EQUATIONS OF LAGRANGE MOTION OF A PARTICLE IN A SPIRAL SCREW DEVICE

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ABSTRACT
The paper describes the relative motion of a particle under the action of a rotating spiral in a cylindrical body using the Lagrange equation of the 2nd kind. The diagram of the decomposition of the normal reaction of the wire turn of the helix to the constituents is given and described in detail. The equation for calculating the angle between the normal reaction of the surface of the wire turn of the spiral and the axis, which characterizes the geometric characteristics of the spiral, the body and the size of the particles of material in the device, is considered. The equations of motion taking into account the generalized force are given. As a result, an expression is obtained that describes the relative motion of a particle along an inclined transporter in the steady state.

Keywords: spiral, particle of bulk material, equations of motion, schemes of forces acting on the particle.


1. INTRODUCTION
To calculate and design inclined spiral-screw conveyors, it is necessary to have data on the nature of the functional connection between their parameters and the kinematic elements of the motion of the transported material and individual particles. Such a connection established only for the case when the particle of the transported material maintains a steady pattern of
motion and moves along the inner surface of the cylindrical conveyor casing in the axial direction. This case is private. Widely spiral conveyors have a wider use, in which the movement of material particles occurs not only in the axial direction, but also in the direction perpendicular to it, and the particle makes movement along the surface of the conveyor shell along the curve of the line.

For such a case, these relationships are establish only in the most general mathematical form and have not found application for engineering calculations of spiral screw conveyors. Therefore, it becomes necessary for such connections established for practical use in a particular spiral screw device [1, 2].

2. MATERIALS AND METHODS OF RESEARCH

Suppose that there is a spiral screw device inclined to the vertical at an angle $\beta$, consisting of a spiral, which forms, perpendicular to the axis of the helix and a cylindrical shell. In this case, we assume that the casing is stationary, and the spiral rotates about its axis inside the casing with a constant angular velocity $\omega$. If at the initial moment of time the particle of material is in the lower part of the casing, then after a while it will turn out to be a tightened upward frictional force, arising between the particle and the surface of the spiral. Moving along the surface of the casing, both in the axial and perpendicular directions to it, curvilinear character of motion

To describe the relative motion of a particle under the action of a rotating spiral in a cylindrical shell, we use the Lagrange equations of the second kind.

One active gravity acts on the particle $G=mg$, and since the bonds are not ideal, the friction forces of the particle against the wall of the casing $F_1$ and the coil of the helix $F_2$.

For fixed coordinates, we take the right Cartesian coordinate system, where Oz coincides with the axis of the spiral, tilted at an angle $\beta$ to the vertical rotated around the axis Ox.

Since the motion of a particle occurs along a helical line with one degree of freedom, its position determined by one generalized coordinate, directed along the helical line of the spiral $s$ [3, 4]. The origin at the point at $\gamma=0$, where $\gamma$ the angle between the axis Oy and the projection of the radius vector of the particle on the plane Oxy.

We write the Lagrange equation of the second kind for one generalized coordinate $s$:

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{s}} \right) - \frac{\partial T}{\partial s} = Q,$$

where $T$ – the kinetic energy of the particle; $Q$ – the generalized force corresponding to the generalized coordinate $s$.

To determine the generalized force, we give the particle a possible displacement corresponding to the generalized coordinate $s$. We calculate the sum of the work of the active force and the frictional forces on the displacement:

$$\sum \delta A(F_k) = (\sum F_{ks}) \delta s,$$

where $\sum \delta A(F_k)$ – the sum of the works of active force and frictional forces; $\sum F_{ks}$ – the projection of all forces on the direction $s$. 
Figure 1 Forces acting on a particle located on the helical surface of a spiral

The normal reaction $N_1$ acting on the particle on the side of the spiral screw makes an angle $\theta$ with the perpendicular to the helical inclined line, and the perpendicular in turn is the angle $\alpha$ with the $Oz$ axis [5, 6, 7]. The sliding friction force is direct to the side opposite to the motion, and is located on the line of the vector $\nu_0$:

$$F'_1 = f'_1 \cdot N_1,$$

where $f'_1$ – the coefficient of friction of the particle about the helix.

