INVERSE KINEMATICS ANALYSIS AND SIMULATION OF 5DOF ROBOT MANIPULATOR

AldoomShareef\textsuperscript{1,2}, Ji-Ping Zhou\textsuperscript{2}, Hong Miao\textsuperscript{3}, Hui Shen\textsuperscript{4}

\textsuperscript{1}Mechanical Engineering Department, Faculty of Engineering Science, University of Nyala, Nyala, South Dar Fur, Sudan
\textsuperscript{2,3,4}Mechanical Engineering College, Yangzhou University, Yangzhou, Jiangsu province, P.R. China

ABSTRACT

Where the control of the robot manipulator depends on the inverse kinematics solution, so this paper present solution of the inverse kinematics of 5DOF robot manipulator following the D-H method to setting up, the coordinates frame of the robot’s manipulator, the forward kinematics and schematic diagram of the robot, then the validity of the solution method is confirmed by adopted examples and simulations, matlab is used for the verification, this work is still in primary stages but verification is important for the following stages. These obtained results are not only for this robot, but also can be used in robots have a similar mechanical structure.

Keywords: Inverse Kinematics, Matlab, Robot Manipulator, Simulation.

1. INTRODUCTION

Along history human seeks to use robots in many fields special that is hazardous, hot, heavy and dangerous for the human being; and by advances in technology of control, computer science in last decades, robots become popular for using in more fields than in previous and every day more than previous. [1], [2]

Due to ageing people in some countries like Japan will lead to lack of workers who decreases the economy wheel speed, due to lack of workers in some seasons in which some fruits, vegetables and leaves of some trees ripped and in need for harvesting leads people to think about using robots in this field to solve a like problems.[3]

In China at season of harvesting some kinds of tea appears a lack of workers and the income
for tea harvesters is very low, which pushes workers to depart this part time job. Therefore, the objective of this work is to design and manufacture robot manipulator who can be capable of harvest tea leaves for some kinds of trees.

The problems face robot design is starting from configuring the structure of robot, forward kinematics, inverse kinematics, finding singularities, trajectory path to control.

The forward kinematics problem is when the kinematical data are known for the joint coordinates and are utilized to find the data in the base Cartesian coordinates frame.

The inverse kinematics problem is when the kinematics data are known for the end-effector in Cartesian space and kinematic data are needed in the joint space, and also the forward kinematics is defined as transformation from joint space to Cartesian space where as Inverse Kinematics is defined as transformation from Cartesian space to joint space.[4, 8]

While the forward kinematics equations not only are quietly easy but also leads to unique solution, but the inverse kinematics equations’ solutions are more difficult, complicated and may leads to several solutions.[9]

Solving the inverse kinematics is introduced by many researches in two methods either analytical or in geometrical approach. The analysis of manipulator in primary stages is useful for path tracking of the manipulator with pick-and-place applications.

An iterative algorithm method for solving the inverse kinematics for five degrees of freedom manipulator with offset wrist is introduced where that iterative minimize the iteration numbers.[10, 11, 12]

2. DESCRIPTION OF THE 5DOF SERIAL MANIPULATOR

This work is in primary stage and designed to use for harvesting some kinds of tea leaves

**Manipulator unit design:** The manipulator consists of five servomotors to be capable reaching objects on the three dimensions. Mechanically, the manipulator consists of link1 =20cm, link2 =120cm, link3 =100cm, and the wrist which is link4=32cm, which is holding the end effector. The angle between link three and link four is always 90 degrees, because the end effector assumed to be vertical when performing the action of cutting the target.

First servomotor (base servomotor) rotates from -500 ~ 500; the second servomotor rotates from -300 ~ 1900; the third servomotor rotates from -1300 ~ 1700 and the fourth servomotor rotates from -400 to 900; the fifth servomotor can rotate approximately 3600.

The end effector is in a scissor shape, one pairs of it is actuated by a servomotor rotates a cam attached shaft to enable cutting the tea leaves.

