MHD FLOW AND HEAT TRANSFER FOR THE UPPER CONVECTED MAXWELL FLUID OVER A STRETCHING SHEET WITH VISCOUS DISSIPATION

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ABSTRACT

In the present work, the effect of MHD flow and heat transfer within a boundary layer flow on an upper-convected Maxwell (UCM) fluid over a stretching sheet is examined. The governing boundary layer equations of motion and heat transfer are non-dimensionalized using suitable similarity variables and the resulting transformed ordinary differential equations are then solved numerically by shooting technique. For a UCM fluid, a thinning of the boundary layer and a drop in wall skin friction coefficient is predicted to occur for higher the elastic number which agrees with the results of Hayat et al. The objective of the present work is to investigate the effect of various parameters like elastic parameter $\beta$, magnetic parameter $Mn$, Prandtl number $Pr$ and Eckert number $Ec$ on the temperature field above the sheet.

Key words: Eckert number, Elastic Parameter, Magnetic parameter, UCM fluid.

1. INTRODUCTION

The studies of boundary layer flows of Newtonian and non-Newtonian fluids over a stretching surface have received much attention because of their extensive applications in the field of metallurgy and chemical engineering particularly in the extrusion of polymer sheet from a die or in the drawing of plastic films. During the manufacture of these sheets, the melt issues from a slit and is subsequently stretched to achieve the desired thickness. Such investigations of magnetohydrodynamics (MHD) flow are very important industrially and have applications in different areas of research such as petroleum production and metallurgical processes. The magnetic field has been used in the process of purification of molten metals from non-metallic inclusions. The study of flow and heat transfer caused by a stretching surface is of great importance in many
manufacturing processes such as in extrusion process, glass blowing, hot rolling, and manufacturing of plastic and rubber sheets, crystal growing, continuous cooling and fibers spinning. In all these cases, a study of flow field and heat transfer can be of significant importance because the quality of the final product depends to a large extent on the skin friction coefficient and the surface heat transfer rate (Fox, V.G., Ericksen [11], Siddappa and Abel [12]). Sarpakaya [1] was the first researcher to study the MHD flow of a non-Newtonian fluid. Prandtl’s boundary layer theory proved to be of great use in Newtonian fluids as Navier-Stokes equations can be converted into much simplified boundary layer equation which is easier to handle. Crane [2] was the first among others to consider the steady two-dimensional flow of a Newtonian fluid driven by a stretching elastic flat sheet. Further many authors (Refs. Grubka et.al [3], Dutta et.al [4], Jeng et.al [5], Chakrabarti et.al [6] and Abel et.al [7]) have extended the Cranes work.

![Schematic showing flow above a stretching sheet](image-url)

**Fig.1. Schematic showing flow above a stretching sheet**

Generally it is observed that rheological properties of a material are specified by their constitutive equations. The simplest constitute equation for a fluid is a Newtonian one and the governing equation for such a fluid is the Navier-Stokes equation. But in many fields, such as food industry, drilling operations and bio-engineering, the fluids, rather synthetic or natural or mixtures of different stuffs such as water, particles, oils, red cells and other long chain of molecules. This combination imparts strong non-Newtonian characteristics to the resulting liquids. In these cases, the fluids have been treated as non-Newtonian fluids. To have a better control on the rate of cooling, in recent years it has been proposed that it might be advantageous for water to be made more or less viscoelastic, say, through the use of polymeric additives (Andersson [9]). Recently, Liu [21] have investigated heat and mass transfer for a hydromagnetic flow over a stretching sheet.

Although there is no doubt about the importance of the theoretical studies cited above, but they are not above reproach. For example, the viscoelastic fluid models used in these works are simple models such as second-order model and/or Watler’s B model which are known to be good only for weakly elastic fluids subject to slow and/or slowly-varying flows (Bird et al [10]). A non-Newtonian second grade fluid does not give meaningful results for highly elastic fluids (polymer melts) which occur at high Deborah numbers [18-19]. Therefore, the significance of the results
reported in the above works are limited, at least as far as polymer industry is concerned. Obviously, for the theoretical results to become of any industrial significance, more realistic viscoelastic fluid models such as upper-convected Maxwell model or Oldroyd-B model should be invoked in the analysis. Indeed, these two fluid models have recently been used to study the flow of viscoelastic fluids above stretching and non-stretching sheets but wadehith no heat transfer effects involved (Pahlavan et al[14] and Renardy[15]). Some researchers, Pahlavan et al[14], Renardy[15], Aliakbar et al[18], Rajgopal [20] have done the work related to UCM fluid by using HAM- method and the researcher Conte et al [19] have studied UCM fluid by using numerical methods with no heat transfer.

