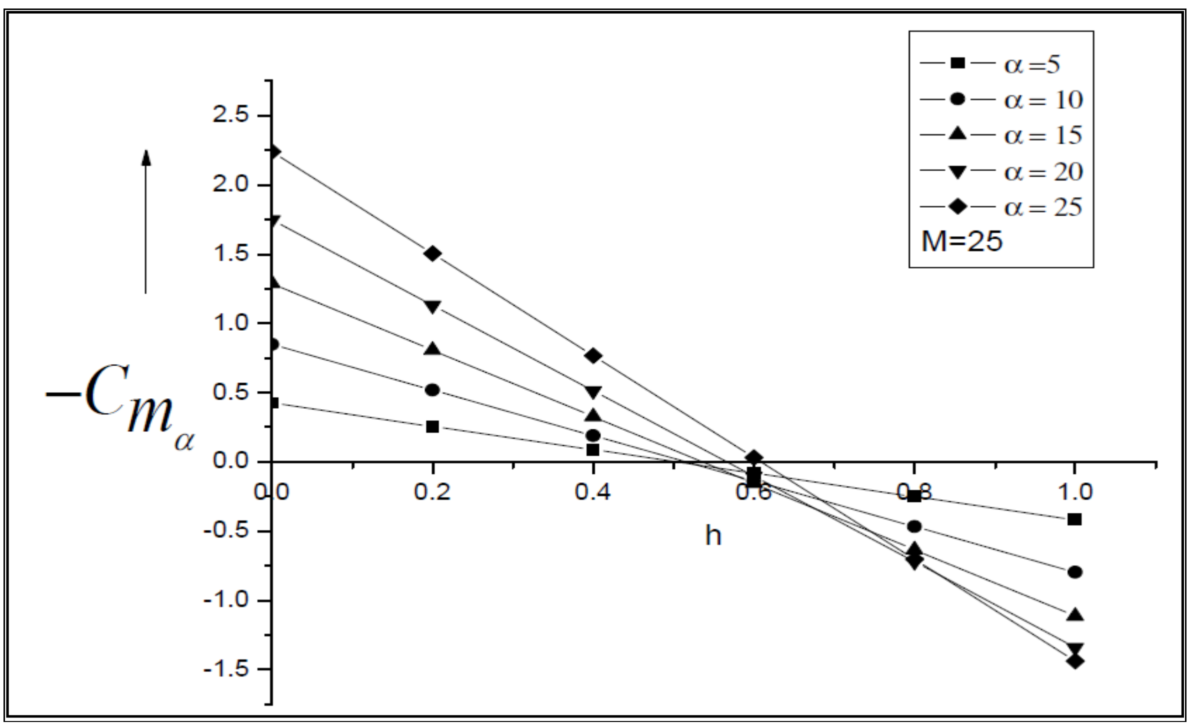

Fig. 17: Variation of Stiffness derivative with pivot position at M = 25

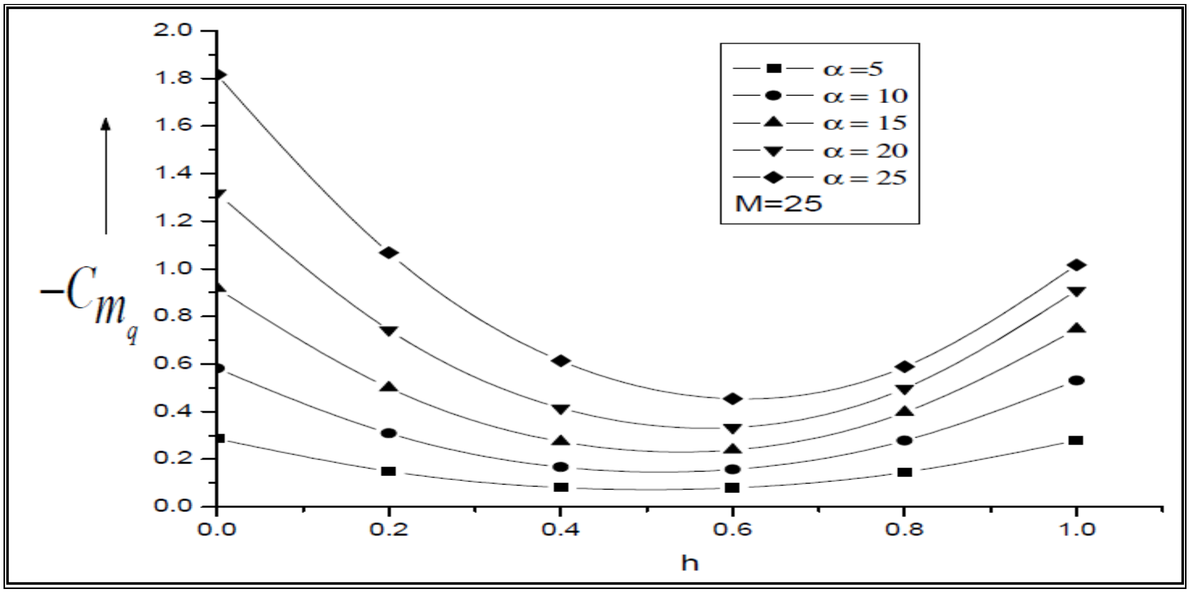

Fig. 18: variation of Damping derivative with pivot position at M = 25

Results for Mach number 20 are presented in Fig .15 and 16. As far as the stiffness derivative is concerned there is no much change since the Mach number is very high and the principle of Mach number independence exists. From Fig. 16 it is seen that the value of the damping derivative is very high for the entire range of the semi vertex angle. As far as the numerical value of the damping derivative is concerned when we compare the value of damping derivative at Mach number 20 with that at Mach number 10, it is found that the values are just doubled at Mach 20 as compared to Mach 10. For the lower values of the semi vertex angles namely 5 & 10 degrees the trend is flat in nature and the minima can be considered any where 40 % to 60 % from the nose due the nature of the curve. However, for higher values of semi vertex angles namely 15, 20, and 25 degrees the minima has shifted towards the right and is around 60 % from the nose of the wedge. The same trend is seen for Mach Number 25 shown in Fig. 17 and Fig.18.

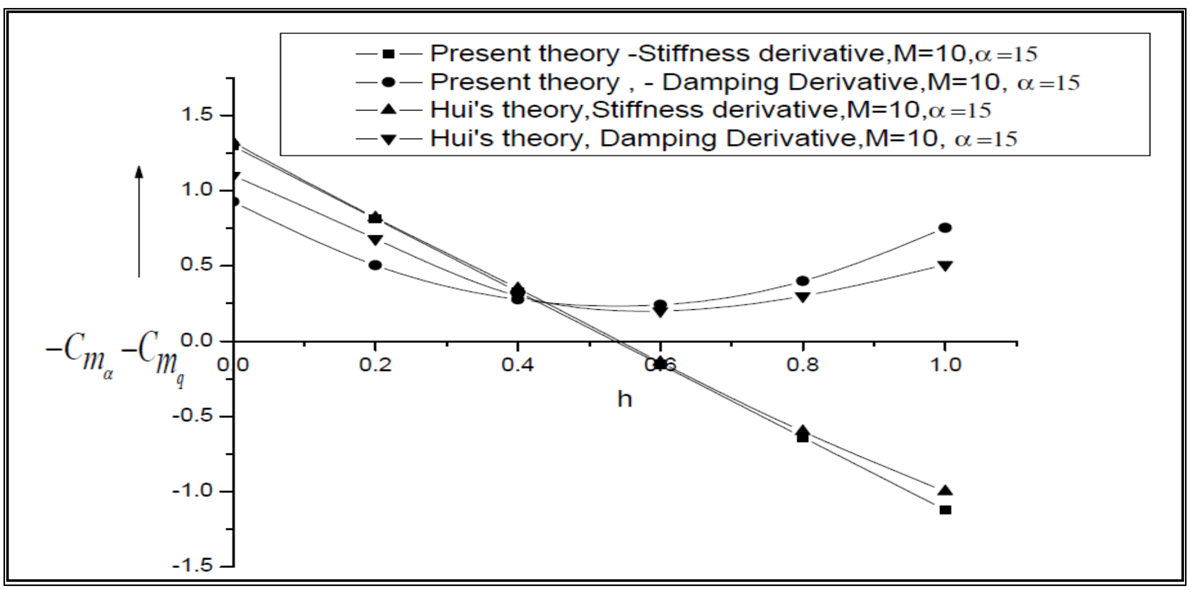

Fig. 19: Variation of Stability derivatives with pivot position at M = 10

Stiffness and damping derivatives in pitch calculated by the present theory have been compared with Liu and Hui [10] in Fig. 19. The stiffness derivative shows good agreement. The difference in the damping derivative is attributed to the present theory being a quasi-steady one whereas Liu et al give an unsteady theory which predicts $C_{m\theta.}$ The present work invokes strip theory arguments. Hui et al. [11] also use strip theory arguments whereby the flow at any span wise station is considered equivalent to an oscillating flat plate flow; this is calculated by perturbing the known steady flat plate flow (oblique shock solution) which serves as the 'basic flow' for the theory. For a pitching wing the mean incidence is the same for all 'strips' (irrespective of span wise location) and hence there is a single 'basic flow' which Hui et al. have utilized to obtain closed form expression for stiffness and damping derivatives. Their theory is valid for supersonic as well as hypersonic flows; whereas the present theory also gives closed form expressions for Stiffness & damping derivatives in pitch. Liu and Hui's [10] theory is more accurate than of Hui et al [11] as far hypersonic flow is concerned.
The present theory is simpler than both Liu and Hui [10]and Hui et al [11] and brings out the explicit dependence of the derivatives on the similarity parameters S_1.

CONCLUSION

Present theory demonstrates its wide application range in angle of incidence and the Mach number. The theory is valid only when the shock wave is attached and the effect of Lee surface has been neglected. The present theory could be handy at the initial design stage of the Aerospace Vehicles. Effect of viscosity and wave reflection is been neglected. The present theory is simple and yet gives good results with remarkable computational ease.

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