3. FORWARD KINEMATICS AND SCHEMATIC DIAGRAM OF THE MANIPULATOR AND LINKS PARAMETERS

This robot manipulator arm is consisted of five links actuated by five servomotors at five joints as illustrated in figure (1)
Figure (1) shows the proposed manipulator structure and link parameters

The forward kinematics determines the functional relationship between the joints’ variables and the end effector position and orientation.

<table>
<thead>
<tr>
<th>Joint No.</th>
<th>Link length (a_i, \text{mm})</th>
<th>Link twist angle (\alpha_i^\circ)</th>
<th>Link offset (d_i, \text{mm})</th>
<th>Joint rotation angle (\theta_i^\circ)</th>
<th>Range of joint angle rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>90</td>
<td>(d_i)</td>
<td>(\theta_1)</td>
<td>-50 to 50</td>
</tr>
<tr>
<td>2</td>
<td>L2</td>
<td>0</td>
<td>0</td>
<td>(\theta_2)</td>
<td>-30 to 190</td>
</tr>
<tr>
<td>3</td>
<td>L3</td>
<td>0</td>
<td>0</td>
<td>(\theta_3)</td>
<td>-130 to 170</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>90</td>
<td>0</td>
<td>(\theta_4)</td>
<td>-40 to 90</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>(d_5)</td>
<td>(\theta_5)</td>
<td>0 to 360</td>
</tr>
</tbody>
</table>

Where the four parameters associated with the link are:

- \(a_i\) is the link length measured from \(z_i\)to\(z_{i+1}\),
- \(d_i\) is the link offset distance measured from \(x_i\)to\(x_{i+1}\) along \(z_i\),
- \(\alpha_i\) is the link twist angle between \(z_i\)and\(z_{i+1}\) to be measured about \(x_i\),
- \(\theta_i\) is the joint rotation angle between \(x_i\) and \(x_{i+1}\) to be measured about \(z_i\).

According to the D-H method; the homogeneous coordinates’ transformation matrix for the consecutive two robot’s links is written in the following matrix form.[5]:

\[
T_i^{i-1} = \begin{bmatrix}
    c\theta_i & -ca_i\theta_i & sa_i\theta_i & a_i\theta_i \\
    sa_i & ca_i & -s\theta_i & 0 \\
    0 & s\theta_i & c\theta_i & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix}
\]

(1)

Where:

\[
c\theta_i = \cos \theta_i ca_i = \cos \alpha_i s\theta_i = \sin \theta_i sa_i = \sin \alpha_i
\]
By substituting the link’s parameters from the table (1) in equation (1) the links’ transformation matrices can be obtained as follows:

Transformation of the first link

\[
T_1^0 = \begin{bmatrix}
c_1 & 0 & s_1 & 0 \\
s_1 & 0 & -c_1 & 0 \\
0 & 1 & 0 & d_1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Transformation of the second link

\[
T_2^1 = \begin{bmatrix}
c_2 & -s_2 & 0 & L_2 \cdot c_2 \\
s_2 & c_2 & 0 & L_2 \cdot s_2 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Transformation of the third link

\[
T_3^2 = \begin{bmatrix}
c_3 & -s_3 & 0 & L_3 \cdot c_3 \\
s_3 & c_3 & 0 & L_3 \cdot s_3 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Transformation of the fourth link

\[
T_4^3 = \begin{bmatrix}
c_4 & 0 & s_4 & 0 \\
s_4 & 0 & -c_4 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Transformation of the fifth link

\[
T_5^4 = \begin{bmatrix}
c_5 & -s_5 & 0 & 0 \\
s_5 & c_5 & 0 & 0 \\
0 & 0 & 1 & d_5 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Hence, consequently, the forward kinematics of the Robot manipulator arm that determines the position and the orientation of the tool or the end effector according to the joint variables which could be obtained by multiplying transformation matrices of the manipulator in series as follows:

\[
T_5^0 = T_1^0 \cdot T_2^1 \cdot T_3^2 \cdot T_4^3 \cdot T_5^4 \quad (2)
\]

The end effector position is at \( T_5^0 \)

\[
T_5^0 = T_{\text{tool}}^{\text{base}} = \begin{bmatrix}
    n_x & O_x & a_x & p_x \\
    n_y & O_y & a_y & p_y \\
    n_z & O_z & a_z & p_z \\
    0 & 0 & 0 & 1
\end{bmatrix}
\]