The focal point in the present work is to investigate MHD flow and heat transfer for the Upper Convected Maxwell fluid over a stretching sheet with viscous dissipation. To achieve this goal, use will be made of a recent analysis carried out by Hayat et al. [13] in which the velocity field above the sheet was calculated for MHD flow of an UCM fluid with no heat transfer involved using homotopy analysis method (HAM). To the best of our knowledge, no numerical solution has previously been investigated for MHD flow and heat transfer of a UCM fluid above a stretching sheet. Such investigation has important applications in polymer industry. Also the boundary layer flow over a stretching surface is often encountered in many engineering disciplines.

2. GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

The equations governing the heat transfer and momentum between a stretching sheet and the surrounding fluid (see fig.1) can be significantly simplified if it can be assumed that boundary layer approximations are applicable to both momentum and energy equations. Although this theory is incomplete for Viscoelastic fluids, but has been recently discussed by Renardy [15], it is more plausible for Maxwell fluids as compared to other viscoelastic fluid models. For MHD flow of an incompressible Maxwell fluid resting above a stretching sheet, the equations governing transport of heat and momentum can be written as Sadegy et al [16] and Pahlavan et al[17].

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)
\]

\[
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \lambda \left[ u \frac{\partial^2 u}{\partial x^2} + v \frac{\partial^2 u}{\partial y^2} + 2 \nu v \frac{\partial^2 u}{\partial x \partial y} \right] - \frac{\sigma B_0^2}{\rho} u, \quad (2)
\]

where \( B_0 \) is the strength of the magnetic field, \( \nu \) is the kinematic viscosity, \( \mu \) is dynamic viscosity of the fluid and \( \lambda \) is the relaxation time Parameter of the fluid. As to the boundary conditions, we are going to assume that the sheet is being stretched linearly. Therefore the appropriate boundary conditions on the flow are

\[
u = Bx, \quad v = 0 \quad \text{at} \quad y = 0, \quad \text{and} \quad u \rightarrow 0 \quad \text{as} \quad y \rightarrow \infty \quad (3)
\]

where \( B > 0 \) is the stretching rate. Here \( x \) and \( y \) are, respectively, the directions along and perpendicular to the sheet, \( u \) and \( v \) are the velocity components along \( x \) and \( y \) directions. The flow is caused solely by the stretching of the sheet, the free stream velocity being zero. Eqs. (1) and (2) admit a self-similar solution of the form.
where superscript ' denotes the differentiation with respect to $\eta$. Clearly $u$ and $v$ satisfy Eq. (1) identically. Substituting these new variables in Eq. (2), we have

$$f''' - M^2 f'(f')' + f'' + \beta (2ff'' - ff''') = 0,$$

(5)

Here $M$ and $\beta$ are magnetic and elastic parameters.

The boundary conditions (3) become

$$f'(0) = 1, \quad f(0) = 0 \quad \text{at} \quad \eta = 0
\quad f'(\infty) \to 0, \quad f''(0) \to 0 \quad \text{as} \quad \eta \to \infty$$

(6)

3. HEAT TRANSFER ANALYSIS

By using usual boundary layer approximations, the equation of the energy for two-dimensional flow is given by

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p} \left( \frac{\partial u}{\partial y} \right)^2.$$

(7)

where $T$, $\rho$, $c_p$ and $k$ are, respectively, the temperature, the density, specific heat at constant pressure and the thermal conductivity is assumed to vary linearly with temperature. We define the dimensionless temperature as

$$\theta(\eta) = \frac{T - T_w}{T_w - T_\infty}, \quad \text{where} \quad T_w - T_\infty = b \left( \frac{x}{l} \right)^2 \theta(\eta) \quad \text{(PST Case)}$$

(8a)

$$g(\eta) = \frac{T - T_w}{b \left( \frac{x}{l} \right)^2 \frac{1}{k} \sqrt{\frac{v}{b}}}, \quad \text{where} \quad T_w - T_\infty = D \left( \frac{x}{l} \right)^2 \sqrt{\frac{v}{b}} \quad \text{(PHF Case)}$$

(8b)

The thermal boundary conditions depend upon the type of the heating process being considered. Here, we are considering two general cases of heating namely, (i) Prescribed surface temperature and (ii) prescribed wall heat flux, varying with the distance.