The angle $\theta$ between the normal reaction of the surface of the helical wire turn and the $Oy$ axis characterizes the geometric characteristics of the spiral, casing and the particle size of the material in the device and is determined by the formula:

$$\theta = \arcsin \left( \frac{r + r_3 - r_1 - r_2}{r_1 + r_3} \right).$$

where $r$ – the inner radius of the casing, m; $r_1$ – the radius of the particle, m; $r_2$ - outer radius of the spiral screw, m; $r_3$ – the radius of the wire, m.

The normal reaction acting on the particle from the inner surface of the casing is at the radius of the casing, and the frictional force $F_2 = f_2 \cdot N_2$ has a direction opposite to the absolute velocity vector, and makes an angle $\beta$ with the axis ($\varphi$), where $f_2$ – the coefficient of friction of the particle on the inner surface of the shell.

We unfold the screw line on the plane tangent to the surface of the shell (Figure 2). Equations of motion found from conditions

The generalized force is the coefficient:

$$Q = \sum F_{ks} = F_1 \cos(\alpha + \varphi) - F_2 - G_2 \sin \alpha + G_1 \cos \alpha,$$

where $F_1$ – the friction force of the particle on the inner surface of the casing, $H$; $N_1$- the force acting on the particle from the shell side, $H$; $F_2$ – the sliding friction of the particle on
the helix surface, H; \( N_2 \) – the force that acts on the particle from the side of the helix surface, H;

The reaction \( N_1 \) acts on the particle from the side of the helix's surface, makes an angle with the axis \( Oz \) (Figure 2), the sliding friction force directed to the side opposite to the displacement, and is located.

![Figure 2](image)

**Figure 2** Scheme of velocities and forces acting on the particle in the plane of the generalized coordinate \( s \) perpendicular to the plane \( Ox\ y \)

\[
F_1 = f_1 \cdot N_1 ,
\]

where \( f_1 \) – the coefficient of friction of a particle of material about the surface of the helix.

The reaction \( N_2 \) acts on the particle from the side of the shell wall surface is determined by the inertia force due to the rotation speed of the particle located vertically of the absolute velocity and makes an angle \( \beta \) with the \( \phi \) axis [8, 9, 10]:

\[
F_2 = f_2 \cdot N_2 ,
\]

where \( f_2 \) – the coefficient of friction of the particle on the inner surface of the shell.

\[
F_1 = G \cdot f_1 \left( \frac{(v_a \cos \varphi)^2}{rg} + \tan \theta + \sin \beta \cos \gamma \right)
\]

\[
F_2 = f_2 \left( G_2 \cdot \cos \alpha + G_1 \cdot \sin \alpha + F_1 \sin(\alpha + \varphi) \right)
\]

\[
G_2 = G \cos \beta \quad G_1 = G \cdot \sin \beta \sin \gamma
\]

Define the angle \( \gamma \) at some time \( t \) from the beginning of the movement the particle has a coordinate \( s \) the spiral will turn by an angle \( \omega t \) in the opposite direction. Consequently, the angle determines the position of the particle with respect to the fixed coordinate system (axis \( Oy \)) will be:

\[
\gamma = \omega t - s \cdot \cos \alpha / r .
\]
Expressed $\sin(\alpha + \varphi)$ and $\cos(\alpha + \varphi)$ through the relative and portable speeds. From Figure 2, by the sine theorem:

$$\sin(\alpha + \varphi) = \omega r \sin \alpha / \nu.$$  

By the cosine theorem:

$$\nu_a = \sqrt{\omega^2 r^2 - 2\omega r \dot{s} \cos \alpha + \dot{s}^2}.$$  

Then, taking into account (10):

$$\sin(\alpha + \varphi) = \omega r \sin \alpha / \sqrt{\omega^2 r^2 - 2\omega r \dot{s} \cos \alpha + \dot{s}^2},$$

where $\omega r$ – the velocity of the particle; $\omega$ – angular velocity of the spiral; $\dot{s}$ – the relative velocity of the particle.

$$\cos(\alpha + \varphi) = \sqrt{1 - \sin^2 (\alpha + \varphi)} = \left( \omega r \cos \alpha - \dot{s} \right) / \sqrt{\omega^2 r^2 - 2\omega r \dot{s} \cos \alpha + \dot{s}^2}.$$  