Where \( n \) in the first column indicates for the orthogonal vector, \( O \) in the second column indicates for the orientation vector, \( a \) in third column indicates for the approach vector and \( P \) in the fourth column indicates the position vector of the end effector. [6]
\[ n_x = c_1 * c_5 * c_{234} + s_1 * s_5 \]
\[ n_y = s_1 * c_5 * c_{234} - c_1 * s_5 \]
\[ n_z = c_5 * s_{234} \]
\[ O_x = -c_1 * c_{234} * s_5 + s_1 * c_5 \]
\[ O_y = -s_1 * c_5 * c_{234} - c_1 * c_5 \]
\[ O_z = -s_5 * s_{234} \]
\[ a_x = c_1 * s_{234} \]
\[ a_y = s_1 * s_{234} \]
\[ a_z = -c_{234} \]
\[ p_x = c_1 * (L_2 * c_2 + L_3 * c_{23} + d_5 * s_{234}) \]
\[ p_y = s_1 * (L_2 * c_2 + L_3 * c_{23} + d_5 * s_{234}) \]
\[ p_z = d_1 + (L_2 * s_2 + L_3 * s_{23} - d_5 * c_{234}) \]

Where:
\[ s_a = \sin a \quad s_{abc} = \sin (a + b + c) \quad c_a = \cos a \quad c_{abc} = \cos (a + b + c) \]

Through the equations (2) and (3), any position for the end effector frame can be mapped in the base frame. [7]

4. INVERSE KINEMATICS

Inverse kinematics problem is to find the joint variables corresponding to the end effector position and orientation, [6]

For finding the theta one and five by equating the elements (1, 4) and (2, 4) from equations (2) and (3) respectively, \( \theta_1 \) can be obtained.

\[ \theta_1 = \tan^{-1} \left( \frac{p_y}{p_x} \right) \quad (5) \]

And by the same way equating the elements (3, 1) and (3, 2) from equations (2) and (3) respectively, \( \theta_5 \) can be obtained.

\[ \theta_5 = \tan^{-1} \left( -\frac{a_z}{n_z} \right) \quad (6) \]

For finding theta three multiply both sides of equations (2) and (3) by \( [T_1^0]^{-1} \)

\[ [T_1^0]^{-1} * T_5^0 = T_1^1 * T_3^2 * T_4^3 * T_5^4 = [T_1^0]^{-1} * T_{tool}^{base} \quad (7) \]

And by the same way equating elements (1, 4) and (2, 4) of equation (7) respectively and arrange the equations, \( \theta_3 \) can be obtained.

\[ (L_2 * c_2 + L_3 * c_{23} + d_5 * s_{234}) = p_x * c_1 + p_y * s_1 \quad (8) \]

\[ (L_2 * s_2 + L_3 * s_{23} - d_5 * c_{234}) = p_z - d_1 \quad (9) \]

\[ c_3 = \frac{(p_x * c_1 + p_y * s_1 - d_5 * s_{234})^2 + (p_z - d_1 + d_5 * c_{234})^2 - L_2^2 - L_3^2}{2 * L_2 * L_3} \]
\[ s_3 = \pm \sqrt{1 - c_3^2} \]

\[ \theta_3 = \tan^{-1}(s_3/c_3) \quad (10) \]

Also from elements (1, 1) and (2, 1) in equation (7) \( \theta(two+three+four) \) can be obtained.

\[ \theta_{234} = \tan^{-1}\left(\frac{n_x}{n_x*c_1+n_y*s_1}\right) \quad (11) \]

For \( \theta_4 \) by another arrangement to equations (8) and (9), \( \theta_4 \) can be obtained as follows

\[ (L_3 * c_23 + d_5 * s_{234}) = p_x * c_1 + p_y * s_1 - L_2 * c_2 \quad (8) \]

\[ (L_3 * s_{23} - d_5 * c_{234}) = p_z - d_1 - L_2 * s_2 \quad (9) \]