3.1. Governing equation for the prescribed surface temperature case (PST - Case)

For this heating process, the prescribed temperature is assumed to be a quadratic function of $x$ is given by
\[ u = Bx, \quad v = 0, \quad T = T_w(x) = T_0 - T_c \left( \frac{x}{l} \right)^2 \quad \text{at} \quad y = 0. \quad (9) \]

Where \( l \) is the characteristic length. Using (4), (5) and (9), the dimensionless temperature variable \( \theta \) given by (8a), satisfies

\[ \text{Pr} \left[ 2 f' \theta' - \theta' f - Ec f'' \right] = \theta^*, \quad (10) \]

Where \( \text{Pr} = \frac{\mu c_p}{k} \) is the Prandtl number and \( Ec = \frac{\nu^2 l^2}{AC_p} \) is the Eckert number. The corresponding boundary conditions are

\[ \theta(0) = 1 \quad \text{at} \quad \eta = 0 \]
\[ \theta(\infty) = 0 \quad \text{as} \quad \eta \to \infty \quad (11) \]

### 3.2. Governing equation for the prescribed heat flux case (PHF - Case)

The power law heat flux on the wall surface is considered to be a quadratic power of \( x \) in the form

\[ u = Bx, \quad -k \left( \frac{\partial T}{\partial y} \right)_w = q_w = D \left( \frac{x}{l} \right)^2 \quad \text{at} \quad y = 0 \]

\[ u \to 0, \quad T \to T_\infty \quad \text{as} \quad y \to \infty. \quad (12) \]

Here \( D \) is constant, \( k \) is thermal conductivity. Using (4), (5) and (12), the dimensionless temperature variable \( g \) given by (8b), satisfies

\[ \text{Pr} \left[ 2 g' g - g' f - Ec f'' \right] = g^*, \quad (13) \]

The corresponding boundary conditions are \( g'(\eta) = -1, \quad g(\infty) = 0. \quad (14) \)

The rate of heat transfer between the surface and the fluid conventionally expressed in dimensionless form as a local Nusselt number and is given by

\[ Nu_x \equiv -\frac{x}{T_w - T_\infty} \left( \frac{\partial T}{\partial y} \right)_{y=0} = -x \sqrt{\text{Re} \theta'(0)} \quad (15) \]

Similarly, momentum equation is simplified and exact analytic solutions can be derived for the skin-friction coefficient or frictional drag coefficient as

\[ C_f \equiv \frac{\frac{\mu}{\rho} \frac{\partial u}{\partial y}}{y(\rho Bx)^2} = -f^*(0) \frac{1}{\sqrt{\text{Re}_x}} \quad (16) \]

where \( \text{Re}_x = \frac{\rho Bx^2}{\mu} \) is known as local Reynolds number
4. NUMERICAL SOLUTION

We adopt the most effective shooting method (see Refs.. Conte et al [19]) with fourth order Runge-Kutta integration scheme to solve boundary value problems in PST and PHF cases mentioned in the previous section. The non-linear equations (5) and (10) in the PST case are transformed into a system of five first order differential equations as follows:

\[
\begin{align*}
\frac{df_0}{d\eta} &= f_1, \\
\frac{df_1}{d\eta} &= f_2, \\
\frac{df_2}{d\eta} &= \frac{\left( f_1 \right)^2 + M^2 f_1 - f_0 f_2 - 2\beta f_0 f_1 f_2}{1 - \beta f_0^2}, \\
\frac{d\theta_0}{d\eta} &= \theta_1, \\
\frac{d\theta_1}{d\eta} &= \Pr \left[ 2 f_1 \theta_0 - \theta_1 f_0 - \text{Ec} f_0^2 \right].
\end{align*}
\]

(17)

Subsequently the boundary conditions in (6) and (11) take the form,

\[
\begin{align*}
f_0(0) &= 0, \quad f_1(0) = 1, \quad f_1(\infty) = 0, \\
f_2(0) &= 0, \quad \theta_0(0) = 0, \quad \theta_0(\infty) = 0.
\end{align*}
\]

(18)

Here \( f_0 = f(\eta) \) and \( \theta_0 = \theta(\eta) \). The aforementioned boundary value problem is first converted into an initial value problem by appropriately guessing the missing slopes \( f_2(0) \) and \( \theta_1(0) \). The resulting IVP is solved by shooting method for a set of parameters appearing in the governing equations with a known value of \( f_2(0) \) and \( \theta_1(0) \). Once the convergence is achieved we integrate the resultant ordinary differential equations using standard fourth order Runge–Kutta method with the given set of parameters to obtain the required solution.

5. RESULTS AND DISCUSSION

The exact solution do not seem feasible for a complete set of equations (5)-(10) because of the non linear form of the momentum and thermal boundary layer equations. This fact forces one to obtain the solution of the problem numerically. The effect of several parameters controlling the velocity and temperature profiles obtained are compared with Hayat et.al[13], shown graphically and discussed briefly.
Figs. 2(a) and 2(b) shows the effect of magnetic parameter $M$, in the absence of Elastic parameter (at $\beta = 0$) on the velocity profile above the sheet. An increase in the magnetic parameter leads in decrease of both $u$- and $v$- velocity components at any given point above the sheet. This is due to the fact that applied transverse magnetic field produces a drag in the form of Lorentz force thereby decreasing the magnitude of velocity. The drop in horizontal velocity as a consequence of increase in the strength of magnetic field is observed.