The speed of rotation of the particle (Figure 2):

$$\nu \cos \varphi = \omega r - \dot{s} \cos \alpha.$$  

Substituting (5), (7), (8) with regard for (9), (11), (12), (13) in (3), we obtain [11]:

$$Q_s = G \cdot \left\{ f_1 \left( \left( \omega r - \dot{s} \cos \alpha \right)^2 / (rg) + \tan \theta + \sin \beta \cos \left( \omega t - s \cdot \cos \alpha / r \right) \right) \right\}$$

$$- \sin \beta \left( f_2 \cos \theta \sin \alpha - \cos \alpha \right) \sin \left( \omega t - s \cdot \cos \alpha / r \right) - \cos \beta \left( f_2 \cos \theta \cos \alpha - \sin \alpha \right) \}$$

The kinetic energy of the particle:

$$T = \frac{1}{2} \frac{G}{g} \nu^2.$$  

Substituting (10) into (15) we obtain:

$$T = \frac{1}{2} \frac{G}{g} \left( \omega^2 r^2 - 2\omega r \dot{s} \cos \alpha + \dot{s}^2 \right).$$  

Differentiating this expression according to (1), we calculate the corresponding quantities:

$$\frac{\partial T}{\partial \dot{s}} = \frac{G}{g} \left( \dot{s} - \omega r \cos \alpha \right).$$  

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{s}} = \frac{G}{g} \ddot{s}.$$
Equations of Lagrange Motion of a Particle in a Spiral Screw Device

\[ \frac{\partial T}{\partial s} = 0. \]  

(19)

After substituting the expression from (14) and the derivatives (18), (19) into the Lagrange equation (1) and simplifying, we obtain the differential equation of particle motion:

\[ \frac{\ddot{s}}{g} = G \cdot \left\{ f_1 \left( (\omega r - \dot{s} \cos \alpha)^2 / (rg) + \tan \theta + \sin \beta \cos \left( \omega t - s \cdot \cos \alpha / r \right) \right) \right\} \cdot \left( (\omega r (\cos \alpha - f_2 \cos \theta \sin \alpha) - \dot{s}) / \sqrt{\omega^2 r^2 - 2 \omega \dot{r} \cos \alpha + s^2} \right) - \sin \beta \cdot \left( f_2 \cos \theta \sin \alpha - \cos \alpha \right) \sin \left( \omega t - s \cdot \cos \alpha / r \right) - \cos \beta \left( f_2 \cos \theta \cos \alpha - \sin \alpha \right) \}

(20)

3. RESULTS OF THE RESEARCH

Equation (20) describes the relative motion of a particle along an inclined spiral-helical device. This equation allows, in a steady state, to analyze the motion characteristics of the particle as a function of the angle of inclination and the number of revolutions.

Figure 3 shows the results of calculations for a spiral screw conveyor with the following characteristics:

- \( f_1 = 0.5 \) - coefficient of friction of the particle about the wire of the helix;
- \( f_2 = 0.5 \) is the coefficient of friction of the particle against the casing body;
- \( r = 0.0205 \) m - inner radius of the shell;
- \( \omega = 50 \) s\(^{-1}\) - rotational speed of the spiral;
- \( \beta = 60^\circ \) - the angle of inclination of the conveyor to the vertical;
- \( d = 0.004 \) m - diameter of the helix wire;
- \( r_1 = 0.002 \) m is the average radius of the particle;
- \( r_2 = 0.02 \) m is the radius of the helix;
- \( S = 0.0035 \) m is the pitch of the helix of the helix.

**Figure 3** Changes in axial velocity and particle motion as a function of time with the selected characteristics of the spiral-screw device:

- \( z(t_1) \) is the axial displacement of the particle; \( \upsilon_0(t_1) \) is the axial displacement of particles
It can be seen from Figure 3 that the unsteady mode of moving bulk material in a device with a working member in the form of a spiral screw lasts less than two seconds, then the steady state of motion sets in and the velocity of material movement becomes constant.

4. CONCLUSION

After all the mathematical transformations, the differential equation of motion of the particle was obtained in Lagrange's equation, which describes the relative motion of a particle along an inclined spiral-helical device in the established mode of material transfer. In this case, the steady motion of the particle, depending on the angle of inclination and the number of revolutions, can be divided into 3 ranges: in the first range the particle moves uniformly progressively parallel to the Z axis; in the second range, the particle cannot move rectilinearly, slides inside the helical screw, with the angle of inclination critical, the third diapason at an angle greater than n and moving along a helical line.

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