Square both sides and add them

\[ s_4 = \left(\frac{[p_x * c_1 + p_y * s_1 - L_2 * c_2]^2 + [p_z - d_1 - L_2 * s_2]^2 - d_5^2 - L_3^2}{2 * L_3 * d_5}\right) \]

\[ c_4 = \pm \sqrt{1 - s_4^2} \]

\[ \theta_4 = \tan^{-1}(s_4/c_4) \quad (12) \]

Theta two

By another arrangement to (8) and (9)

\[ (L_2 * c_2 + L_3 * c_{23}) = p_x * c_1 + p_y * s_1 - d_5 * s_{234} \quad (13) \]

\[ (L_2 * s_2 + L_3 * s_{23}) = p_z - d_1 + d_5 * c_{234} \quad (14) \]

Multiply both sides of equation (13) by \( L_3 * s_3 \) and both sides of (14) by \( (L_2 + L_3 * c_3) \) and arrange them to obtain

\[ s_2 = \frac{(L_2 + L_3 * c_3) * (p_z - d_1 + d_5 * c_{234}) - L_3 * s_3 * (p_x * c_1 + p_y * s_1 - d_5 * s_{234})}{(L_2^2 + L_3^2 + 2 * L_2 * L_3 * c_3)} \]

Do similarly to obtain

\[ c_2 = \frac{L_3 * s_3 * (p_z - d_1 + d_5 * c_{234}) + (L_2 + L_3 * c_3) * (p_x * c_1 + p_y * s_1 - d_5 * s_{234})}{(L_2^2 + L_3^2 + 2 * L_2 * L_3 * c_3)} \]

Then \( \theta_2 \)

\[ \theta_2 = \tan^{-1}\left(\frac{(L_2 + L_3 * c_3) * (p_z - d_1 + d_5 * c_{234}) - L_3 * s_3 * (p_x * c_1 + p_y * s_1 - d_5 * s_{234})}{L_3 * s_3 * (p_z - d_1 + d_5 * c_{234}) + (L_2 + L_3 * c_3) * (p_x * c_1 + p_y * s_1 - d_5 * s_{234})}\right) \quad (15) \]
5. DISCUSSION AND SIMULATION

After finding the inverse kinematics equations from the forward kinematics equation, we adopted assumption for some angles for the angles of the robot manipulator to verify the solution of the inverse kinematics equations; table (2) shows the values. Then we substituted these values at transformation matrices of the robot arm, which was mentioned above; the obtained transformations multiplied in series according to equation (2) and after that we used the end effector position transformation matrix for each of the angles’ group as given end effector position transformation matrix and substitute the values in equations (5), (6), (10), (11), (12) and (15) to get the values of table (3) to confirm the validity of our solution.

To confirm the validity of this solution, we used matlab Robotics toolbox for calculating the transformation matrices and the inverse kinematics angles, and next we chose some groups for simulation as 1- 4 - 7 - 11 - 15. In addition, these adopted angles prove the same end effector position as in calculated ones.