Figs. 3(a) and 3(b) show the same effect as said above, but, in the presence of Elastic parameter (at $\beta = 1$). That is, an increase in the magnetic parameter leads in increase of fluid temperature at any given point above the sheet.
Figs. 4(a) and 4(b) show the effect of Elastic parameter $\beta$, in the absence of magnetic number (at $M = 0$) on the velocity profile above the sheet. An increase in the Elastic parameter is noticed to decrease both $u$- and $v$- velocity components at any given point above the sheet.

Figs. 5(a) and 5(b) show the effect of Elastic parameter $\beta$, in the presence of magnetic number (at $M = 1$) on the velocity profile above the sheet. An increase in the Elastic parameter is noticed to decrease both $u$- and $v$- velocity components at any given point above the sheet.
Fig. 6(a). The effect of MHD parameter M on temperature profiles $\theta(\eta)$

Fig. 6(b). The effect of MHD parameter M on temperature profiles $\theta(\eta)$

Figs. 6(a) and 6(b) show the effect of magnetic parameter on the temperature profiles above the sheet for both PST and PHF cases. An increase in the magnetic parameter is seen to increase the fluid temperature above the sheet. That is, the thermal boundary layer becomes thicker for larger the magnetic parameter.

Fig 7(a): Effect of Pr on temperature profile

Fig 7(b): Effect of Pr on temperature gradient

Figs. 7(a) and 7(b) show the effect of Prandtl number on the temperature profiles above the sheet for both PST and PHF cases. An increase in the Prandtl number is seen to decrease the fluid temperature $\theta(\eta)$ above the sheet. That is not surprising realizing the fact that the thermal boundary becomes thinner for larger the Prandtl number. Therefore, with an increase in the Prandtl number the rate of thermal diffusion drops. This scenario is valid for both PST and PHF cases. For the PST case
the dimensionless wall temperature is unity for all parameter values. However, it may be other than unity for the PHF case because of its differing thermal boundary conditions.

Figs. 8(a) and 8(b) show the effect of Eckert number on the temperature profiles above the sheet for both PST and PHF cases. An increase in the Eckert number is seen to enhance the temperature in the fluid, i.e. increasing values of Ec contributes in thickening of thermal boundary layer for effective cooling of the sheet, a fluid of low viscosity is preferable.

A drop in skin friction as investigated in this paper has an important implication that in free coating operations, elastic properties of the coating formulations may be beneficial for the whole process. Which means that less force may be needed to pull a moving sheet at a given withdrawal velocity or equivalently higher withdrawal speeds can be achieved for a given driving force resulting in, increase in the rate of production (Rajgopal [20]). A drop in skin friction with increase in Elastic parameter as observed in Table 1 gives the comparison of present results with that of Hayat et al [13], without any doubt, from this table, we can claim that our results are in excellent agreement with that of Hayat et al. [13].

6. CONCLUSION

The present work analyses, the MHD flow and heat transfer within a boundary layer of UCM fluid above a stretching sheet. Numerical results are presented to illustrate the details of the flow and heat transfer characteristics and their dependence on the various parameters.

We observe that, when the magnetic parameter increases the velocity decreases, also, for increase in Elastic parameter, there is decreases in velocity. The effect of magnetic field and Elastic parameter on the UCM fluid above the stretching sheet is to suppress the velocity field, which in turn causes the enhancement of the temperature.

Also it is observed that, an increase of Prandtl number and Eckert number results in decreasing thermal boundary layer thickness and more uniform temperature distribution across the boundary layer in both the PST and PHF cases.
TABLE 1: Comparison of values of skin friction coefficient \( f''(0) \) with \( Mn = 0.0 \)

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TABLE 2: Comparison values of skin friction coefficient \( f''(0) \) with \( M=0 \) and \( M=0.2 \)

\[
\begin{array}{cccccc}
S & \text{Hayat et al [13]} & \text{Present Results} \\
& M=0.0 & M=0.2 & M=0.0 & M=0.2 \\
0.0 & -1.90250 & -1.94211 & -0.999962 & -1.095445  \\
0.4 & -2.19206 & -2.23023 & -1.101850 & -1.188270  \\
0.8 & -2.50598 & -2.55180 & -1.196692 & -1.275878  \\
1.2 & -2.89841 & -2.96086 & -1.285257 & -1.358733  \\
1.6 & -3.42262 & -3.51050 & -1.368641 & -1.437369  \\
2.0 & -4.13099 & -4.25324 & -1.447617 & -1.512280  \\
\end{array}
\]

TABLE 3: Comparison values of \( -\theta'(0) \) and \( g(0) \) for various values of \( \beta, Mn, Pr \) and \( Ec \)

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