<table>
<thead>
<tr>
<th>No.</th>
<th>( \theta_1 )</th>
<th>( \theta_2 )</th>
<th>( \theta_3 )</th>
<th>( \theta_4 )</th>
<th>( \theta_5 )</th>
<th>( \theta_{234} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>45</td>
<td>30</td>
<td>0</td>
<td>60</td>
<td>30</td>
<td>90</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>15</td>
<td>60</td>
<td>90</td>
<td>60</td>
<td>165</td>
</tr>
<tr>
<td>3</td>
<td>70</td>
<td>50</td>
<td>20</td>
<td>12</td>
<td>17</td>
<td>82</td>
</tr>
<tr>
<td>4</td>
<td>35</td>
<td>20</td>
<td>-113</td>
<td>-23</td>
<td>142</td>
<td>-116</td>
</tr>
<tr>
<td>5</td>
<td>-50</td>
<td>90</td>
<td>-130</td>
<td>-40</td>
<td>0</td>
<td>-80</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>43</td>
<td>-100</td>
<td>10</td>
<td>120</td>
<td>-47</td>
</tr>
<tr>
<td>7</td>
<td>-10</td>
<td>8</td>
<td>-95</td>
<td>30</td>
<td>45</td>
<td>-57</td>
</tr>
<tr>
<td>8</td>
<td>-20</td>
<td>65</td>
<td>110</td>
<td>-20</td>
<td>-33</td>
<td>155</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
<td>-17</td>
<td>37</td>
<td>54</td>
<td>210</td>
<td>74</td>
</tr>
<tr>
<td>10</td>
<td>24</td>
<td>-28</td>
<td>50</td>
<td>48</td>
<td>310</td>
<td>70</td>
</tr>
<tr>
<td>11</td>
<td>50</td>
<td>142</td>
<td>-11</td>
<td>-40</td>
<td>89</td>
<td>91</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>184</td>
<td>-1</td>
<td>-4</td>
<td>91</td>
<td>179</td>
</tr>
<tr>
<td>13</td>
<td>-3</td>
<td>11</td>
<td>90</td>
<td>89</td>
<td>180</td>
<td>190</td>
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<td>14</td>
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<td>12</td>
<td>179</td>
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<td>37</td>
<td>-90</td>
<td>0</td>
<td>271</td>
<td>-53</td>
</tr>
</tbody>
</table>
Table (3): shows the angles obtained from the Inverse Kinematics Equations that validating the inverse kinematics solution

<table>
<thead>
<tr>
<th>No.</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\theta_4$</th>
<th>$\theta_5$</th>
<th>$\theta_{234}$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>29.999</td>
<td>0.0000</td>
<td>60.0001</td>
<td>30.0007</td>
<td>90</td>
</tr>
<tr>
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<td>30.0006</td>
<td>14.9947</td>
<td>60.0061</td>
<td>89.9982</td>
<td>59.9970</td>
<td>164.9989</td>
</tr>
<tr>
<td>3</td>
<td>69.9991</td>
<td>50.0014</td>
<td>19.9962</td>
<td>11.9994</td>
<td>16.9965</td>
<td>81.9972</td>
</tr>
<tr>
<td>4</td>
<td>35.0016</td>
<td>20.0021</td>
<td>-113.0021</td>
<td>-22.9971</td>
<td>141.9992</td>
<td>-115.9971</td>
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<tr>
<td>5</td>
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<td>89.9994</td>
<td>-130.0034</td>
<td>-39.9976</td>
<td>0</td>
<td>-80.0016</td>
</tr>
<tr>
<td>6</td>
<td>19.9996</td>
<td>42.9982</td>
<td>-99.9965</td>
<td>9.9924</td>
<td>120.0004</td>
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</tr>
<tr>
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<td>37.0008</td>
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<td>209.9977</td>
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<td>49.9945</td>
<td>47.9998</td>
<td>310.0008</td>
<td>69.9974</td>
</tr>
<tr>
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<td>50.0004</td>
<td>142.0278</td>
<td>-11.0513</td>
<td>-39.9405</td>
<td>89.0029</td>
<td>91.0359</td>
</tr>
<tr>
<td>12</td>
<td>0.9994</td>
<td>184.1750</td>
<td>-1.3842</td>
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<td>91.0027</td>
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<td>37.0090</td>
<td>-89.9553</td>
<td>0.0267</td>
<td>270.9973</td>
<td>-52.8350</td>
</tr>
</tbody>
</table>

The end-effector position transformation matrix is explained in term of when one angle is varying while other angles are fixed to zero in figure (3).

**Figure (2-a):** Simulation result for group (1) using matlab

**Figure (2-b):** Simulation result for group (4) using matlab
6. CONCLUSION

In this paper the five degree of freedom robot manipulator diagram is shown and D-H homogeneous transformation matrices and forward kinematics are represented, in addition to the inverse kinematics equations are analyzed and solved by a simple method to make control equation simpler and easy, then the inverse kinematics solution is verified by matlab simulation giving the accurate values for end effector position as in forward equation. This analytical method can be used to drive other robot inverse kinematics solution. This gained result can set the basement for next work for this robot and others similar to.
REFERENCES


